Attentional Dynamics Explain the Elusive Nature of Context Effects

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Abstract

Context effects in multi-alternative, multi-attribute choice are pervasive and yet, paradoxically, elusive at the same time. For example, simple changes to the spatial layout of alternatives on the screen can nullify or reverse the effects. Despite the success of dynamic decision models in explaining the occurrence of context effects, a coherent theory for understanding their elusiveness is currently lacking. We introduce a novel theoretical framework that relies on attention modulated comparisons to explain the elusive nature of context effects. We show via simulation that our model produces the attraction, compromise, and similarity effects simply by assuming that more time is spent comparing alternatives that are more similar. However, when more time is spent comparing dissimilar alternatives, model simulations reveal a reversal of the attraction and compromise effects. The empirical support for this model-based prediction is assessed by manipulating similarity-based attention in separate experiments for the three context effects (total $N = 317$). Further, by allowing the spatial organization of information to constrain the attention process, the model can explain changes in context effects induced by display layout. We show that the model's spatial attention mechanism allows it to capture presentation order effects in a reanalysis of previously published data. Finally, we develop a continuous approximation of the full model that permits fitting of choices and response times. In summary, the proposed framework provides a new tool for understanding not only the existence of context effects in choice, but also the attentional factors that lead to null or reversed context effects.

Keywords: multi-alternative decision making, evidence accumulation models, attention, similarity

Attentional Dynamics Explain the Elusive Nature of Context Effects

Contextual sensitivity of decisions, and more specifically the attraction, compromise, and similarity effects, have long been of interest in the study of decision making as examples of violations of principles of classic economic theories of choice. Observance of these violations has prompted the development of numerous theoretical models attempting to explain the decision processes responsible for them, particularly the three previously mentioned context effects. While these theories posit a range of different possible mechanisms, a significant majority incorporate some notion of attention as a critical process influencing the way in which alternatives are evaluated.

We build on this line of research to develop a simple yet powerful model of attention modulated preference formation in multi-alternative, multi-attribute choice. This theory not only accounts for the presence of the classic context effects, but also how changes in attention processes can nullify and even reverse these effects. This theory thus unifies a large body of past research, both empirical and theoretical, that have attempted to understand why context effects arise and why they appear to be simultaneously both universal (appearing in numerous choice domains) and fragile (disappearing or reversing in some studies).

In the following sections, we discuss the concept of contextual sensitivity in choice and describe the three classic context effects. We then provide a brief overview of formal theories that aim to explain the existence of these effects. The quest to model context effects has been challenged by recent findings demonstrating the fragility of these effects. We discuss the evidence for this fragility before introducing our modeling framework, which uses attention processes to explain why context effects sometimes arise and other times disappear or reverse.

Contextual sensitivity and context effects

Many of the decisions we make in our lives involve choices among multiple options that have multiple attributes. Imagine you are planning your next vacation. As part of that process, you need to select a flight, rental car, and hotel room. Each one of these choices involves multiple options (e.g., different airlines) that vary on many different features (e.g., price, departure time, length of layover, seat availability, etc.). The question of how people make decisions in these complex tasks has intrigued researchers in psychology, marketing, neuroscience, and behavioral economics for decades. One of the key findings related to multi-alternative, multi-attribute choice is that people's decisions often display contextual sensitivity. That is, people's choices among existing options can be influenced by the introduction of new alternatives.

To illustrate contextual sensitivity in multi-alternative, multi-attribute choice, consider the scenario of selecting a flight for an upcoming trip. Imagine you are deliberating between two options. One flight (call this flight A) has a good departure time, but is expensive, and the other flight (call this flight B) has an inconvenient departure time, but is more affordable. In this case, there is a trade-off between convenience of departure time and affordability. Now, imagine you learn of a new flight option (call this flight C) that departs at the same time as flight B, but is more expensive then flight B. In this scenario, flight C is not a very good option because it is dominated by flight B. Even though you would never select flight C, it makes flight B look more appealing and you end up selecting flight B over flight A. This phenomenon is known as the attraction effect (Huber et al., 1982) and is one example of a context effect in multi-alternative, multi-attribute choice.

Early research on context effects was aimed at showing violations of principles of classic economic theories of choice such as simple scalability, regularity, and independence of irrelevant alternatives (Huber et al., 1982; Tversky, 1972). After this early work in the 1970s and 1980s, research on context effects in multi-alternative, multi-attribute choice proceeded along two related branches. One branch was concerned with empirical investigations of the effects in a wide variety of tasks and species (e.g., monkeys, honeybees, hummingbirds, etc.). This research painted a picture that context effects were "universal" since they seemed to show up everywhere. However, about a decade ago, research started suggesting the story was much more nuanced. Studies were published showing null or even reversed context effects (for a review, see Spektor et al., 2021). This led to some researchers calling into question the importance of context effects altogether

(Frederick et al., 2014; Huber et al., 2014).

The other main branch of research in this area was concerned with developing formal theories to explain the presence of context effects, leading to the development of numerous cognitive and neural models of these effects. The desire to have a formal theory is rooted in the fact that these effects cannot be explained by classic decision theories (e.g. classic utility models). Thus, the primary goal of modelers was developing theories to account for and explain the existence of context effects, often with a single set of parameters (Trueblood et al., 2015). Most of these models were not developed to explain null or reversed context effects. Thus, we currently do not have a rigorous theoretical framework to explain why reversals and null effects occur.

In this paper, we focus on three well-established context effects: the attraction (Huber et al., 1982), similarity (Tversky, 1972), and compromise (Simonson, 1989) effects. In all three effects, choices between two options, X and Y, are influenced by a third option Z. The three options are defined in terms of two attributes, a_1 and a_2. For the following explanations, let us assume that $X^{a_1} < Y^{a_1}$ and $X^{a_2} > Y^{a_2}$ (see Figure 1). The relationship between the third option Z and X and Y determines the specific type of context effect.

We use the term *target* to refer to the option that is traditionally advantaged with the inclusion of the third option. Likewise, we use the term *competitor* to refer to the option that is traditionally disadvantaged with the inclusion of the third option. In the literature, there are different ways of measuring context effects. In this paper, we use the Relative Choice Share of the Target (RST; Berkowitsch et al., 2014) and provide alternative measures in the supplement. The RST calculates the probability of selecting the target divided by the probability of selecting the target or competitor: $pr(T)/(pr(T)+pr(C))$. More specifically, if Z_X is a third option that enhances X (i.e., X is the target in the choice set $\{X,Y,Z_X\}$) and Z_Y is a third option that enhances Y (i.e., Y is the target in the choice set $\{X,Y,Z_Y\}$), then the RST is defined as

$$RST = \frac{pr(X|\{X,Y,Z_X\}) + pr(Y|\{X,Y,Z_Y\})}{pr(X|\{X,Y,Z_X\}) + pr(Y|\{X,Y,Z_Y\}) + pr(X|\{X,Y,Z_Y\}) + pr(Y|\{X,Y,Z_X\})} \tag{1}$$

A standard context effect occurs when RST > 0.5, which implies that the target is selected more often in the presence of the third option than the competitor.[1]

In the attraction effect, the third option is a dominated decoy that is similar but inferior to either X or Y. Specifically, let A_X be a decoy similar, but inferior to X and A_Y be a decoy similar, but interior to Y as shown in Figure 1. In the attraction effect, the target is the option that is similar to, but dominates the decoy (e.g., X in the choice set $\{X, Y, A_X\}$). Likewise, the competitor is the dissimilar option (e.g., Y in the choice set $\{X, Y, A_X\}$).

In the similarity effect, the third option is similar and competitive to either X or Y. In particular, let S_X be similar and competitive to Y and S_Y be similar and competitive to X as shown in Figure 1. In the similarity effect, the target is the dissimilar option (e.g., X in the choice set $\{X, Y, S_X\}$) and the competitor is the similar option (e.g., Y in the choice set $\{X, Y, S_X\}$). In the similarity effect, the two similar options compete with one another and split the choice shares. The result is that the dissimilar option is selected more often.

In the compromise effect, the third option is an extreme option that makes either X or Y appear as a compromise. Specifically, let C_X be an extreme option such that X appears as a compromise between C_X and Y. In this case, we have the following relationship among attributes: $C_X^{a_1} < X^{a_1} < Y^{a_1}$ and $C_X^{a_2} > X^{a_2} > Y^{a_2}$. Let C_Y be an extreme option such that Y appears as a compromise between C_Y and X. Here we have $C_Y^{a_1} > Y^{a_1} > X^{a_1}$ and $C_Y^{a_2} < Y^{a_2} < X^{a_2}$. In the compromise effect, the target is the compromise option (e.g., X in the choice set $\{X, Y, C_X\}$) and the competitor in the extreme option (e.g., Y in the choice set $\{X, Y, C_X\}$).

Models of context effects

As previously mentioned, researchers studying context effects in multi-alternative, multi-attribute choice have developed formal computational models to explain the underlying cognitive processes that give rise to the effects (for reviews, see Busemeyer et al., 2019;

[1] Note that by including both choice sets where X and Y are targets, we control for the possibility that an individual has a strong preference for either X or Y in our calculation of the RST. A strong attribute bias can also be controlled by using an alternative definition of RST as proposed by Katsimpokis et al. (2022) and discussed in the supplement.

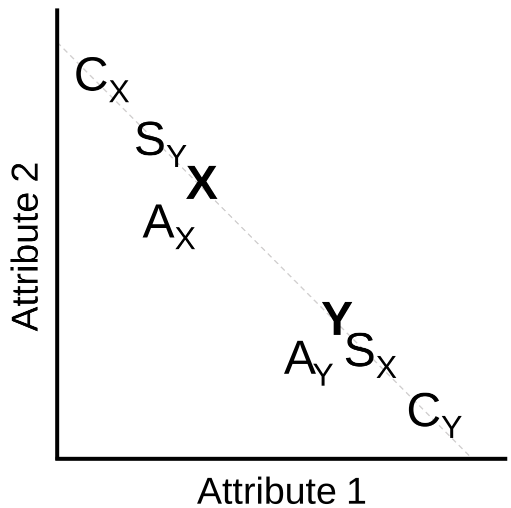

Figure 1
Locations of standard decoys in two-dimensional attribute space: *Two core alternatives (X and Y) are shown along with attraction (A), similarity (S), and compromise (C) decoys. The subscripts denote the option that is traditionally advantaged by the inclusion of the decoy.*

Trueblood, 2022; Wollschlaeger and Diederich, 2020). Many of these models are based on the Evidence Accumulation Modeling (EAM) framework (Ratcliff, 1978) and assume that evidence or preference for different options accumulates over the course of deliberation. Each option is associated with an accumulator that tracks the preference state for that option over time. The final decision is determined by the accumulator with the greatest amount of preference at a fixed time point (external stopping rule) or by the accumulator that reaches a decision threshold first (internal stopping rule). A key advantage of EAM models is their ability to simultaneously account for choices and response times.

Although existing EAM models make different assumptions about the nature of preference accumulation and the cognitive mechanisms that produce the attraction, compromise, and similarity effects, one thing that most of the models have in common is the idea that attention is important for explaining context effects (Trueblood, 2022). In decisions between multiple options with multiple attributes, it is impossible to attend to all of the information at once, and so attention is assumed to fluctuate between different pieces of information over the course of deliberation. Preference accumulates based on the information attended to at each time point. For example, multialternative decision field theory (MDFT; Roe et al., 2001) assumes that at each time point, a particular attribute is stochastically attended to and the value of each option on that attribute is compared to the average of the values of the other options. Similarly, the multiattribute linear ballistic accumulator model (MLBA; Trueblood et al., 2014) assumes that options are compared in a pairwise fashion on each attribute, with attention weights that depend on the similarity of the options involved in a particular comparison. Thus, attention appears to be a critical feature for understanding how context effects arise in multi-alternative, multi-attribute choice. This article will expand on this observation.

The elusive nature of context effects

An interesting but puzzling feature of context effects is they are both universal and fragile. For example, previous research has explored how context influences preferences across a variety of different domains including perception (Trueblood, 2015; Trueblood et al., 2015; Trueblood et al., 2013), inference (Trueblood, 2012), similarity judgments (Yearsley et al., 2021), memory (Maylor & Roberts, 2007), gambles (Farmer et al., 2017; Wedell, 1991), and motor planning decisions (Farmer et al., 2015). This work aimed to answer the question, do context effects, which are often found in consumer choice (Evangelidis et al., 2018; Huber et al., 1982; Simonson, 1989), generalize to other domains? Results show that these phenomena can arise outside of consumer choice (e.g., in perceptual decisions about the size of rectangles; Trueblood et al., 2013). In addition, research in developmental psychology and behavioral ecology has shown that the attraction effect occurs in children (Zhen & Yu, 2016), monkeys (Parrish et al., 2015), honeybees (Latty & Trueblood, 2020; Shafir et al., 2002), hummingbirds (Bateson et al., 2003), and even slime molds (Latty & Beekman, 2011). Thus, there is a sense that contextual sensitivity is a universal property of multi-alternative choice behavior.

Despite the seemingly ubiquitous nature of contextual sensitivity, the three classic effects are also very fragile. First, very few participants produce all three effects within a single experiment even though most people show the effects in isolation (i.e., in separate experiments) (Trueblood et al., 2015). Second, there are large individual differences in the strength and co-occurrence of the effects (Liew et al., 2016). Third, there is evidence that the attraction effect is much stronger in abstract, stylized stimuli as opposed to natural stimuli (Frederick et al., 2014). Fourth, the effects are quite sensitive to task features such as the spatial arrangement and presentation order of the alternatives (Cataldo & Cohen, 2018, 2019; Evans et al., 2021; Spektor et al., 2018). This has led to the recent conclusion that context effects are elusive (for a review, see Spektor et al., 2021).

The present paper introduces a new modeling framework with the goal of explaining the diverse patterns of behavior observed in multi-alternative, multi-attribute choice. This framework builds off of the prior observation that attention, and in particular the way it dynamically alters the evaluation of choice alternatives, has a potentially critical role in producing these patterns of behavior. Whereas past efforts have attempted to produce these specific context effects, our goal here is to not only understand their presence, but also why they appear to be fragile and how they may be manipulated empirically.

The aim of the present paper is to propose a modeling framework that builds on and extends the ideas formalized in existing EAM models. Our goal is not to introduce a new model to compete with existing models in a head-to-head comparison, but rather to use the proposed framework to develop a better understanding of the role of attention in the decision process and what its effects on behavior might look like. With this in mind, we do not build a single model of attention, but rather a more general framework to evaluate different models of attention and attention-related factors. Using this framework, we show that seemingly subtle changes in attention can lead to a complex pattern of standard, null, and reversed context effects. We corroborate the modeling results with new experiments and the reanalysis of published data. In totality, our results support prior studies positing that attention may be (partly) responsible for context effects. We however take a step further and illustrate how either intrinsic or extrinsic changes in the attention process can lead to a wide range of seemingly contradictory empirical observations.

Theoretical Framework for Attention and Decision-making

General Model Description

Based on the rich theory of evidence accumulation in psychology and neuroscience (Ratcliff et al., 2016), we develop a new computational modeling framework of multi-alternative, multi-attribute choice where preferences are constructed over time through a sequence of attention modulated comparisons. In this way, we integrate hypotheses for how attention affects information acquisition with EAM models of preference formation to study their joint effects on decisions. We note that this is a framework rather than a single model and thus it is designed to incorporate a range of different possible assumptions regarding the form of comparisons and attention. Here we briefly describe the overarching idea of this approach with further detail below. In the most general form, we will assume that:

- Evidence (or preference) for an alternative is sampled by comparing it to some referent along a single attribute at any given time. A sequence of comparisons involving different attributes and / or alternatives may occur over time, but at any point in time a single alternative is compared to a single referent on a single attribute.

- This referent could be another of the alternatives (i.e. pairwise comparison), a summary of the choice set along that attribute (e.g., the mean of the choice set), or something formed from past experience. In this article, we will primarily consider pairwise comparisons though the framework can readily accommodate other possibilities.

- Attention modulates which of the set of all possible comparisons is made. For mathematical tractability we will assume that the selection of future comparisons does not depend on the past sequence of comparisons. With this simplifying assumption, the sequence of comparisons can be described by a Markov process. This Markov process can account for a range of different potential factors such as attribute bias (preference for making comparisons on one particular attribute) or presentation format induced biases of attention. History based, non-Markovian attention processes can be included in this framework, but they are more challenging to handle mathematically.

- The time spent on any given comparison is related to the characteristics of the alternative and referent being compared. For example, a decision maker may spend more or less time comparing an alternative and referent that are similar to each other, or alternatively the time spent could be independent of similarity.

General Mathematical Formulation

Here we describe the general mathematical formulation of this framework without discussion of specific assumptions about the choice of referent or how attention is modulated. As noted previously, this is not a single model but rather a framework. Where useful, we will consider 3-alternative, 2-attribute scenarios for notational convenience.

Consider a choice set consisting of alternatives $\{(x_i^1, x_i^2)\}_{i=1:3}$. Here $\vec{x}_i$ indicates an individual alternative (i) with 2 attributes. Denote $\{c_m\}$ to be the set of all possible comparisons that can be made for this choice set. For example, for pairwise comparison where only intra-attribute comparisons are possible, this would consist of three possible comparisons on each attribute. Then in a small interval of time Δt, the preference accumulated for all possible alternatives is dependent on the given comparison being made:

$$P(t+\Delta t) = P(t) + \vec{d}_{c_m}\Delta t. \tag{2}$$

Here P is a vector of preferences for the alternatives and $\vec{d_{c_m}}$ is a comparison dependent rate of accumulation, typically taken to be a simple difference between the alternative and referent along the attribute being attended to in that time interval. For example, if alternatives 1 and 3 are being compared on attribute 2, the rates of accumulation for the three alternatives would be $x_1^2 - x_3^2$, 0, $x_3^2 - x_1^2$ respectively where alternative 2 has a 0 rate of accumulation since it is not being attended to. Further, the rates for alternatives 1 and 3 are the same in magnitude but different in sign since a comparison that favours one option will disfavor the other.

It is clear that in this approach, the specific comparison being made determines the rate of preference accumulation. We thus next describe an attention-based model where stochastic sequences of attention impact preference accumulation. For this discussion, we will consider the case where one alternative is compared to another, though other referents can be readily incorporated with appropriate modification as previously discussed. To model a sequence of comparisons, we need to describe two things: 1) how long each comparison will last and 2) once one comparison is completed, how the next comparison is chosen. For comparison time, we will generally assume that the time spent on a particular comparison is dependent on the similarity between the two items being compared. As an example, more similar options may be compared for longer times since they are more difficult to distinguish (other assumptions are possible). Comparison transitions will be modeled as a probabilistic Markov process, which can be described by a transition matrix. For example, if there are 6 possible comparisons (3 alternatives, 2 attributes) transitions will be modelled by a 6x6 transition matrix P. Figure 2 illustrates the modeling framework and Markov attention process.

Detailed Model Formulation

Next we describe the attention process that governs the sequence of comparisons made in more mathematical detail. Let M_{c_n,c_m} denote the probability that, in the given time interval Δt the decision maker terminates comparison c_n and transitions to the comparison c_m. This is the product of the probability of comparison c_n ending and the probability of the new comparison c_m being chosen conditioned on the prior one ending. Different theories regarding attention can be encoded into these two quantities. We thus first formulate the model in general terms using these two quantities, and in a later section we construct the specific form of those quantities from specific hypotheses.

Let T_{c_n} be the probability of the current comparison ending. For three alternatives and two

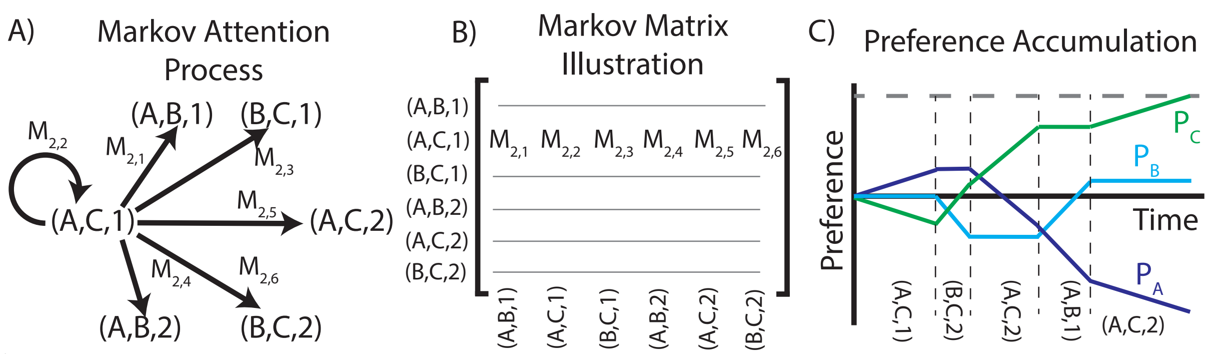

Figure 2

Visualization of the modeling framework: *Panel A illustrates the Markov attention process for the situation where a decision maker is currently comparing options A and C on attribute 1. The arrows indicate the possible comparisons that the decision maker could transition to within the next time point, including the possibility of continuing to compare A and C on attribute 1. Note that the full Markov process includes all possible transitions among the 6 comparison states, but only a subset of these are shown in the figure. Panel B illustrates the 6x6 Markov matrix M that governs the sequence of comparisons. The probabilities $M_{i,j}$ denote the probability of transitioning from comparison i to j. The diagonal elements are the probability of remaining on the current comparison. Panel C illustrates the preference accumulation process for three options A, B, and C modulated by the Markov attention process. In the first time interval, options A and C are being compared on attribute 1 and the preference states P_A and P_C change accordingly. Next, the decision maker compares options B and C on attribute 2 and the preference states P_B and P_C change accordingly. The vertical dotted lines indicate points of transition among comparisons. This process continues until one of the preference states (P_C in the figure) reaches the threshold (horizontal dotted line) triggering a choice.*

attributes, there are 6 such probabilities. Define P_{c_n,c_m} to be a transition matrix describing the probability of transitioning from any one comparison to another, conditioned on a transition occurring. Note this transition probability matrix P is distinct from the preference accumulator variable P defined earlier. In this language,

$$M_{c_n,c_m} = T_{c_n} \cdot P_{c_n,c_m} \tag{3}$$

$$M_{c_n,c_n} = 1 - T_{c_n}. \tag{4}$$

The first relation represents the probability of a transition occurring while the second is the probability of continuing the current comparison. This probabilistic model of attention can be summarized in the following matrix equation

$$M = I + T \cdot P, \tag{5}$$

where the quantities T and P are matrix representations of the above quantities (indexed over the set of possible comparison) that encode factors related to attention. T is a 6x6 diagonal matrix of termination probabilities with the 6 termination probabilities on the diagonal and 0 in all off-diagonal elements. The off-diagonal entries of P are the conditional transition probabilities, and the diagonal entries of P are -1. The resulting matrix M is a Markov process describing sequences of comparisons at the discrete time points $\Delta t, 2\Delta t, 3\Delta t, \ldots, i\Delta t, \ldots$.

Here P encodes any process that affects which comparison will be attended to next while T encodes any factors that determine how much time is spent on a particular comparison. For timing, we assume that the time spent on any particular comparison is a stochastic quantity that is exponentially distributed with a characteristic deliberation time τ_{c_n}. With this in mind and assuming that the simulation time step is sufficiently small,

$$T_{c_n}^{\Delta t} \approx \frac{1}{\tau_{c_n}} \Delta t. \tag{6}$$

The Δt super-script illustrates that this quantity depends on the computational time interval, though we will omit this superscript for notational simplicity. The specification of a basic model within this framework thus requires specifying how the quantities $\{\tau_{c_n}\}$ and $\{P_{c_n,c_m}\}$ depend on factors of interest. We describe a few such specific models in the subsequent sections.

Similarity-based Attention

Here we illustrate how attention effects can be encoded in the deliberation time component of the model ($\{\tau_{c_n}\}$) and how this factor alone can account for classic context effects in multi-alternative, multi-attribute choice. Following Tversky and Simonson (1993) and many other models of context effects (Noguchi & Stewart, 2018; Trueblood et al., 2014; Wollschläger

& Diederich, 2012), we consider a pairwise model where the referent to which an alternative is compared is another alternative in the choice set. A comparison then is determined by the two alternatives (i, j) being compared along with the attribute (a) that is being attended to: $c_n = (i, j, a)$. For further simplicity and to illustrate the effect of this model factor on its own, we will assume that all possible comparisons are equally likely so that P_{c_n,c_m} is the same for all $n \neq m$.

We will consider three possible comparison time models depending on the relationship between similarity and attention (illustrated in Figure 3):

- Similarity Variant: The more similar two alternatives are on the attribute being attended to, the longer they are compared.
- Dissimilarity Variant: The more similar two alternatives are on the attribute being attended to, the less time they are compared.
- Independent Variant: The time spent on a comparison is independent of similarity.

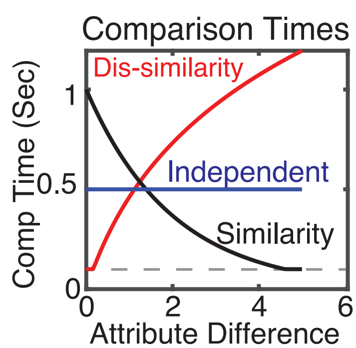

Figure 3
Attribute dependent comparison time for attention variants: *Sample functions illustrating the dependence of comparison times on attribute differences. In the independent attention variant (blue), attention time does not depend on attributes. In the similarity attention variant (black), more similar alternatives are compared for longer times. In the dissimilarity variant, more distinct alternatives are compared for longer times. The dashed gray line at the bottom reflects that most cases will include a minimum comparison time. For the similarity models we use a function of the shape* $\exp(-\lambda|x_i^a - x_j^a|)$ *where* x_i^a *is the value of alternative i on attribute a. For the dissimilarity model, we use a function of the form log*$(1 + \lambda|x_i^a - x_j^a|)$*. These are convenient functions with the appropriate shape.*

To assess the impact of these different similarity-based attention assumptions, we performed a simulation study to explore the presence of context effects in the three-alternative, two-attribute choice domain. To do so, we fixed two alternatives that differ on each attribute but fall along the line of indifference $(0.4, 0.6), (0.6, 0.4)$ and varied the location of a third "decoy" alternative throughout attribute space. We subjected three models, corresponding to the three comparison time scenarios above, to this *in silico* experiment. For the Similarity Variant, we assumed the comparison time distribution for an alternative is exponential with a mean time of $\tau_{c_n} = \exp(-\lambda|x_i^a - x_j^a|)$. For the Dissimilarity variant, we used $\tau_{c_n} = \log(1 + \lambda|x_i^a - x_j^a|)$. And for the Independent Variant, we used $\tau_{c_n} = \lambda$. In the Similarity and Dissimilarity variants, λ is a scale parameter that determines how sensitive the comparison time is to attribute differences. Figure 4 shows that with no additional assumptions, the similarity variant demonstrates all three context effects.

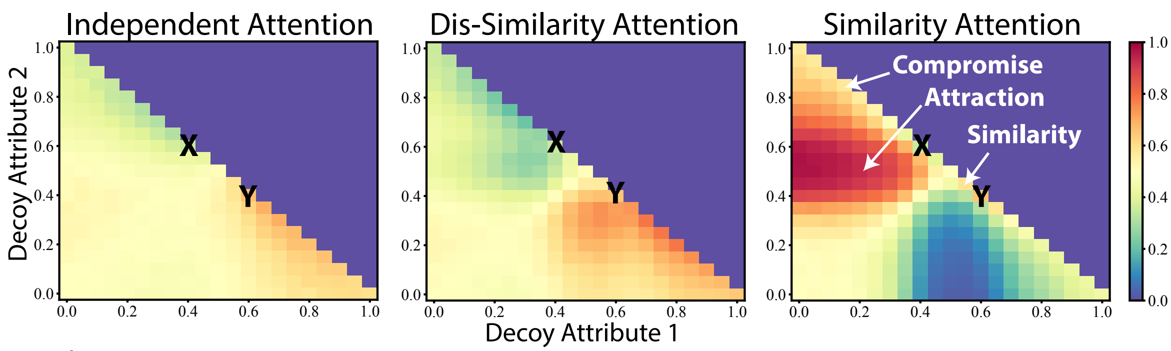

Figure 4

Context effects simulation study: *To study the effect of adding a decoy alternative to a binary choice, we fix the attribute values of two alternatives (x and y shown on the plot) and vary the location of a third "decoy" alternative. For each decoy value, N = 1000 simulations are run and the number of times each alternative $(n(x), n(y), n(d))$ is chosen is recorded. Each plot shows the relative share of alternative x as a function of decoy location $(n(x)/n(x) + n(y))$. The indicated regions on the similarity variant demonstrate it simultaneously accounts for the three classic context effects. For these simulations, we use an attention parameter $\lambda = 5$ and threshold $b = 0.2$.*

Given the relative simplicity of this model (only two free parameters, λ and the accumulation threshold b), we next sought to understand how these effects manifest in the similarity variant. To do so, we first plotted the mean evidence trace as a function of time for each model. We simulated 1,000 independent, stochastic evidence traces for standard attraction, similarity, and compromise choice sets and at each point in time, plot the mean evidence value across all traces (Figure 5 top). In the attraction scenario, it is clear that the evidence trace for the target increases faster than the other alternatives and therefore it is chosen more frequently. The essential reason for this is that with similarity-based attention, proximity of the decoy to the target leads to an increase in the time spent on target favourable comparisons, leading to the target being selected more often.

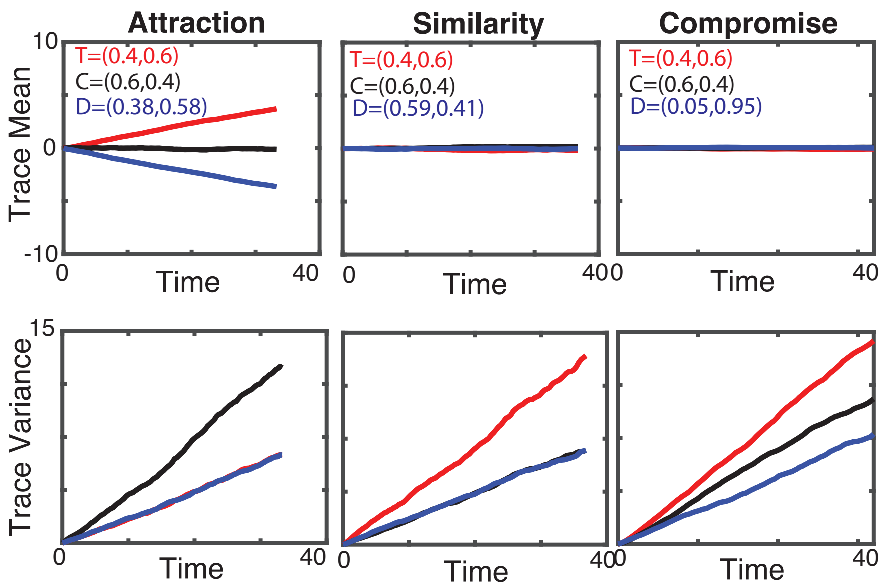

Figure 5

Similarity variant temporal evidence accumulation properties: *To understand the genesis of context effects in the similarity variant, we track the evidence accumulation mean and variance over time for three choice sets indicative of the three context effects. To do so, we simulate 1000 independent trials for each choice set and quantify the mean and variance of the evidence state for each choice at each time (illustrated in the respective panels). For all simulations the attention parameter is* $\lambda = 5$. *No threshold parameter is included since only preference traces are quantified, not choices. The notation* T, C, D *is used here since the respective alternatives are the target, competitor, and decoy for these choice sets. Comparable figures for the independent and dissimilarity models are in the Supplemental Materials.*

The similarity and compromise scenarios however show that the mean evidence traces are flat for all three alternatives. In retrospect, this is sensible since all three alternatives lie on the line of indifference (see Figure 1). For every comparison that is favourable for one alternative, there is an equal and opposite comparison on the other attribute that is equally unfavourable. This leads to a net zero accumulation when averaged over many independent model choices. We next plotted the variance in accumulator traces. For each point in time we calculated the variance on the accumulation traces across all N=1,000 simulations (Figure 5 bottom). Results demonstrate that in the similarity and compromise scenarios, the target exhibits more variability, which breaks the symmetry between the three otherwise equal alternatives. Since the first passage time of a purely diffusive process to a boundary inversely depends on the rate of diffusion (quantified by the plotted variance here), the target is chosen more frequently in both cases.

What is the interpretation of this explanation of the compromise and similarity effects? On average, none of these alternatives is objectively superior and thus there is no net accumulation of evidence when aggregated over many simulated choices. However, the attention process introduces intra-trial variability in the length and sequence of attention segments and therefore accumulation. Due to this variability, it is simply more likely that a favourable sequence of attention segments will arise for the target than for the competitor or decoy, leading to increased choice share. We do note that there is a flip side to this. The same high variance can also lead to an increased chance of a sequence of unfavourable attention segments for the target.

Spatial Attention

We next illustrate how attention processes can be incorporated into the comparison transition component of the framework (the matrix P), which is independent of the comparison time T process just discussed. In anticipation of subsequent modeling and analysis, we discuss two factors that may influence which comparisons are chosen more or less frequently. The first is attribute preference. When multiple attributes are present, it is conceivable and even likely that in many cases attributes will be given different weights in the decision process. One way to incorporate this is through the comparison transition matrix P. For simplicity, we will assume two attributes here and introduce the parameter w encoding the weight for attribute 1 (and consequently $1 - w$ is the weight for attribute 2). Multiplying all elements of the matrix P corresponding to attribute 1 by w (and similarly for attribute 2) naturally incorporates differential preference for attending to these attributes.

Spatial attention, arising from the presentation structure of information, is another factor that may influence decisions. For example, alternatives in different sections of a computer screen (e.g., left versus right) may receive greater attention and the comparison of physically adjacent alternatives may be more frequent. These factors can be readily incorporated into this framework through P as well. We demonstrate this for one scenario, where comparisons of alternatives that are physically adjacent to each other are more common than those separated. Consider the three-alternative, two-attribute scenario. Assume the alternatives are ordered left (L), middle (M), and right (R) when presented. There are three spatial pairs of comparison here (LM, LR, RM). We can define two parameters p_{LM}, p_{MR} indicating the probability of two of those comparisons occurring (with the third being $1 - p_{LM} - p_{MR}$). Combined with the aforementioned attribute weight w, this yields a three parameter model of spatial attention along with attribute preference that can be encoded in the transition matrix P.

Similarity-based Attention Experiments

Our modeling framework predicts the presence of the attraction, compromise, and similarity effects simply from the assumption that more similar alternatives are compared for longer times. It also predicts the reversal of the attraction and compromise effects when dissimilar options are compared for longer times (see Figure 4). In the following section, we present three behavioral experiments that were aimed at testing this model-based prediction. Each experiment involved choices between three alternatives with two attributes. We manipulated similarity-based attention by asking participants to identify the pair of options with the most similar attribute values (similarity condition) or the least similar attribute values (dissimilarity condition) before making a choice. If context effects arise from the increased attention to comparisons between similar alternatives, we should observe standard context effects in the similarity condition and reversed attraction and compromise effects in the dissimilarity condition. We test this hypothesis for the attraction, compromise, and similarity effects in separate experiments.

Experiment 1: Attraction Effect

Method

Participants

99 participants (women = 61, men = 37, unreported = 1; age: M = 39.89 , SD = 14.93) completed the experiment, which was approved by the Institutional Review Board at Vanderbilt University. Participants were recruited via Amazon Mechanical Turk using the CloudResearch platform with the intent of having approximately 50 participants per condition. Half of the participants (N = 50) were randomly assigned to the similarity condition and the other half (N = 49) were assigned to the dissimilarity condition. The sample size was determined prior to starting the experiment and pre-registered on AsPredicted.org (#69295). The data was analyzed only after all data had been collected. All participants were compensated $0.50 for the completion of the study.

Materials

Participants were asked to make investment decisions about businesses, based upon ratings from two financial advisors. Ratings ranged between 0 to 100 with 0 implying a very low likelihood of a good return on investment (ROI) and 100 indicating a very high likelihood of a good ROI. The attribute values of each option were based on those used in Trueblood (2012).

Four types of businesses were used to examine the attraction effect: two focal options (X and Y) and two range-frequency attraction decoys (A_x and A_y). The two decoy options were similar, but inferior to their associated focal option (e.g., A_x was similar, but inferior to X). In addition to choice sets examining the attraction effect, the experiment also included filler choice sets where there was a clearly superior option (i.e., a business that dominated the others on both attributes). In total, there were 10 attraction effect trials with the A_x decoy, 10 attraction effect trials with the A_y decoy, and 10 filler trials. For the attraction effect trials, the specific attribute values for each option varied slightly across trials. The mean and standard deviation of the attribute values for the attraction effect options are provided in Table 1.

Table 1
Summary of attribute values used in Experiment 1 for the attraction effect

	Rating 1		Rating 2	
Option	Mean	SD	Mean	SD
X	34.30	1.59	65.90	1.55
Y	67.40	1.31	32.80	1.01
A_x	28.90	1.97	60.40	1.65
A_y	61.60	1.07	27.70	1.49

Procedures

To examine the effect of similarity- and dissimilarity-based attention mechanisms on context effects, participants were randomly assigned to one of two between-subject conditions (similarity or dissimilarity) at the beginning of the experiment.

At the start of the experiment, participants were told that they would be making investment decisions about businesses. They were told that on each trial, they would see three different businesses and to select the one that they would like to invest in. They were also told that they would be given ratings from two financial advisors. The ratings were described to participants as providing the likelihood of a good return on investment (ROI), with a rating of 0 implying a very low likelihood of a good ROI and a rating of 100 implying a very high likelihood of a good ROI. Additionally, participants were told that the evaluations of businesses can vary from advisor to advisor. Thus, two different financial advisors can provide different ratings. Participants were told to treat the ratings from both advisors equally.

At the beginning of each trial, participants were presented with three businesses and their ROI ratings from two advisors (Figure 6). For example, on trials testing the attraction effect, the three businesses consisted of the focal options X and Y and a decoy option (either A_x or A_y). The options and their attributes were displayed in a table with the different options in different rows and the two attributes in separate columns. The row locations of the options were randomized.

Before making their choice on each trial, all participants completed a similarity judgment task (see Figure 6 for an example). In the similarity condition, participants were asked to identify

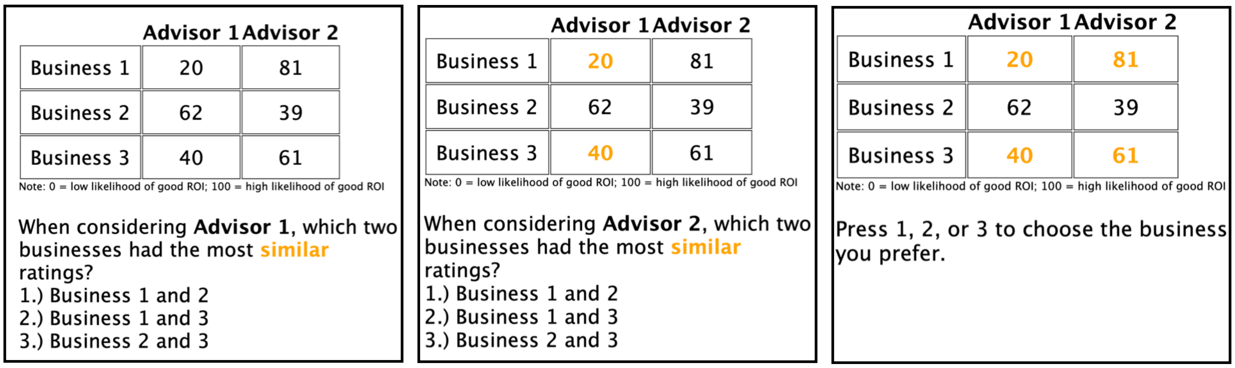

Figure 6

Example of a trial in Experiments 1-3. On a given trial, participants first judged the similarity (or dissimilarity) of the options along attribute 1. Next, they judged the similarity (or dissimilarity) of the options along attribute 2. Finally, they made a decision about their preferred investment. After their choice, the participants proceeded to the next trial. An example of a compromise effect trial in the similarity condition is shown.

the pair of options having the most similar values on each attribute, separately. First, they were queried about attribute 1 (i.e., Advisor 1) followed by attribute 2 (i.e., Advisor 2). Participants received feedback following each judgment. After both judgments, the participants selected the business they preferred (Figure 6). In the dissimilarity condition, participants were asked to identify the pair of options having the most dissimilar values on each attribute prior to the investment decisions. Similar to the similarity condition, participants were asked about attribute 1 before attribute 2 and received feedback on each judgment. In both conditions, the attribute values of the correct pair of options were colored for the participants after they made their pairwise similarity/dissimilarity judgment. Thus, the most similar/dissimilar options were shown in colored font when participants were making their investment decision. After making their decision, participants proceeded to the next trial. Participants did not receive feedback about their decision. The order of the trials were fully randomized.

Results

No participants were excluded from the analyses. We used a hierarchical Bayesian model to test for the presence of the attraction effect in the similarity and dissimilarity conditions separately. The model estimates the *relative choice share for the target* (RST, Equation 1; Berkowitsch et al., 2014), defined as the number of times the target is selected divided by the number of times the target plus the competitor are selected. If the RST values are greater than 0.5, then that is evidence of an attraction effect. An analysis using a modified version of RST (Katsimpokis et al., 2022) yielded nearly identical results and is provided in the Supplemental Materials, along with analyses of the attraction effect for the two focal options (X and Y) separately.

The model we used is similar to the one used in Trueblood (2015) and Trueblood et al. (2015). It is a beta-binomial model where the number of times the target is selected follows a binomial distribution: $Binomial(\theta, n)$, where θ represents the probability that the target is selected and n is the number of times the target plus the competitor are selected. We estimated a separate θ parameter for each person. We also assumed that the person-specific θ parameters were drawn from population-level beta distributions with mean (μ) and concentration (κ) parameters. We modeled the similarity and dissimilarity conditions separately, thus there were separate μ and κ parameters for the two conditions. The priors for the hyper-parameters were set to be relatively vague, with $\mu \sim Beta(1,1)$ and $\kappa \sim |N(0,10)|$. See the Supplemental Materials for the graphical model. The model was fit using the python package PyMC3 (Salvatier et al., 2016). Four Markov chain Monte Carlo (MCMC) chains with 5000 samples each were used to estimate the posterior distributions. All chains converged with R-hat values less than 1.1.

Table 2 shows the posterior mean and the 95% highest posterior density (HPD) interval for the μ parameter, estimated separately for the two conditions. If the HPD is above 0.5, then that suggests that the target was selected more than the competitor, indicating an attraction effect. As shown in the table, an attraction effect is observed in the similarity condition, but not the dissimilarity condition. The RST value is below 0.5 for the dissimilarity condition, suggesting a reversed attraction effect in that condition. We note that the 95% HPD includes 0.5 and thus there is only weak evidence of a reversal in the dissimilarity condition.

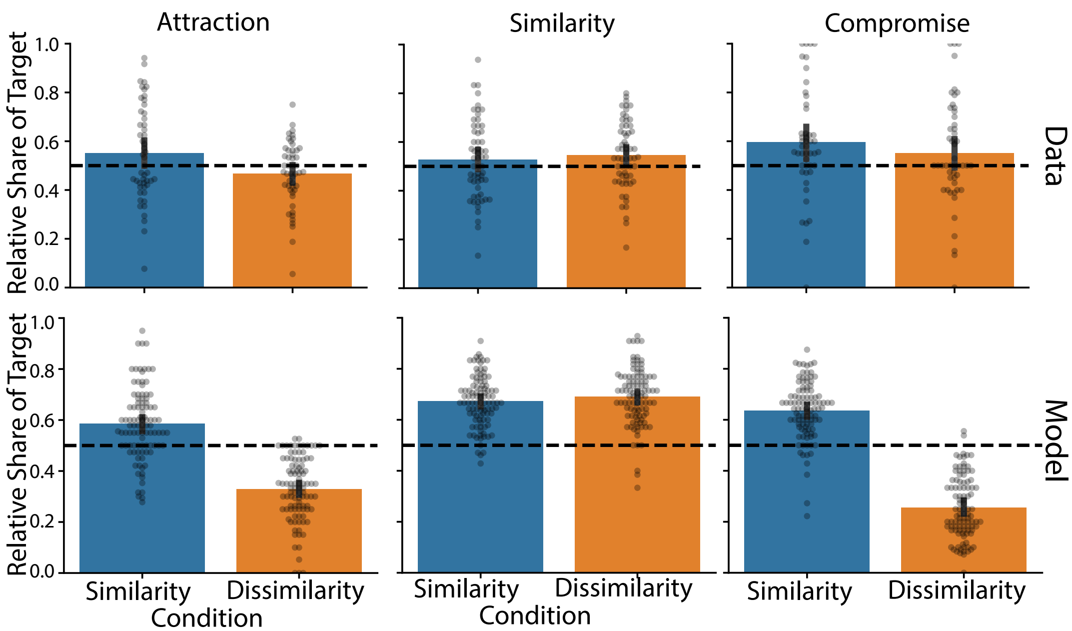

Figure 7

Behavioral (top) and modeling (bottom) results for Experiments 1-3: *Behavioral results show the relative choice share of the target for two conditions where attention was directed either towards similar options or dissimilar options for three context effects: attraction (Experiment 1), similarity (Experiment 2), and compromise (Experiment 3). Bars show the mean RST, error bars show the 95% confidence interval, and grey dots show the individual RST values. For model results, we simulated a synthetic version of the experiment with the actual experimental stimuli values. We constructed 100* in silico *"participants" by drawing 100 random collections of the parameters* (b, w, λ). *Each of these simulated participants completed each of the experimental choices and the relative share of target was computed the same as with the data. Each dot indicates the RST for each of these in silico participants while the bar indicates the mean RST. Analysis and plotting conventions are the same as with the data. The parameter sampling ranges were* $b = [0.1, 0.3]$, $w = [0.3, 0.7]$, *and the attention parameters were* $\lambda = [0, 5]$ *for the attraction and similarity choice sets and* $\lambda = [5, 10]$ *for the compromise choice sets.*

In order to compare the behavioral results to predictions from the modeling framework, we performed an in silico experiment. Using both the similarity and dissimilarity model variants, we simulated 100 "participants" from each model variant by randomly selecting 100 parameter sets (b, w, λ). The parameter ranges were set to be $[0.1, 0.3]$ for b, $[0.3, 0.7]$ for w, and $[0, 5]$ for λ. Each in silico participant completed the 20 attraction effect trials from the behavioral experiment (the filler trials were not included). For each simulated participant, we calculated the RST, similar to the behavioral data. The results are shown in the bottom left panel of Figure 7. As shown in the figure, a standard attraction effect occurs for the similarity model variant and a reversed attraction effect (repulsion effect) occurs for the dissimilarity model variant. The model predictions are inline with the behavioral data where an attraction effect occurs in the similarity condition, but not the dissimilarity condition.

Table 2
Results for Experiments 1-3

	Attraction		Similarity		Compromise	
Condition	Mean	95% HPD	Mean	95% HPD	Mean	95% HPD
Similarity	0.553	0.50-0.61	0.530	0.48-0.57	0.616	0.55-0.68
Dissimilarity	0.466	0.42-0.51	0.544	0.50-0.59	0.563	0.50-0.62

Experiment 2: Similarity Effect

Method

Participants

120 participants (women = 63, men = 56, age: M = 40.84 , SD = 13.60) completed the study online via Amazon Mechanical Turk using the CloudResearch platform. Similar to Experiments 1, we intended to have approximately 50 participants per condition. 59 participants completed the similarity condition and 61 participants completed the dissimilarity condition. The sample size was determined prior to starting the experiment and pre-registered on AsPredicted.org (#69809). We note the that the actual sample size was larger than the pre-registered sample size due to an error during participant recruitment. The data was analyzed only after all data had been collected. All participants were compensated $0.50 for the completion of the study.

Materials

Similar to Experiment 1, participants were asked to choose their preferred business based on the ROI ratings from two financial advisors. There were four options used to examine the similarity effect: two focal options and two similarity decoys (S_x and S_y). Like Experiment 1, this experiment also included filler choice sets where there was a clearly superior option. In total, there were 10 similarity effect trials with the S_x decoy, 10 similarity effect trials with the S_y decoy, and 10 filler trials. For the similarity effect trials, the specific attribute values for each option varied slightly across trials. The mean and standard deviation of the attribute values for the similarity effect options are provided in Table 3 and are identical to those used in Trueblood (2012).

Table 3
Summary of rating values used in Experiment 2 for the similarity effect

	Rating 1		Rating 2	
Option	Mean	SD	Mean	SD
X	34.80	0.89	74.90	1.07
Y	54.60	0.68	55.10	0.72
S_x	56.00	0.67	53.70	1.16
S_y	33.00	1.05	76.70	0.95

Procedures

The procedure for Experiment 2 was identical to Experiment 1. Participants were randomly assigned to either the similarity or dissimilarity condition at the beginning of the experiment, and then completed 30 randomized trials. On each trial, participants started with attribute-wise similarity/dissimilarity judgments about the businesses based on their assigned condition. After making these judgments, participants selected the business that they preferred and then proceeded to the next trial.

Results

No participants were excluded from the analyses. Similar to Experiment 1, we used a hierarchical Bayesian model to test for the presence of the similarity effect in the similarity and dissimilarity conditions separately. The approach was identical to Experiment 1 where we estimated the RST values using a beta-binomial model. An analysis using a modified version of RST (Katsimpokis et al., 2022) yielded nearly identical results and is provided in the

Supplemental Materials, along with analyses of the similarity effect for the two focal options (X and Y) separately.

The model specifications for Experiment 2 were also identical to Experiment 1. Namely, the priors for the hyper-parameters were set to be relatively vague, with $\mu \sim Beta(1,1)$ and $\kappa \sim |N(0,10)|$. The model was fit using the python package PyMC3 (Salvatier et al., 2016). Four Markov chain Monte Carlo (MCMC) chains with 5000 samples each were used to estimate the posterior distributions. All chains converged with R-hat values less than 1.1.

Table 2 shows the posterior mean and the 95% HPD interval for the μ parameter, estimated separately for the two conditions. As shown in the table, both conditions have an RST value greater than 0.5. We note that the 95% HPD includes 0.5 for the similarity condition, thus there is only weak evidence for a similarity effect in that condition.

Next we performed an in silico experiment using the modeling framework, similar to Experiment 1. We simulated 100 participants each from the similarity and dissimilarity model variants by randomly selecting 100 parameter sets within the following ranges: $b \in [0.1, 0.3]$, $w \in [0.3, 0.7]$, and $\lambda \in [0, 5]$. Each in silico participant completed the 20 similarity effect trials from the behavioral experiment (the filler trials were not included), and we calculated the resulting RST for each participant. As shown in the middle bottom panel of Figure 7, standard similarity effects occur for both model variants. These results are similar to the data were we see RST values above 0.5 in both conditions.

Experiment 3: Compromise Effect

Method

Participants

99 participants (women = 35, men = 64, age: M = 39.79 , SD = 13.37) were recruited from Amazon Mechanical Turk using the CloudResearch platform and completed the study online. Similar to Experiments 1 and 2, we intended to have approximately 50 participants per condition. 49 participants completed the similarity condition and 50 participants completed the

dissimilarity condition. The sample size was determined prior to starting the experiment and pre-registered on AsPredicted.org (#71161). The data was analyzed only after all data had been collected. All participants were compensated $0.50 for the completion of the study.

Materials

Similar to Experiments 1 and 2, participants in Experiment 3 made investment decisions based on the ROI ratings from two financial advisors. There were four options used to examine the compromise effect: two focal options and two compromise decoys (C_x and C_y). Like the other experiments, Experiment 3 also included filler choice sets where there was a clearly superior option. In total, there were 10 compromise effect trials with the C_x decoy, 10 compromise effect trials with the C_y decoy, and 10 filler trials. For the compromise effect trials, the specific attribute values for each option varied slightly across trials. The mean and standard deviation of the attribute values for the compromise effect options are provided in Table 4. Note that the attribute values of the compromise decoys are 2-units closer to the target than to the competitor, so that the answer to the similarity ratings is unique on each trial.

Table 4

Summary of rating values used in Experiment 3 for the compromise effect

	Rating 1		Rating 2	
Option	Mean	SD	Mean	SD
X	38.60	1.43	62.40	1.62
Y	62.20	2.36	38.80	1.94
C_x	17.00	3.50	84.00	3.94
C_y	83.80	4.89	17.20	4.39

Procedures

The procedure for Experiment 3 was identical to Experiments 1 and 2. Participants were randomly assigned to either the similarity or dissimilarity condition at the beginning of the experiment, and then completed 30 randomized trials. On each trial, participants first made two attribute-wise similarity/dissimilarity judgments based on their condition, and then they selected their preferred business.

Results

No participants were excluded from the analyses. Similar to Experiments 1 and 2, we used a hierarchical Bayesian model to test for the presence of the similarity effect in the similarity and dissimilarity conditions separately. The model specifications for Experiment 3 were also identical to Experiments 1 and 2. The model was fit using the python package PyMC3 (Salvatier et al., 2016). Four Markov chain Monte Carlo (MCMC) chains with 5000 samples each were used to estimate the posterior distributions. All chains converged with R-hat values less than 1.1. Table 2 shows the posterior mean and the 95% HPD interval for the μ parameter, estimated separately for the two conditions. As shown in the table, a compromise effect occurred in both the similarity and dissimilarity conditions. An analysis using a modified version of RST (Katsimpokis et al., 2022) yielded nearly identical results and is provided in the Supplemental Materials, along with analyses of the compromise effect for the two focal options (X and Y) separately.

Next we performed an in silico experiment using the modeling framework, similar to Experiments 1 and 2. We simulated 100 participants each from the similarity and dissimilarity model variants by randomly selecting 100 parameter sets within the following ranges: $b \in [0.1, 0.3]$, $w \in [0.3, 0.7]$, and $\lambda \in [5, 10]$. We altered the λ range for these compromise simulations since we found it necessary for the compromise effect to arise (in later analysis we will fit this parameter to data to avoid this hand tuning). Each in silico participant completed the 20 compromise effect trials from the behavioral experiment (the filler trials were not included), and we calculated the resulting RST for each participant. As shown in the right bottom panel of Figure 7, a standard compromise effect occurs for the similarity model variant and a reversed compromise effect occurs for the dissimilarity model variant. The results for the similarity model variant are inline with the data where we observe a standard compromise effect in the similarity condition. However, the results for the dissimilarity model variant do not match the data. In the modeling, we observe a reserved compromise effect, but in the data we observe a standard compromise effect.

To further examine the compromise effect in the two conditions, we performed exploratory analyses of the behavioral data, examining the impact of option location on the effects (these analyses were not pre-registered). In all three experiments, we randomized the row location of the three options. Thus across trials, the location of the two similar (dissimilar) options in the similarity (dissimilarity) condition varied. Specifically, the two similar (dissimilar) options could be located in (1) the top and middle rows, (2) the top and bottom rows, or (3) the middle and bottom rows. We reran the beta-binomial model for both conditions treating the three locations separately.

As shown in Table 5, a compromise effect occurs in the similarity condition when the similar options (i.e., the target and decoy) are either in the top and middle rows or in the middle and bottom rows. That is, the effect occurs when the target and decoy are adjacent. A null compromise effect is observed in the similarity condition when the similar options are in the top and bottom rows. In this case, the competitor is the middle option.

For the dissimilarity condition, a null compromise effect is observed when the two dissimilar options (i.e., the competitor and decoy) are either in the top and middle rows or in the middle and bottom rows, inline with model predictions. It is only when the dissimilar options are in the top and bottom rows that a compromise effect is observed. In this case, the target option is in the middle row.

We hypothesize that these patterns of results reflect the role of spatial biases in attention. In particular, options in the middle row are enhanced due to the ease of comparisons with adjacent options. In the similarity condition, when the competitor is the middle option (i.e., similar options are top and bottom), we see an increase in choices for the competitor, nullifying the compromise effect. Likewise, in the dissimilarity condition, when the target is the middle option (i.e., dissimilar options are top and bottom), we see an increase in choices for the target, leading to a standard compromise effect. Thus, we believe that both similarity-based and spatial attention contributed to the effects in this experiment and likely explain the mismatch in modeling and behavioral results for the dissimilarity condition.

Table 5
Results for Experiments 3

	Similarity Condition		Dissimilarity Condition	
Location	Mean	95% HPD	Mean	95% HPD
Top & Middle	0.623	0.53-0.71	0.511	0.42-0.60
Top & Bottom	0.553	0.45-0.66	0.646	0.57-0.72
Middle & Bottom	0.659	0.56-0.75	0.528	0.43-0.62

Discussion of Experiments 1-3

Experiments 1-3 examine the impact of similarity-based attention on the manifestation of context effects. We observe that by simply encouraging participants to attend to similar versus dissimilar options, we can modulate the strength of the effects. Further, the changes in context effects we observe in the behavioral data generally follow model predictions. However, we note that the modeling results in Figure 7 show more pronounced effects (both standard and reversed), than the data.

There are several possible explanations for the differences between the data and model predictions. First, for the model simulations, we randomly selected parameters from a large range of possible values. In reality, some of those parameter sets might not reflect the “best" parameters for our participants, as would be discovered through quantitative fitting. We did not attempt to quantitatively fit the data from these experiments because the number of trials was too low for choice-response time modeling. Instead we discuss quantitative fitting in a later section using data from Trueblood et al. (2015).

Another possibility for the differences between the data and model predictions relates to the strength of our behavioral manipulations. In our experiments, attention was manipulated through a simple judgment task where participants either selected the most similar options on each attribute or the most dissimilar options on each attribute. It is possible that this judgment task did not entirely shift attention in the desired way. That is, participants in the similarity condition could still compare dissimilar options if they desired and vice versa.

Finally, we believe that there are multiple ways that attention impacts context effects. In particular, exploratory analyses revealed strong effects of spatial location on the compromise effect in Experiment 3. It is likely these spatial effects were present in all three experiments and impacted the strength of the context effects. Thus, while our goal was to manipulate similarity-based attention, spatial attention likely played an important role as well. The model predictions did not account for possible spatial biases. Rather our goal with the modeling was to examine the specific influence of similarity-based attention on the context effects. The absence of spatial attention biases in the modeling could have led to differences between the modeling and data.

Despite differences between the data and modeling results, we find the similarity in qualitative patterns compelling. We believe these results demonstrate that similarity-based attention contributes to the elusive nature of context effects. In the next section, we explore the role of spatial attention and context effects in greater detail.

Spatial Attention: Reanalysis of Trueblood et al. 2015

In Trueblood et al. (2015) the attraction, compromise, and similarity effects were examined using the perceptual paradigm developed in Trueblood et al. (2013) where participants made decisions about the size of rectangles. In addition to studying the three effects within participants, the Trueblood et al. (2015) study also manipulated the left to right placement of the options on the screen. Each choice set had three alternatives: target (T), competitor (C), and decoy (D). Thus, participants saw a total of six distinct orders in the experiment: TCD, TDC, CTD, DTC, CDT, and DCT. The main objective of the present analysis is to examine the three effects across the six orders. The original analysis in Trueblood et al. (2015) did not take these different orders into consideration. However, Evans et al. (2021) did use this data to examine the attraction effect for the various orders. Here we also examine the compromise and similarity effects and include the attraction effect for completeness. Below we provide an overview of the methods and refer the reader to Trueblood et al. (2015) for a full description.

Method

Seventy-five participants took part in the experiment in exchange for course credit. On each trial, participants were shown three rectangles and asked to choose the one with the largest area. The rectangles were solid black and appeared on a white background. The height and width of the rectangles were varied in order to generate choice sets for the attraction, compromise, and similarity effects (similar to attributes of price and quality in studies of context effects with consumer products). For each choice set, the two focal rectangles had equal area, but one rectangle had larger height and smaller width than the other.

Participants completed 720 trials: 160 with attraction effect choice sets, 160 with similarity effect choice sets, 160 with compromise effect choice sets, and 240 filler trials where one rectangle was clearly the largest. For the context effects choice sets, half of the choice sets included a decoy targeting one of the two focal options and the other half of the choice sets included a decoy targeting the other focal option. The trials were fully randomized. The left to right placement of the options was also randomized, resulting in six distinct orders.

Results

No participants were excluded from the analyses. Similar to Experiments 1-3, we used a hierarchical Bayesian model to test for the presence of the three context effects across the six different orders. The model specifications were similar to Experiments 1-3. The model was fit using PyMC3 (Salvatier et al., 2016). Four Markov chain Monte Carlo (MCMC) chains with 5000 samples each were used to estimate the posterior distributions. All chains converged with R-hat values less than 1.1. Table 6 shows the posterior mean and the 95% HPD interval for the μ parameter, estimated separately for the three effects and six orders.

Results show that for some spatial orders standard effects emerge, but for other orders, null or reversed effects occur (see both Table 6 and Figure 8). We hypothesize that the observed

Table 6
Results for the reanalysis of Trueblood et al. 2015

	Attraction		Similarity		Compromise	
Order	Mean	95% HPD	Mean	95% HPD	Mean	95% HPD
TCD	0.424	0.39-0.46	0.516	0.48-0.56	0.467	0.44-0.50
TDC	0.587	0.54-0.63	0.6	0.56-0.64	0.561	0.52-0.60
CTD	0.563	0.53-0.60	0.604	0.57-0.64	0.569	0.53-0.61
DTC	0.631	0.59-0.67	0.642	0.60-0.68	0.586	0.55-0.63
CDT	0.499	0.46-0.54	0.546	0.50-0.59	0.475	0.44-0.51
DCT	0.369	0.33-0.41	0.469	0.43-0.51	0.413	0.38-0.45

differences in RST values across the different orders reflect biases in spatial attention. In particular, we suggest that greater attention is allocated to adjacent options as compared to non-adjacent options. In particular, when the target is the middle option (i.e., CTD, DTC), it is adjacent to both the decoy and competitor. In this case, the target receives more attention and thus accumulates evidence faster than the other options, leading to standard context effects. Likewise, when the competitor is the middle option (i.e., TCD, DCT), it receives more attention, resulting in null or reversed context effects. In the remaining two cases (i.e., TDC, CDT), results are mixed. For TDC, all three standard effects occur. However, for CDT, only a standard similarity effect occurs. We believe the difference here is the left versus right placement of the target. When the target is placed in the leftmost position, we believe it receives enhanced attention because English (the primary language of our participants) is a left-to-right writing system.

To further examine the impact of spatial attention biases on context effects, we performed a model simulation exercise using the similarity model variant. For these simulations, we allowed for the effects of spatial location on attention allocation. In particular, we assumed that the comparison of adjacent options is ~ 2.5 times more likely than comparison of non-adjacent options. For these simulations, we used artificial stimuli values, not stimuli values from the task. Our goal here is to simply show that the modeling framework can qualitatively produce the observed behavioral patterns. In a later section, we quantitatively fit and evaluate the modeling framework using this data. The attribute values for the simulations were $T = (0.4, 0.6)$,

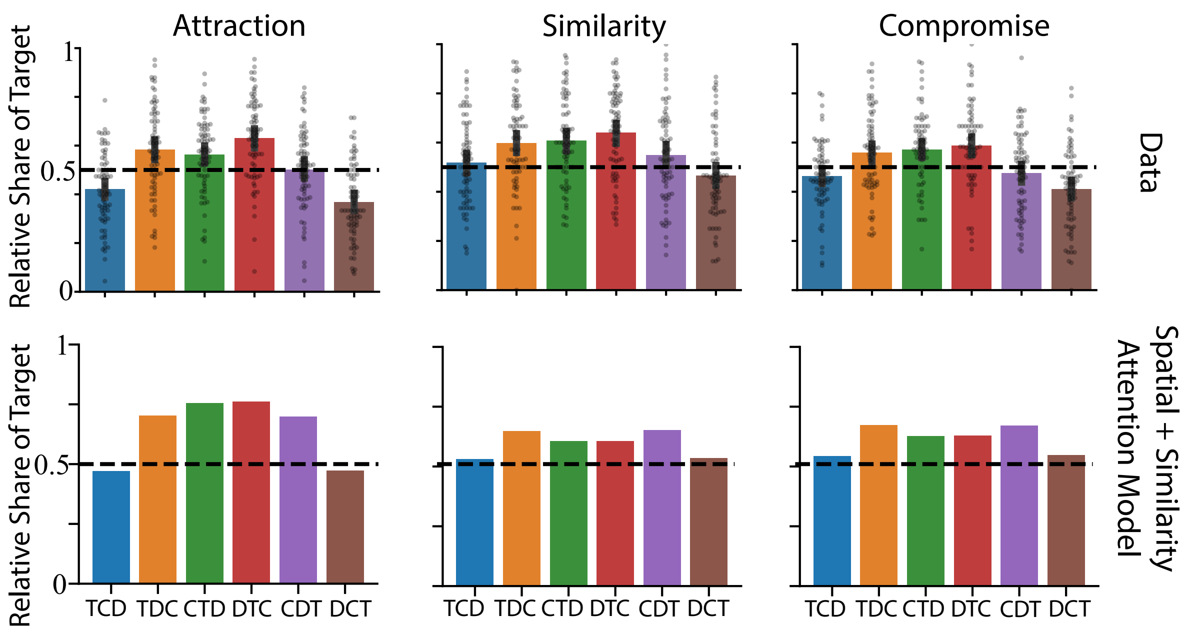

Figure 8

Reanalysis of Trueblood et al. 2015: *Behavioral results show the relative choice share of the target for six different spatial orders for three context effects. Bars show the mean RST, error bars show the 95% confidence interval, and grey dots show the individual RST values. For the model, we simulated synthetic data from the similarity attention variant accounting for effects of spatial location of the alternatives on attention. Specifically, we assumed that comparison of adjacent alternatives is* ~ 2.5 *times more likely than comparison of non-adjacent alternatives. The alternative attribute values were in each case* $T = (0.4, 0.6)$, $C = (0.6, 0.4)$ *with* $D_A = (0.3, 0.5)$, $D_C = (0.2, 0.8)$, $D_S = (0.55, 0.45)$. *For the parameters, the attribute weight was* $w = 0.5$, *the threshold* $b = 0.2$ *and the attention parameter was* $\lambda = 2.5$ *for the attraction and similarity effects and* $\lambda = 7.5$ *for the compromise effect.*

$C = (0.6, 0.4)$ with $D_A = (0.3, 0.5)$, $D_S = (0.55, 0.45)$, $D_C = (0.2, 0.8)$, where D_A, D_S, and D_C are the decoys for the attraction, similarity, and compromise effects respectively. In these simulations, we fixed the attribute weight at $w = 0.5$, the threshold at $b = 0.2$ and the attention parameter at $\lambda = 2.5$ for the attraction and similarity effects and $\lambda = 7.5$ for the compromise effect (a larger value of λ more robustly produces the compromise effect).

Modeling results show a similar qualitative pattern as the data. In particular, we observe standard effects when the target is the middle option and reversed / null effects when the competitor is the middle option. Since the model does not bias left versus right placement of options (as might occur because of a left-to-right writing system), the simulations are symmetric and do not capture the observed differences in TDC and CDT. We note that left / right biases are easy to incorporate into the model if desired.

Discussion

The reanalysis of Trueblood et al. (2015) revealed that the display layout of options had a large impact on the presence / absence of context effects. We hypothesized that changes in display layout might impact how attention is allocated in the task and thus alter context effects. To this end, we conducted model simulations examining how biases in spatial attention influence context effects. The modeling results were qualitatively consistent with the behavioral findings, suggesting that simple biases in attention can have large impacts on choice behavior.

Quantitative Evaluation of the Modeling Framework

To further evaluate the importance and influence of similarity and spatial based attention factors in these decisions, we quantitatively fit four versions of this model to the full choice-RT data from Trueblood et al. (2015). We toggle similarity and spatial factors in and out of the framework (resulting in four models). For the similarity factor, we used the same exponential dependence of deliberation time on similarity; more similar comparisons are compared for longer duration. When this factor is not included, all comparisons are distributed equally.

For spatial attention, we included two additional parameters (p_{LM}, p_{RM}) that alter the weights for different comparisons. This accounts for potential biases for adjacent options and left versus right comparisons. When this factor is not present, all comparisons are equally probable. The most extensive version of this model containing both factors has the following seven estimable parameters: threshold (b), attribute weight (w), non-decision time (t_{nd}), a drift rate scaling parameter (v_{scale}), the similarity decay parameter (λ), and the two spatial probability parameters (p_{LM}, p_{RM}). The drift rate scaling parameter is a methodological parameter that converts attribute units into the preference domain. This is necessary for mathematical consistency (the left and right side of the preference updating equation must have the same units) and is discussed further in the Supplemental Materials. The simplest version of the model that omits both factors contains only four parameters (b, w, t_{nd}, v_{scale}).

Continuous model extension and model fitting procedure

The fully stochastic version of this model can in principle be fit to choice-RT data using simulation based methods such as the Probability Density Approximation (PDA) method (Holmes, 2015; Holmes and Trueblood, 2018; Holmes et al., 2016). This is computationally cumbersome however. We thus develop a continuous approximation to this model that utilizes Stochastic Differential Equations in place of full stochastic simulations. Full details of the description, derivation, and validation of this are in the Supplementary Materials. We briefly note however that this approximation is theoretically grounded. It is essentially the asymptotic limit that results from taking the simulation time step in the stochastic model $\Delta T \to 0$.

The end result of this approximation is a model of the form

$$d\vec{P} = v_{scale} * \vec{v}dt + \vec{\sigma}d\vec{w}, \tag{7}$$

where P is a time varying vector of preferences, $\vec{v}$ is a vector of accumulation rates, $\vec{\sigma}$ is a vector of variances, and v_{scale} is the previously mentioned methodological scaling parameter. Note that $(\vec{v}, \vec{\sigma})$ are not estimable parameters. They are fully analytically constructed from the attention process (see Supplemental Materials). $\vec{v}$ represents the average rate of accumulation properly averaged over all possible attention states. $\vec{\sigma}$ represents the variability in that accumulation rate, accounting for the intrinsic variability of the attention process.

This approximation to the stochastic model is an independent race (IR) to threshold model. That is, it is three stochastic accumulators independently racing to the choice threshold, where the first one to reach that threshold is the chosen alternative. The significant benefit of this simplification is that IR models have known, analytically solvable choice-RT distributions based on the Wald / Inverse Gaussian distribution (see Supplemental Materials). This dramatically simplifies the choice-RT fitting process without sacrificing any of the core characteristics of the modeling framework.

To fit this model, we use a Bayesian approach to fit full choice-RT data, accounting for the spatial order of presented alternatives. Following prior studies, we use Differential Evolution MCMC (DEMC) (Turner et al., 2013). Prior to fitting these models, parameter recovery was demonstrated (Supplemental Materials) using the same experimental design and stimuli as the experiment itself.

Results

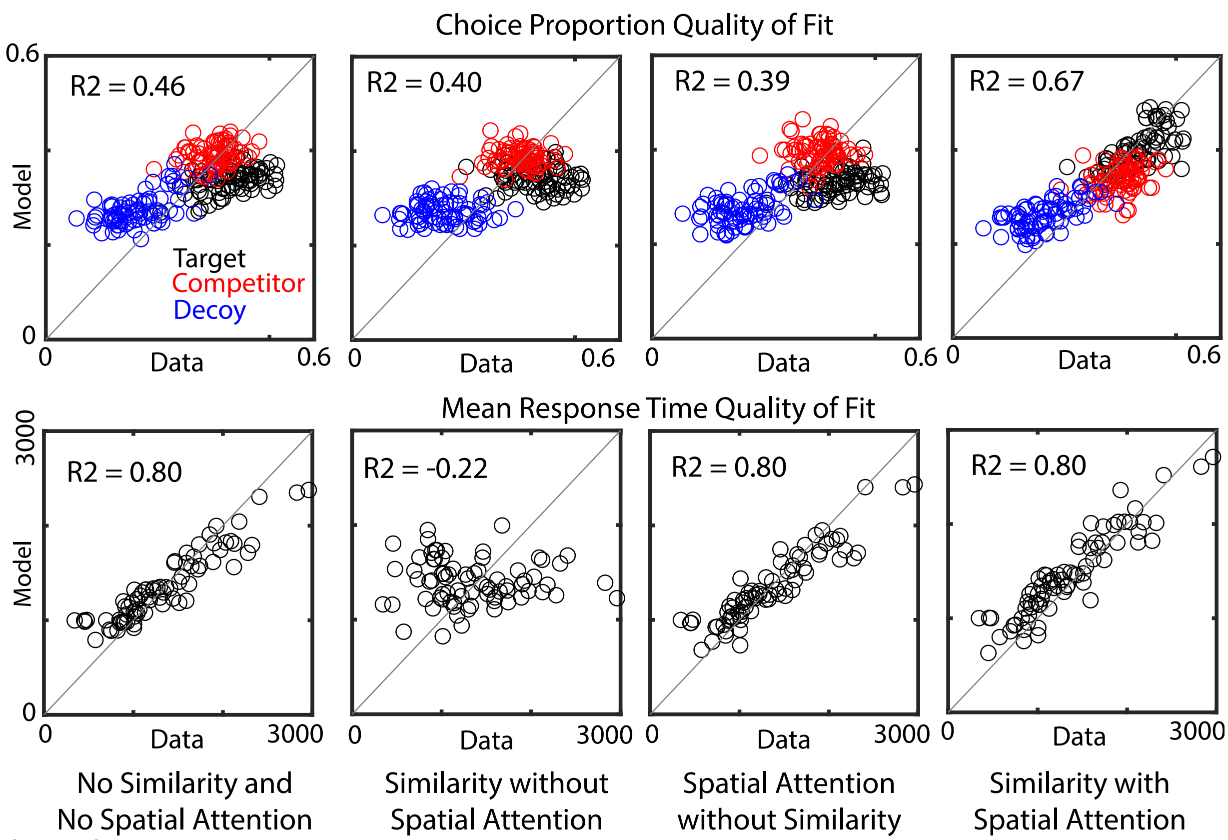

Figure 9

Choice - RT fitting for Trueblood et al. 2015 data with spatial ordering: *For each of the four indicated models, we fit the choice - RT data from Trueblood et al. 2015 as described in the methods. To assess quality of fit, we plot the choice proportion and mean response time predicted by the model for each participant against the same quantities from the data. R-squared values quantify this quality of fit and the gray lines indicated the line of agreement. Similar results for the dissimilarity variants are in the Supplemental Materials.*

Figure 9 illustrates the quality of fit for the four model variants by comparing choice proportions and mean RTs derived from the posterior mean parameter sets for each individual with those from data. Results illustrate that both similarity and spatial attention factors are needed to provide a reasonable account of the data. When only one attention factor is present, the model performs quantitatively worse. More importantly, the single factor models predict higher competitor than target choice proportions, which is qualitatively contrary to data.

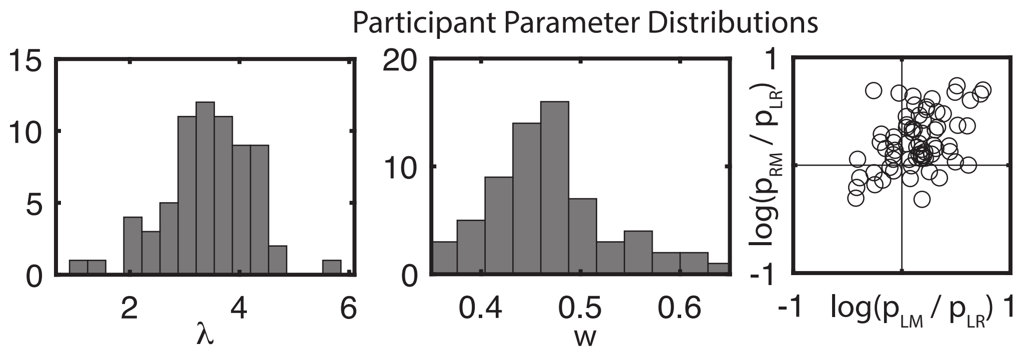

Figure 10
Participant parameters for similarity plus spatial attention model: *For each of the experimental participants, posterior mean parameter values were calculated. Histograms for λ and w are shown and the joint values of spatial attention parameters are shown.*

We further analyzed the distribution of participants' posterior mean parameter values for the model with both similarity and spatial attention included (Figure 10). Results show the attribute weight is generally close to 0.5, which is reasonable since the two attributes in question are length and width of rectangles. The similarity decay parameter λ is significantly different from 0. Finally, participants broadly show an adjacency bias, but not a substantial bias for comparisons involving left or right options.

General Discussion

Context effects in choice, particularly the attraction, compromise, and similarity effects, have received considerable attention over the last several decades. Yet despite being observed across multiple cognitive domains (Farmer et al., 2015; Farmer et al., 2017; Maylor & Roberts, 2007; Trueblood, 2012, 2015; Trueblood et al., 2013; Wedell, 1991; Yearsley et al., 2021) and across multiple species (Bateson et al., 2003; Latty & Trueblood, 2020; Parrish et al., 2015; Shafir et al., 2002), they have proven to be simultaneously fragile (Spektor et al., 2021). In this paper, we introduced a novel modeling framework that explains the elusive nature of context effects.

Using this framework, we demonstrate how the presence, absence, and reversal of classic context effects can be explained by variations in the attention process during evidence accumulation.

The idea that attention is important for understanding context effects is shared by most dynamic models of context effects (Trueblood, 2022). These models have, in many cases, assumed that alternatives are compared to each other in a pairwise fashion, that comparisons between more similar alternatives receive greater weight, and that all alternatives and attributes cannot be simultaneously attended to. Our work extends and generalizes these prior theories. Specifically, the framework described here assumes that only one comparison on one attribute can be attended to at any given time but that the sequence of comparisons is governed by a structured attention process. Rather than assume a fixed attention process (i.e. similarity-based), we explore a range of different attention processes and their consequences on choices. Results show that when more similar alternatives are compared for longer periods of time, the classic context effects arise. However, if that attention processes is altered (e.g., dissimilar alternative are compared for longer periods of time), the effects can disappear or reverse, as has been found in a number of recent empirical studies (Cataldo & Cohen, 2018, 2019; Evans et al., 2021; Spektor et al., 2018). Importantly, our framework also allows for intrinsic factors, such as an individual's attribute biases, and extrinsic factors, such as the spatial arrangement of options in the choice display, to enter into the attention process. In sum, our work illustrates that attentional dynamics are key to understanding why context effects can be simultaneously general and fragile.

We validated our modeling framework using a combination of behavioral experiments and a reanalysis of previously published data. Results from Experiments 1-3 showed that by simply encouraging participants to attend to similar versus dissimilar options, we were able to modulate the strength of the three effects in ways that were generally consistent with model predictions. To test whether attentional biases arising from the spatial layout of items may alter context effects, we reanalyzed data from Trueblood et al. (2015). In that study the attraction, similarity, and compromise effects occurred when the target was the middle option, whereas null or reverse effects occurred when the competitor was the middle option. Our results indicate this is a consequence of people's preference for comparing spatially adjacent options more than nonadjacent options. This result in particular illustrates how easily experimental factors can alter empirical findings. In summary, our findings illustrate the critical role of attention in the generation of context effects. Further, this modeling approach provides a way to disentangle different factors that may influence attention and their potential effects on observations.

Relationships to Other Modeling Approaches

Our modeling framework builds on ideas from previous theories of context effects, particularly the idea that attention plays an important role in multi-alternative, multi-attribute choice (Trueblood, 2022). Although our goal was not to introduce a new model to test against others in a head-to-head comparison, it is informative to compare and contrast the way that attention operates in the present framework with the way that attention operates in other dynamic choice models.

Most dynamic choice models assume that individuals shift a window of attention throughout deliberation to evaluate various options. Here we highlight three models, MDFT (Roe et al., 2001), MLBA (Trueblood et al., 2014), and multialternative decision by sampling (MDbS; Noguchi and Stewart, 2018), to illustrate the relationship between the current framework and existing theories. We refer readers to Trueblood (2022) for a more in depth review of the role of attention in dynamic context effects models. In MDFT, the window of attention includes all options along a single attribute. In MLBA and MDbS, attention is allocated to a pair of options along a single attribute. In MDFT and MDbS, the window of attention fluctuates stochastically over time, whereas MLBA assigns deterministic attention weights to the set of pairwise comparisons. The present framework assumes an explicit sequence of comparisons over time (specifically pairwise comparisons in the model variants studied here), with a Markov process to describe the probabilities of switching to different comparisons.

Additionally, both MDbS and MLBA incorporate some notion of similarity-based attention. In MDbS, the probability of evaluating the value of an option on a particular attribute is proportional to the similarity between that option's value and other values in working memory, including values retrieved from long-term memory (e.g., memory of prices from past experience). In MLBA, the attention weight for each pairwise comparison depends on the similarity of the values being compared. The present framework allows for similarity to affect the amount of time spent on each comparison. For example, in the similarity model variant, greater similarity among options leads to longer deliberation time for the comparison of those options.

We also point out some key differences between the current framework and previous modeling approaches. The current framework is stochastic rather than ballistic as is the case for MLBA. MDbS utilizes ordinal comparisons between options instead of allowing the size of the difference between options to influence the rate of evidence accumulation as in our framework. MDFT incorporates several additional mechanisms not currently included in our framework such as distant-dependent lateral inhibition and leakage. Finally, while both MLBA and MDFT have been quantitatively tested using choice-response time data, we are unaware of such testing of MDbS. Doing so would likely require the development of a continuous approximation of the model similar to what was done here and in Evans et al. (2019).

Another important distinction between our framework and past approaches is that we show that a single mechanism, namely similarity-based attention, is sufficient for producing the attraction, compromise, and similarity effects (see Figure 4). In other dynamic models, additional mechanisms are required to produce at least one of the three effects. For example, MDFT accounts for the attraction and compromise effects using distance-dependent lateral inhibition (Roe et al., 2001), MLBA accounts for all three effects using a combination of similarity-based attention and asymmetric weighting of advantages and disadvantages (Trueblood et al., 2014; Tsetsos et al., 2015), and MDbS accounts for the similarity effect by assuming that people tend to ignore relatively small differences (Noguchi & Stewart, 2018). Our results show that while these factors may be present, they are not necessary and attention alone can lead to the contextual sensitivities observed in the classic effects.

Our framework explicitly models the attention process during deliberation. Another approach is to use empirical measures of visual attention (i.e., gaze data) as input to the evidence accumulation process, as in the attentional drift diffusion model (aDDM; Krajbich et al., 2010). These two approaches are very different: One makes *a priori* assumptions about the way in which attention fluctuates between different pieces of information and how it influences choice, while the other uses exogenous measures of visual attention to inform the model without explaining why gaze is allocated that way in the first place. Few models have attempted to explain the allocation and timing of visual fixations during the deliberation process (for an example in simple value-based choice, see Callaway et al., 2021). The aDDM has proven to be quite useful as a tool for understanding the moderating effects of visual attention on value-based decision making (e.g., Smith and Krajbich, 2018, 2019; for a review, see Krajbich, 2019). It does not however attempt to understand the process responsible for the allocation of attention. Recently, gaze-dependent evidence accumulation models have also been leveraged to study context effects in multi-alternative, multi-attribute choice (Molter et al., 2022). However, because attention is treated as an exogenous variable in the aDDM and other gaze-dependent models, the models make no predictions for the effect that different spatial layouts, for example, will have on attention and (consequently) decision making. The behavior of these models depends on the eye movement data that gets fed into them. In contrast, the present framework allows us to incorporate different theories about the role of spatial and other presentation-related factors into the model's attention process in order to examine their predicted effects on choice behavior. In the Future Directions section, we return to the topic of eye-tracking measures and how they might be used to supplement our modeling framework.

An Alternative Explanation of the Elusiveness of Context Effects

Like other dynamic decision models, our theoretical framework relies on the integration and accumulation of relative comparisons during deliberation to explain the emergence of context effects (Busemeyer et al., 2019). Moreover, it uses attentional dynamics during the deliberation process to explain the elusiveness of context effects. An alternative explanation, suggested by

Spektor et al. (2021), is that the elusiveness of context effects is due to noisiness in the subjective representation of the choice set. Certain features of the choice task, such as the spatial arrangement of the options or the presentation modality of the attributes, could make it more difficult for decision makers to obtain an accurate representation of the options. Spektor et al. suggest that when decision makers first encounter a choice, there is some degree of uncertainty or noise in the representation of the options. Although the uncertainty gradually diminishes over the course of deliberation, if the level of uncertainty is high, dominance relationships and attribute orderings will be more difficult to detect, inhibiting the attraction and compromise effects and potentially leading to reversals of these effects. On the other hand, the similarity effect is less likely to be impacted by representational noise because any difficulties in detecting dominance relationships would likely influence the competitor and decoy as opposed to the target. If the level of uncertainty is low, the attraction and compromise effects should not be impacted and standard effects are anticipated.

Currently, the idea of representational noise has not been incorporated into a formal model of decision making. Future work could explore whether it is possible to extend our framework to allow for uncertainty in the attribute values that diminishes over time. We note that there are many ways of adding noise to models and care must be taken to make sure that the addition of noise does not result in model tractability issues. As of now, our framework explains the elusive nature of context effects purely through subtle changes in attention during the deliberation process, treating the attribute values as given.

Benefits of the Current Modeling Approach

We have stressed that our proposed model is a framework in the sense that it encompasses many possible model variants. For example, we showed how three different assumptions about the relationship between similarity and deliberation time could be incorporated into the framework. The similarity variant, which assumes that the more similar two options are, the longer they are compared, provided the best qualitative account of participants' choices when they were encouraged to attend to similar options. The dissimilarity variant, which assumes a negative relationship between similarity and deliberation time, best matched the empirical patterns when participants were encouraged to attend to dissimilar options. It is possible that under different circumstances, attention may be allocated differently. Further, attribute biases and spatial attention mechanisms can be encoded in the conditional transition matrix that contains the probabilities of switching to different comparisons. A model variant in which comparisons between adjacent options are more probable than comparisons between nonadjacent options was able to account for spatial order effects present in previously published data (Trueblood et al., 2015). The flexibility of our modeling framework is one of its primary advantages, and there are many other possible modifications that could be made.

Another advantage of the current modeling framework is that it can be fit to choice-RT data and that its parameters are recoverable (see Supplemental Materials). This is important, as other dynamic models of context effects have been shown to have relatively poor parameter recovery (Evans et al., 2019). Complex models with many parameters often have strong parameter trade-offs, where increases in one parameter can be compensated by decreases in another (e.g., Miletić et al., 2017). When this occurs, the parameters are said to be weakly identified and it is no longer possible to reliably interpret the parameter estimates. Because our model exhibits good recovery of parameters, we can use the parameters to draw inferences about the underlying cognitive processes mediating choice behavior.

Future Directions

Future studies should collect process data to examine the assumptions of the proposed framework. Eye tracking methods have proven to be particularly useful for this purpose. For example, eye tracking studies have shown that in multi-alternative, multi-attribute choice tasks, pairs of options are compared on single attributes at a time (Noguchi & Stewart, 2014), pairs of similar options are attended to more frequently than pairs of dissimilar options (Noguchi & Stewart, 2014, 2018), and the inclusion of asymmetrically dominated decoys focuses attention on the target option and the dominance attribute (Marini et al., 2020). Alternative methods, such as the hidden-attribute protocol, can also be used to shed light on process mechanisms (Marini et al., 2022). An obvious application of these methods to the current framework would be to test whether people do indeed transition more frequently between adjacent options than nonadjacent options, and whether the ratio of the transition frequencies between adjacent and nonadjacent options is associated with spatial order effects in choice, as predicted by the modeling framework.

More generally, this framework can serve as a platform to understand the consequences of different attention-related factors on decision making and under what circumstances those factors may be important. A significant strength of our approach is that it allows the researcher to explicitly model attention as part of the decision process. For example, this model could be extended to account for potential sequential dependencies in attention. For mathematical tractability, we have assumed that the conditional probabilities of transitioning to different comparisons do not depend on the past sequence of comparisons. This is likely overly restrictive and can be expanded upon in future work. Alternative forms of comparison can also be investigated. We have for simplicity assumed comparisons are within-attribute and between two items in the choice set, but comparison against other exemplars may be possible. This is just a sampling of possible extensions.

Conclusions

Context effects in multi-alternative, multi-attribute choice are widely documented and yet, paradoxically, quite fragile. Their elusiveness challenges previous theories of context effects, which have largely focused on explaining the existence of the effects. In this paper, we introduced a novel computational modeling framework that relies on attention modulated comparisons to explain the fragile nature of context effects. The flexibility of our framework makes it possible to test a number of different theories about the role of attention in the deliberation process. Further, we developed and validated a tractable approximation of the full model that can be used to fit choices and response time data. Using this framework, we demonstrate how attention may be critical in understanding the simultaneous generality and fragility of these effects. Generality arises from the properties of attention allocation, while fragility is a result of the ease with which that attention can be altered either intentionally or unintentionally. In summary, the proposed framework is a powerful tool for understanding the elusive nature of context effects through the lens of attentional dynamics during deliberation.

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Supplemental Materials for "Attentional Dynamics Explain the Elusive Nature of Context Effects"

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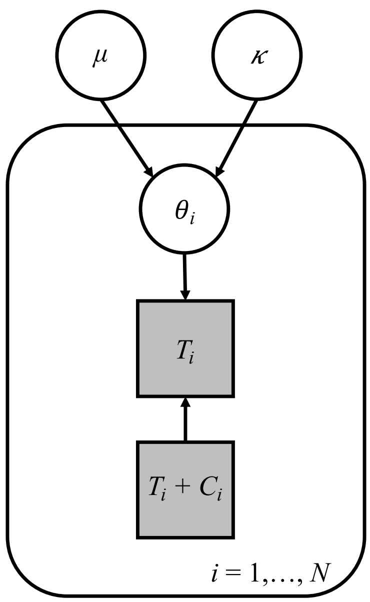
Supplemental Materials for "Attentional Dynamics Explain the Elusive Nature of Context Effects"

Bayesian Model of the Relative Choice Share of the Target (RST)

Figure 1 illustrates the graphical model used to estimate the Relative Choice Share of the Target (RST) in Experiments 1-3 as presented in the main text. The same model was also used in the reanalysis of Trueblood et al., 2015.



$\mu \sim \text{Beta}(1,1)$

$\kappa \sim |N(0,10)|$

$a = \mu\kappa$

$b = (1\text{-}\mu)\kappa$

$\theta_i \sim \text{Beta}(a, b)$

$T_i \sim \text{Binomial}(\theta_i,\, T_i + C_i)$

Figure 1
Hierarchical Bayesian graphical model used for testing the presence of the attraction, compromise, and similarity effects in Experiments 1-3 and the reanalysis of Trueblood et al. (2015).

Comparing Two Versions of RST

The model in Figure 1 is based on the traditional RST measure, in which the frequency of target selections is summed across the ternary choice sets $\{X, Y, Z_X\}$ and $\{X, Y, Z_Y\}$. Recently, Katsimpokis et al. (2022) demonstrated that the conventional RST can lead to biased inferences when the total number of target and competitor selections differs across the two choice sets. This can happen when one of the two decoys (Z_X or Z_Y) is strongly preferred, as sometimes occurs with compromise or similarity decoys. To mitigate this problem, the authors proposed an "equal weights" version of RST defined as

$$RST_{EW} = 0.5 \cdot \left(\frac{P(X|\{X,Y,Z_X\})}{P(X|\{X,Y,Z_X\}) + P(Y|\{X,Y,Z_X\})} + \frac{P(Y|\{X,Y,Z_Y\})}{P(X|\{X,Y,Z_Y\}) + P(Y|\{X,Y,Z_Y\})} \right) \quad (1)$$

Note that RST_{EW} is the simple average of the RST calculated within each choice set.

Following Katsimpokis et al. (2022), we constructed a hierarchical Bayesian beta-binomial model for RST_{EW} by estimating separate θ parameters for the two choice sets (Figure 2). Each individual's θ parameters were assumed to be drawn from population-level beta distributions with separate mean (μ) and concentration (κ) parameters. A group-level estimate of RST_{EW} can be computed by averaging the μ parameters across the two choice sets, i.e., $\mu_{EW} = 0.5 \cdot (\mu_1 + \mu_2)$. We modeled the similarity and dissimilarity conditions separately in Experiments 1-3. The model was fit in JAGS via the runjags R package (Denwood, 2016). Four Markov chain Monte Carlo (MCMC) chains with 5000 samples each were used to estimate the posterior distributions. All chains converged with R-hat values less than 1.1.

Table 1 shows the posterior means and 95% highest posterior density (HPD) intervals for the μ_{EW} parameters across conditions. If μ_{EW} is greater than 0.5, then that is evidence of a positive context effect. The results were nearly identical to the results for the traditional RST presented in the main text. The only qualitative difference is that the HPD for the dissimilarity condition in the similarity effect experiment overlapped with 0.5 using RST_{EW}, whereas it was completely above 0.5 using traditional RST. However, the overall conclusions were unaffected by the type of RST selected.

Additional Results for Experiments 1-3

In addition to the RST, we also examined the strength of context effects in Experiments 1-3 for each focal option separately. For this analysis, we calculated the change in choice shares

Table 1
Results for Experiments 1-3 using the equal weights version of RST

	Attraction		Similarity		Compromise	
Condition	Mean	95% HPD	Mean	95% HPD	Mean	95% HPD
Similarity	0.556	0.51-0.60	0.509	0.47-0.56	0.618	0.57-0.66
Dissimilarity	0.467	0.42-0.51	0.532	0.49-0.57	0.564	0.52-0.61

of each focal alternative ($\Delta P_a,\ a = X,Y$; Wedell, 1991). This measure compares the choice proportion of a particular focal alternative in the ternary choice set when that particular focal alternative is the target and when it is not. Thus, ΔP_a is defined for each focal option separately:

$$\begin{aligned} \Delta P_X &= P(X|\{X,Y,D_x\}) - P(X|\{X,Y,D_y\}), \\ \Delta P_Y &= P(Y|\{X,Y,D_y\}) - P(Y|\{X,Y,D_x\}), \end{aligned} \tag{2}$$

where D_x and D_y are decoy options. When $\Delta P_a > 0$, this indicates a standard context effect for the particular focal alternative.

Similar to the main text, no participants were excluded from the additional analyses. We used a hierarchical Bayesian model to estimate ΔP_X and ΔP_Y. The graphical model is shown in Figure 3. It is a beta-binomial model where the number of times a focal option is selected follows a binomial distribution: $Binomial(\theta, n)$, where θ represents the probability that the option is selected and n is the number of times a particular of context effect choice set appears. Specifically, the parameter θ is estimated separately for the choice sets where the focal option is the target and for the choice sets where the focal option is the competitor. We also assumed that the person-specific θ parameters were drawn from population-level beta distributions with mean (μ) and concentration (κ) parameters. The difference in the μ parameters is the group-level estimate of ΔP_a. We modeled the similarity and dissimilarity conditions separately, thus there were separate estimates of ΔP_a for the two conditions. The model was fit using the python package PyMC3 (Salvatier et al., 2016). Four Markov chain Monte Carlo (MCMC) chains with

5000 samples each were used to estimate the posterior distributions. All chains converged with R-hat values less than 1.1.

Table 2 shows the posterior mean and the 95% highest posterior density (HPD) interval for ΔP_X and ΔP_Y, estimated separately for the two conditions. If the HPD is above 0, then that suggests that the target was selected more than the competitor, indicating a context effect. As shown in the table, the results are similar to those obtained using the RST.

Table 2
Estimates of ΔP_X and ΔP_Y for Experiments 1-3

	Attraction		Similarity		Compromise	
Condition	Mean	95% HPD	Mean	95% HPD	Mean	95% HPD
Similarity ΔP_X	0.085	0.01-0.16	-0.026	-0.10-0.04	0.228	0.14-0.32
Similarity ΔP_Y	0.084	0.00-0.17	0.103	0.03-0.18	0.158	0.06-0.26
Dissimilarity ΔP_X	-0.066	-0.15-0.02	0.022	-0.04-0.08	0.116	0.02-0.21
Dissimilarity ΔP_Y	-0.044	-0.12-0.04	0.100	0.03-0.17	0.123	0.03-0.21

Continuous Model Extension

For purposes of efficiency, parameter estimation, and response time modeling, it will be useful to have a continuous, stochastic differential equation (SDE) approximation of the discrete model described in the main article. We show here that the model can be approximated by a SDE of the form,

$$dP_j = \bar{v}_j dt + \sigma_j dw, \tag{3}$$

where P_j represents the preference for alternative j, $\bar{v}_j$ is the "average" accumulation rate (averaged over all possible comparisons), σ_j is the amplitude of the stochastic variability, and dw is a standard Gaussian white noise increment. The key is to determine the values of $\bar{v}_j$ and σ_j based on the discrete process described above. We note that in many historical models, great care is taken in determining what the average accumulation rates $(\bar{v}_j)$ should be for a race model while noise parameters are often taken to be free parameters to be estimated or fixed at some value. Here both will be derived as a properties of the discrete model formulated previously. We first derive the average accumulation rates $\bar{v}_j$ and subsequently derive a relationship between the variances σ_j.

Approach

In order to derive these quantities, consider the following simplified scenario. Fix a time interval T and divide it into smaller increments of time increment Δs. Then there are $N_{\Delta s} = T/\Delta s$ time increments in the full unit of time. Define S_{c_m} to be the number of those time steps spent on comparison c_m. By determining the statistical properties of these random variables (mean and variance in particular), we can construct $(\bar{v}_j, \sigma_j)$.

The average accumulation rate for alternative j over that full unit of time would be

$$v_j = \sum_{\{c_m \text{ involving } j\}} \frac{S_{c_m}}{N_{\Delta s}} (x_j^{a_k} - x_i^{a_k}), \tag{4}$$

where (j, i, a_k) are the indices associated with comparison c_m. (i, j) indicates the alternatives being compared while a_k is the attribute being attended to. This average drift rate will be a stochastic quantity ($v_j \sim N(\bar{v}_j, \sigma_j)$) whose mean is

$$\bar{v}_j = \sum_{\{c_m \text{ involving } j\}} \left\langle \frac{S_{c_m}}{N_{\Delta s}} \right\rangle (x_j^{a_k} - x_i^{a_k}) \tag{5}$$

and variance is

$$(\sigma_j)^2 = \sum_{\{c_m \text{ involving } j\}} \mathrm{Var}\left(\frac{S_{c_m}}{N_{\Delta s}}\right) (x_j^{a_k} - x_i^{a_k})^2. \tag{6}$$

Note that in deriving the variance for the drift, the assumption that S_{c_m} are independent is required. This is of course not the case since those quantities must sum to $N_{\Delta s}$. However we were unable to find a way around this. We have performed simulation studies to verify that this representation of the variance is a good approximation to the simulated variance from the discrete process. We also note that the resulting SDE race model exhibits the same characteristics as the discrete model (e.g. exhibits all context effects for the same parameters). Thus while this approximation is not strictly correct, it does not appear to have a substantive effect on the variance estimates and provides a drastic simplification that allows us to write an analytic estimate for that variance.

Derivation of Average Accumulation Rates

The average rate of preference accumulation for alternative j is given by

$$\bar{v}_j = \sum_{\{c_n \text{ involving } j\}} f_{c_n} \left(x_j^{a_k} - x_i^{a_k} \right), \tag{7}$$

where $f_{c_n} = \langle S_{c_n} / N_{\Delta s} \rangle$ is the asymptotic fraction of each unit time spent on comparison c_n, the sum is taken over all comparisons involving alternative j, and a_k is the attribute on which comparison c_n is based. The vector $\vec{f} = (f_{c_1}, ..., f_{c_{last}})$ is the "stationary vector" of the Markov chain M. The stationary vector satisfies $\vec{f} M = \vec{f}$ and therefore $\vec{f}$ can be calculated by finding the left eigenvector of the matrix M associated with the eigenvalue 1.

The elements of $\vec{f}$ represent the asymptotic probability that at any instant in time the comparison c_n is being attended to. Thus, v_j represents the weighted average of the accumulation rates for all comparisons where the weights are determined by the Markov process M. Notice that while the Markov chain M was dependent on the size of the time increment Δs, these asymptotic probabilities do not. This in combination with Equ. (5) completes the derivation of $\bar{v}$.

Derivation of Variances (σ_j)

What we are going to derive here is as follows. Suppose you have a given unit of time T which contains $N_{\Delta s} = T / \Delta s$ time steps. Above we derived the expected fraction of those steps that will be spent on each possible comparison. That estimate will however have a variance. What is that variance? This is what we will derive. There is little intuition behind this derivation and thus we simply provide the details.

Let S_{c_m} be the number of steps (of the $N_{\Delta s}$ available) of the random walk spent on comparison c_m. Then S_{c_m} is normally distributed according to $S_{c_m} \sim N(N_{\Delta s} f_{c_m}, N_{\Delta s} \sigma_{c_m})$. In order to determine σ_{c_m}, define F to be a square matrix where each row is an identical copy of the stationary state row vector $\vec{f}$ derived above. Further define $Z = [I - M + F]^{-1}$ where I is the appropriately sized identity matrix. This is often referred to as the "Fundamental Matrix" for the Markov chain. Note that invertibility is guaranteed since this is a regular, ergodic Markov process. Then

$$\sigma_{c_m}^2 = \text{Var}\left(\frac{S_{c_m}}{N_{\Delta s}}\right) = \frac{2f_{c_m}z_{c_m,c_m} - f_{c_m} - f_{c_m}^2}{N_{\Delta s}}. \quad (8)$$

We note that while there is an apparent dependence of σ_{c_m} on Δs, for sufficiently small increments, this converges to an asymptotic value. We have validated that this variance estimate is a good approximation to the actual variance by comparing it to results of brute force simulations (Figure 4). There, we simulated the Markov process many times, calculated the true variance $\sigma_{c_m}^2$, and compared it to the approximate value. Results show good agreement, validating this approximation. With these approximations for $\sigma_{c_m}^2$, we can now fully construct the required noise parameters in Equ. (6).

Full Description of the Continuous Model

Any time one constructs a race to threshold model such as this, there is usually a dimensional mismatch. The left and right sides of an equation must have the same units. Consider the first term $\bar{v}_j dt$ in Equ. (3). The units of this term are measured in units of the attribute value. However, P and b (the threshold) are measured in units of preference. When only choice proportions are being modeled, this is not a significant issue, but when modeling response times it must be accounted for. One way to do so is to introduce a scaling parameter v_{scale} that maps attribute strength into the preference domain. Note that this is a single, condition independent scaling parameter that applies to all attribute values. With this scaling parameter, we can update the model in Equ. (3) as

$$dP_j = v_{scale} * \bar{v}_j dt + \sigma_j dw, \quad (9)$$

where all relevant quantities on the right hand side of this expression are described in Eqs. (5), (6). Unless otherwise stated, $v_{scale} = 1$ will be used for all simulation studies.

To ensure that this resulting continuous model exhibits the same choice behavior as the full discrete model, we simulated the same context effect numerical experiment as in the main text. Results (Figure 5) show that the models indeed predict the same choice behavior, validating this continuous approximation of the full discrete model.

Continuous Model Choice-RT Likelihood Function

The resulting model in Equ. (9) is an independent race (IR) model. Each alternative has a single preference state accumulator, which accumulates independently of the other alternatives, until one reaches the decision threshold b, triggering a choice. A primary benefit of this SDE formulation over the full stochastic formulation is that the IR model has an analytic solution for its choice-RT distribution (see Wikipedia).

Consider a single accumulator

$$dx_j = v_j dt + \sigma_j dw \tag{10}$$

with $x_j(0) = 0$. The first passage time to a threshold $b > 0$ is

$$f_j(t) = IG(b/v, (b/\sigma)^2), \tag{11}$$

where IG is the Inverse Gaussian distribution's probability density function (PDF). That is, f_j describes the hitting time distribution for accumulator j, in the absence of any other accumulators. This is however not the quantity of interest. The probability of one accumulator hitting $x = b$ at time t without any other accumulator hitting prior to that is needed. This is the *first* passage time. This can be readily calculated as

$$FPT_j(t) = f_j(t) \prod_{i \neq j} (1 - F_i(t)), \tag{12}$$

where $F_i(t)$ is the cumulative distribution function (CDF) associated with the PDF $f_i(t)$. This is precisely the probability that accumulator j completes at time t with no other accumulator completing prior to that.

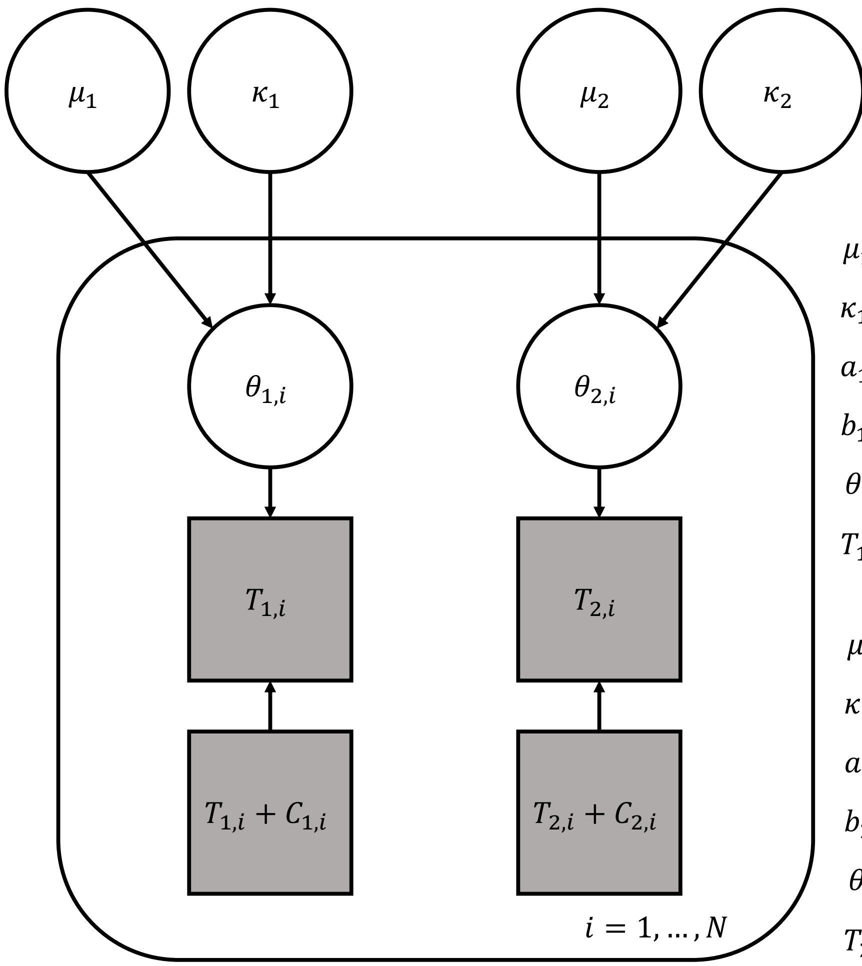
We can thus calculate the choice-RT first passage time distributions for this SDE model in two steps. First, construct the necessary drift rates and variances from the specified model and input stimuli parameters. Using these composite quantities, calculate the FPT_j using the PDF and CDF for the inverse Gaussian.

Fitting Methods

We fit each of the four choice-RT models to data from Trueblood et al., 2015 using a custom, Matlab based implementation of Differential Evolution Monte Carlo (DEMC, Turner et al., 2013). Fits were performed at the individual level. In models where spatial attention parameters were included, the ratios $p_{LM}/p_{LR}, p_{RM}/p_{LR}$ were log-sampled to ensure even sampling of values below and above one. Standard practice for DEMC is to use 3 chains per parameter. We thus used 21 chains (7 parameters). The following priors were used for the parameters: $b \sim U(0,10)$, $t_{nd} \sim U(0.05,1)$, $w \sim U(0.1,0.9)$, $v_{scale} \sim U(0,20)$, $\lambda \sim U(0,10)$, $\log_{10}(p_{LM}/p_{LR})$ and $\log_{10}(p_{RM}/p_{LR}) \sim N(0,1)$, where U,N indicate Uniform and Normal distributions.

Parameter Recovery

To illustrate parameter recovery for this model and fitting approach, we simulated data directly out of stimuli sets seen by one participant. All filler trials were removed, leaving 480 stimuli sets factored over similarity, compromise, and attraction effects as well as the six different spatial orderings. We simulated a single trial from each stimuli set with the parameters $b = 1, w = 0.5, t_{nd} = 0.15, \lambda = 3, v_{scale} = 10, p_{LM}/p_{LR} = 1.6, p_{RM}/p_{LR} = 1.2$. This results in 480 choice-RT pairs, matching the structure of the experimental data. Results in Figure 6 demonstrate good recovery of parameters.



$\mu_1 \sim \mathrm{Beta}(1,1)$

$\kappa_1 \sim |N(0,10)|$

$a_1 = \mu_1 \kappa_1$

$b_1 = (1 - \mu_1)\kappa_1$

$\theta_{1,i} \sim \mathrm{Beta}(a_1, b_1)$

$T_{1,i} \sim \mathrm{Binomial}(\theta_{1,i}, T_{1,i} + C_{1,i})$

$\mu_2 \sim \mathrm{Beta}(1,1)$

$\kappa_2 \sim |N(0,10)|$

$a_2 = \mu_2 \kappa_2$

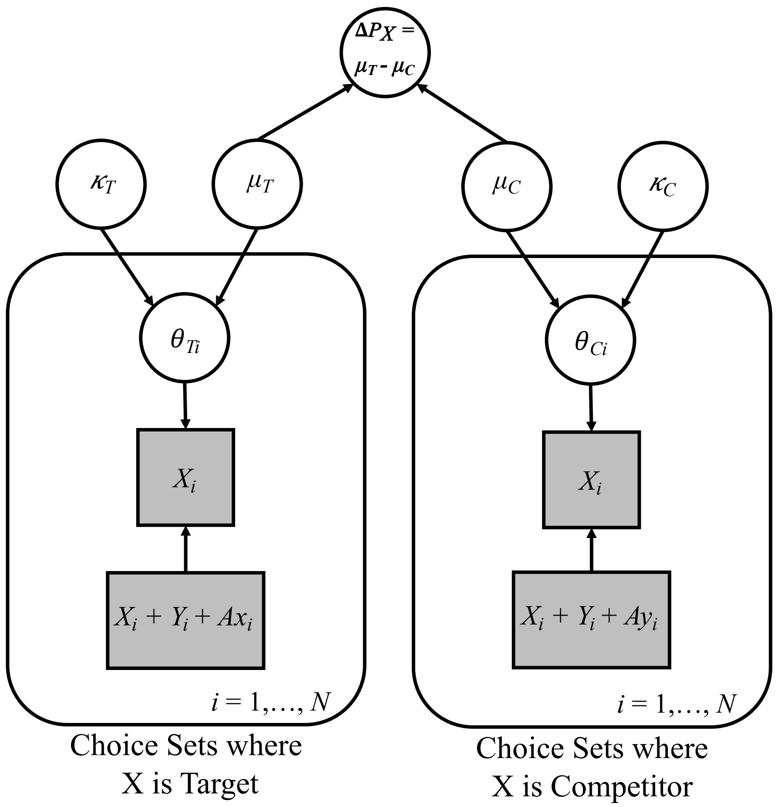
$b_2 = (1 - \mu_2)\kappa_2$

$\theta_{2,i} \sim \mathrm{Beta}(a_2, b_2)$

$T_{2,i} \sim \mathrm{Binomial}(\theta_{2,i}, T_{2,i} + C_{2,i})$

Figure 2

Hierarchical Bayesian graphical model for the equal weights version of the relative choice share of the target (RST_{EW}). The two choice sets $\{X, Y, Z_X\}$ and $\{X, Y, Z_Y\}$ are denoted respectively by the subscripts 1 and 2.



$\mu_T, \mu_C \sim \text{Beta}(1,1)$

$\kappa_T, \kappa_c \sim |N(0,10)|$

$a_T, a_C = \mu_j \kappa_j$

$b_T, b_C = (1- \mu_j)\ \kappa_j$

$\theta_{Ti}, \theta_{Ci} \sim \text{Beta}(a_j, b_j)$

$X_i \sim \text{Binomial}(\theta_{ji}, X_i + Y_i + A_i)$

Figure 3

Hierarchical Bayesian graphical model used for estimating ΔP_a for Experiments 1-3. The estimation of ΔP_X for the attraction effect choice sets is shown. Estimation of ΔP_X for the compromise and similarity effects is similar. Estimation of ΔP_Y for all three effects is similar.

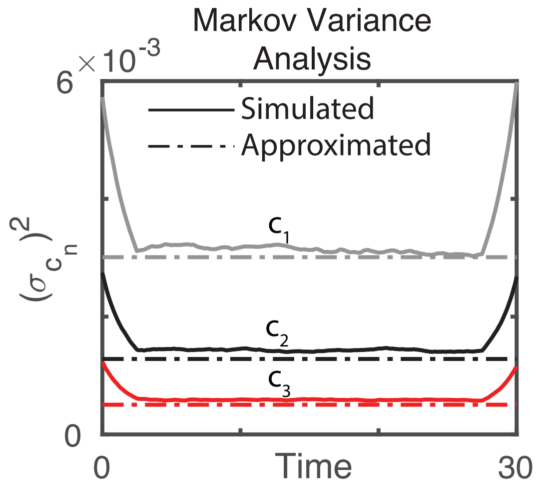


Figure 4

Markov Model Variance Approximation. *Here we validate the continuous model variance approximation for* Σ_{c_n} *in Equ.* (8)*. We fix the relevant parameters at* $w = 0.5, \lambda = 0.5$ *and simulate* $N_{acc} = 10,000$ *identical copies of the Markov state transition process with a time step of* $\Delta s = 20ms$ *using a two attribute, ternary choice set consisting of* $\{(3,6),(6,4),(5,5)\}$*. Thus there are six possible comparisons that can be made (3 alternatives and 2 attributes). From the direct simulations we calculated* $\mathrm{Var}(S_{c_m}/N_{\Delta s})$ *(solid lines) as well as the approximated variance in Equ.* (8) *(dashed lines). Note that in order to get an estimate for the mean time spent in each comparison state for each Markov state trace, we averaged over a* $T = 5sec$ *time interval. Thus the simulated variances exhibit defects near the start and end time due to this averaging. For clarity we show agreement for only the first three comparisons, though similar agreement is found for the remaining three. We also performed similar simulations with* $\Delta s = 2ms$ *and found similar results. These results validate the approximation for* Σ_{c_n} *in Equ.* (8)*.*

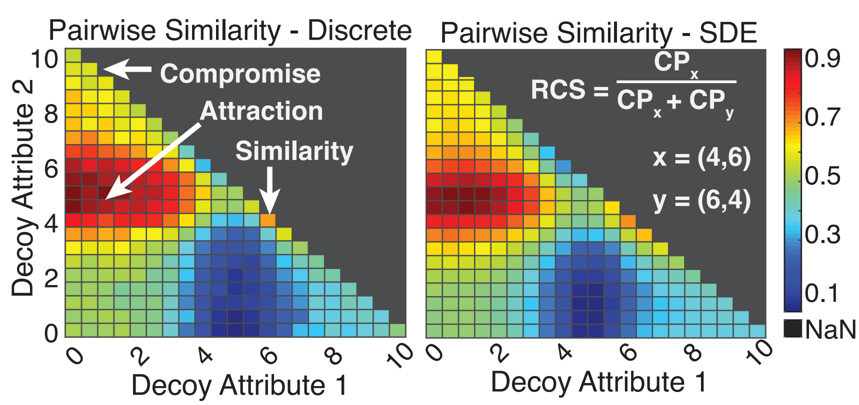


Figure 5

Discrete and continuous models capture context effects. ***a)*** *Decoy plane depicting how moving a third alternative around attribute space influences the relative share for the target (T / T + C) when the target and competitor are held fixed. Arrows point to regions that demonstrate various context effects. Parameters are fixed at values shown in Panel b. Model parameters are set to* $w = 0.5, b = 2, \lambda = 0.5, coll = 0$*.*

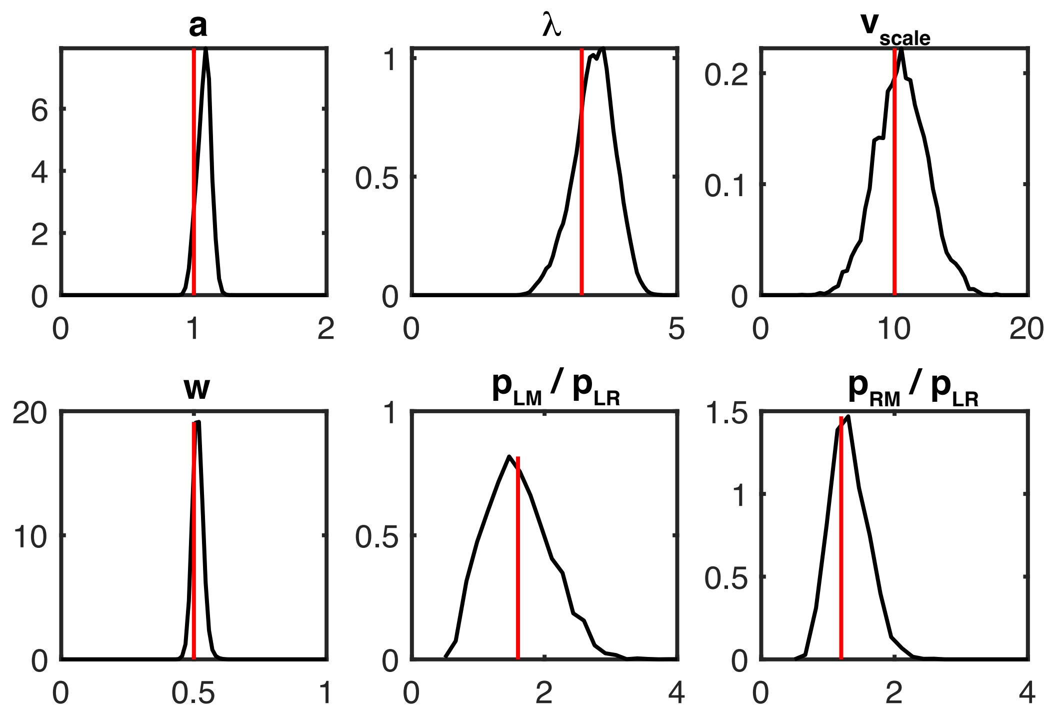


Figure 6

Parameter recovery for Trueblood et al. 2015 model fitting. *To assess whether parameters of the models fit to the Trueblood et al. 2015 data are recoverable, we simulated a synthetic data set from the most complicated model with both spatial attention and a similarity attention process. Simulation parameters were* $b = 1, w = 0.5$, $t_{nd} = 0.15$, $\lambda = 3$, $v_{scale} = 10$, $p_{LM}/p_{LR} = 1.6$, $p_{RM}/p_{LR} = 1.2$ *using the exact stimuli values from one of the experimental participants as inputs. Results show the posterior distributions for 6 of the 7 parameters (non-decision time was similarly recovered). Red vertical lines indicate the parameter values used for the simulation.*

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