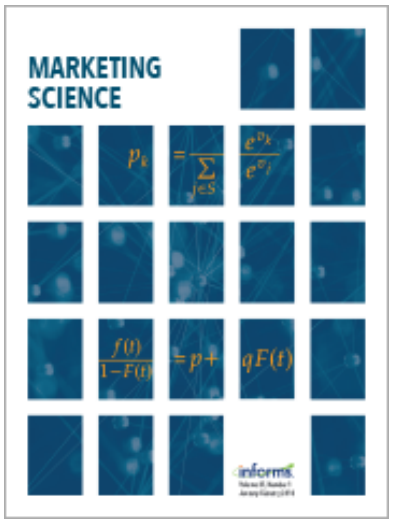

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The Company That You Keep: When to Buy a Competitor's Keyword

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In search advertising, brand names are often purchased as keywords by the brand owner or a competitor. We aim to understand the strategic benefits and costs of a firm buying its own brand name or a competitor's brand name as a keyword. We model the effect of search advertising to depend on the presence or absence of a competitor's advertisement on the same results page. We find that the quality difference between the brand owner and the competitor moderates the purchase decision of both firms. Interestingly, in some cases, a firm may buy its own brand name only to defend itself from the competitor's threat. It is also possible that the brand owner, by buying its own branded keyword, precludes the competitor from buying the same keyword. Our result also implies that the practice of bidding on the competitor's brand name creates a prisoner's dilemma, and thus both firms may be worse off, but the search engine captures the lost profits. We also discuss the difference in our results when the search is for a generic keyword instead of a branded keyword. Finally, we find some empirical support for our theory from the observation of actual purchase patterns on Google AdWords.

Keywords: search advertising; branded keywords; brand advertising; keyword selection; analytical model
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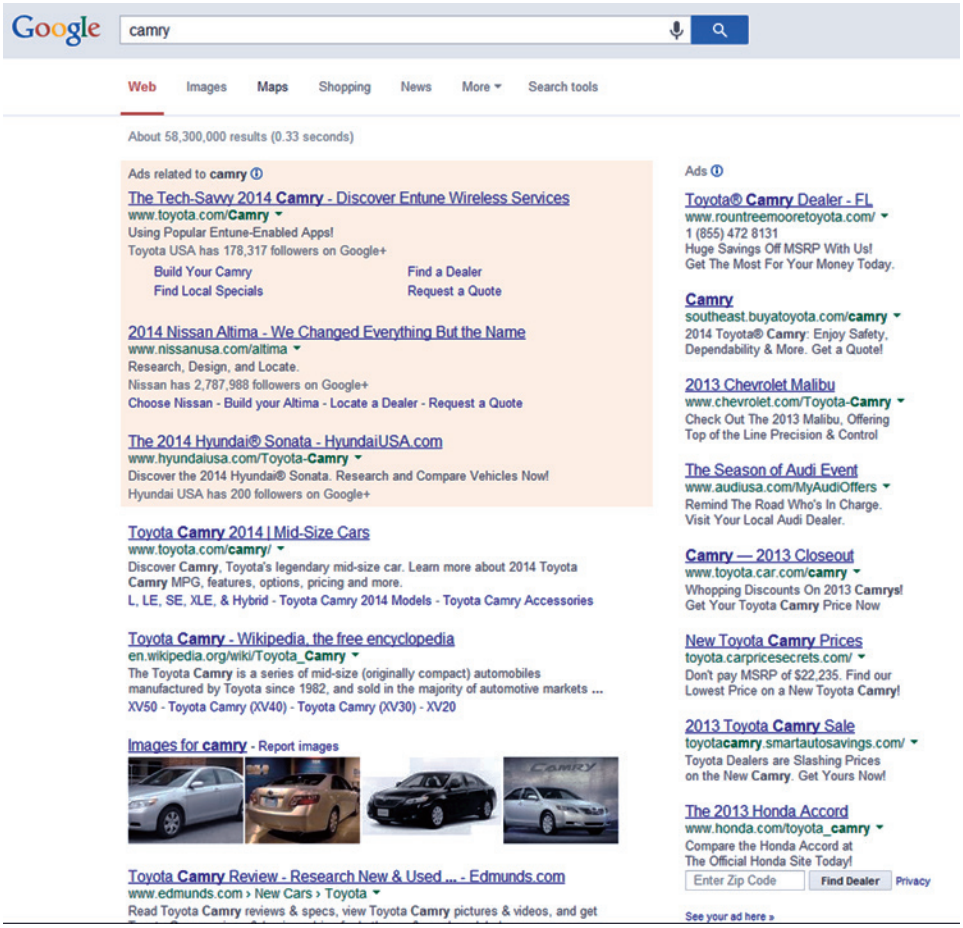
1. Introduction

Consumers often use specific brand names as keywords in conducting online searches during their purchase process. For example, the keyword "Camry" averages about 2.7 million searches per month, and "Chevrolet" averages 25 million per month. Thus, brand names are often highly sought-after keywords in keyword search advertising auctions, and in practitioner circles, there is much discussion about the merits of a firm buying its own and competitors' brand names as keywords.[1] In this paper, we examine the strategic benefits and costs of bidding for not only one's own brand name but also a competitor's brand name as a keyword, knowing that the competitor can do the same. We start with the observation that if the two firms buy the same keyword, then the two advertising messages will be in close proximity. As a result, the consumer will not only be exposed to the two messages but also consciously or unconsciously compare the two brands, leading the consumer to change his or her perceptions of the quality of the two brands. To systematically investigate the competitive implications of such changes, we draw from two behavioral literatures, one on the normal exposure effect for advertising (e.g., Zajonc 1968, Ferraro et al. 2009, Nedungadi et al. 1993) and the second on assimilation-contrast effect framework to account for the possibility of comparisons (e.g., Hovland et al. 1957, Sherif and Hovland 1961, Mussweiler 2003). By incorporating these psychological aspects of consumer behavior in an analytical model, we provide a new perspective for analyzing the impact of keyword search advertising.

This model allows us to answer a number of interesting questions about competing firms' strategies for branded keyword choice in search advertising. For example, if you type "Camry" in Google's search box, you will see a link to Toyota's website appear on the top of the organic search section of the search results page and another link to Toyota's website along with brief advertising copy in the sponsored section of the search results page. In addition, a sponsored link to Nissan Altima will also appear. (See Figure 1 for a

[1] Examples include Search Engine Watch (http://searchenginewatch.com/article/2066504/3-Reasons-to-Spend-Money-on-Branded-Terms), Search Engine Land (http://searchengineland.com/the-complete-guide-to-bidding-on-competitor-brand-names-trademarked-terms-118576), and SEOmoz (http://www.seomoz.org/blog/introducing-branded-keyword-rules-and-metrics), all accessed on February 4, 2013.

Figure 1 Example of Google Search Results Page



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screen capture.) Instances such as this raise questions such as why we observe some well-known firms buying their own brand names as keywords and why some lesser-known firms buy the brand names of their more famous competitors as keywords, whereas others do not. In this paper, we examine these strategic issues by building an analytical model of duopoly that includes (1) a model of consumer behavior reflecting different levels of search activity and the aforementioned psychological factors affecting brand choice and (2) an auction model that captures the firms' decision on whether or not to buy the two firms' branded keywords. Because the assimilation-contrast effects are a key element in our model of how consumers respond when exposed to keyword search advertising, we start by giving some prima facie evidence that such effects exist in a keyword search advertising setting. Later, we also provide empirical support for some of the implications of our model.

1.1. Impact of Search Advertising on Consumer Perceptions

Our example is based on an online experiment of 95 subjects who participated for monetary payment. The participants were exposed to a search engine results page showing results of a search performed on the keyword "Sony TV." We used a 2×2 between-subject design in which two factors were Sony buying the Sony keyword (yes, no) and a lower-end competitor, Haier, buying the Sony keyword (yes, no). After seeing the search result page, the subjects rated the quality of the two brands on a seven-point scale (where 1 = lowest possible quality available and 7 = best quality available). (See Figure 2 for examples of the actual stimuli.) The results are shown in Table 1. As expected, Sony's perceived quality rating when neither firm advertised was substantially higher than Haier's, i.e., 5.15 compared with 3.26 ($t = 11.21$, $p < 0.01$). Looking first at the quality rating for Sony, we see that it increases when participants see Sony's advertisement. In the case where Haier does not advertise, we see the rating increases from the base rating of 5.15 to 5.58 ($t = 1.73$, $p < 0.05$), and when Haier also advertises, the rating is even higher; i.e., it increases to 5.77 ($t = 2.50$, $p < 0.01$). Similar comparisons show benefits to Haier when its advertisement is present. Now the perceived quality increases from 3.26 to 4.18 ($t = 3.48$, $p < 0.01$) when Sony does not advertise. When Sony's

Figure 2 Stimuli Used in the Online Survey

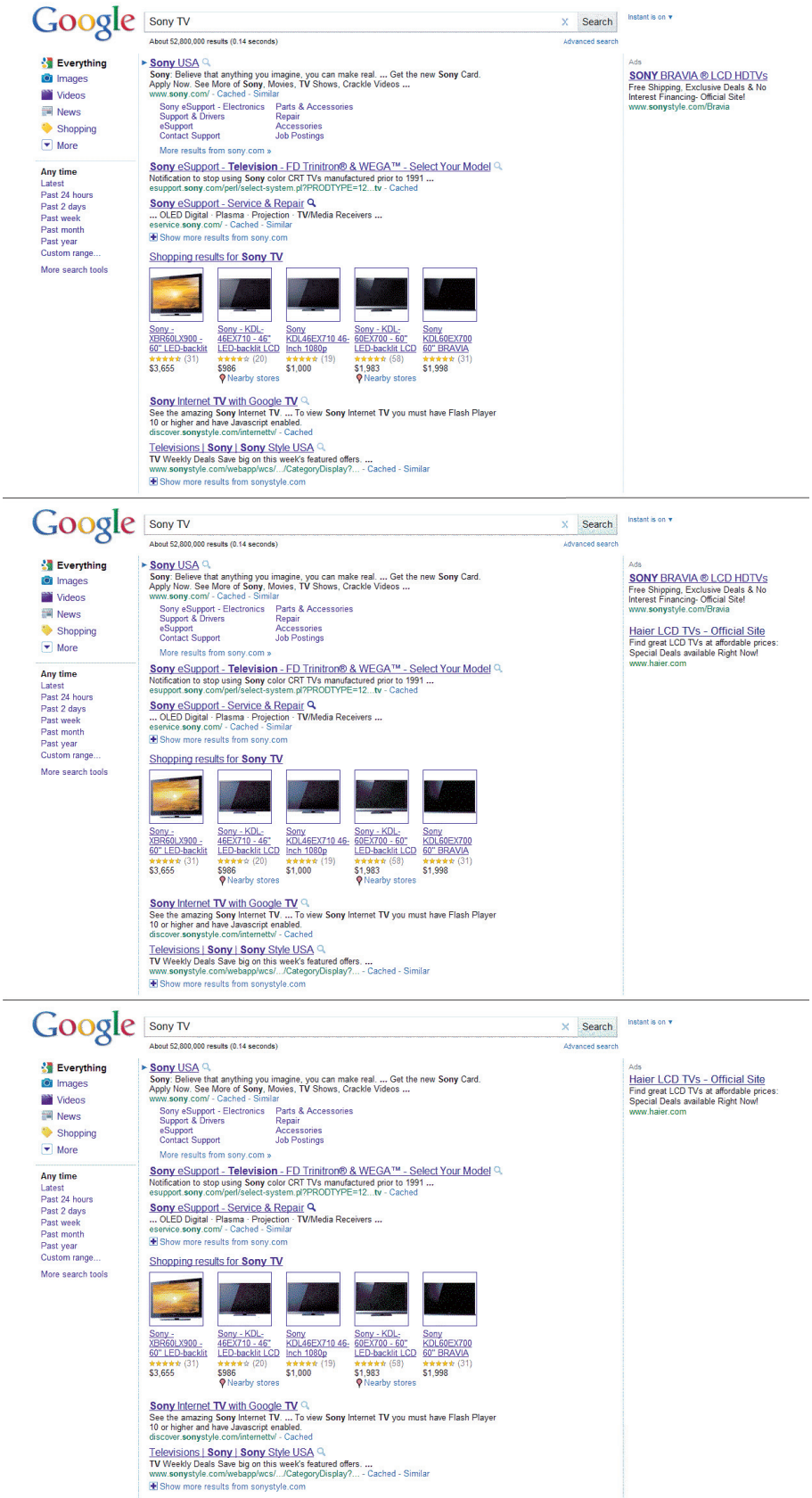

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Table 1 Perceived Quality of Sony and Haier by Advertising Scenario

	Haier's advertising decision	
Sony's advertising decision	No	Yes
No	5.15 (Sony) 3.26 (Haier)	5.59 (Sony) 4.18 (Haier)
Yes	5.58 (Sony) 2.90 (Haier)	5.77 (Sony) 3.57 (Haier)

advertisement is also present, the perceived quality rating still increases, but it is significantly less than in the previous case; i.e., it decreases from 4.18 to 3.57 ($t = -1.95$, $p < 0.05$). Why do we see Sony's and Haier's quality perceptions go up when they advertise? Why do we find this increase is larger (smaller) for Sony (Haier) when the two advertisements are in close proximity?

We address these questions by first calling upon the vast literature on advertising effects and the behavioral literature on context effects and then building a model that partitions the impact of keyword search advertising into two different effects: exposure and context. The exposure effect is the typical effect of advertising that captures the change in consumers' perceptions of brand quality after being exposed to the brand's advertisement. This change may be due to subtle effects, such as mere exposure (Zajonc 1968) and the nonconscious processing effect (Ferraro et al. 2009), or the more conscious effects resulting from the information gain from the advertising text in the results page (Nedungadi et al. 1993) and/or the consumer learning after clicking on a sponsored link and visiting the advertiser's website.

The context effect comes about when consumers, consciously or unconsciously, compare the two brands. It has been shown empirically that in such situations objects (brands) seen as being very different will be perceived to be even more different by consumers after seeing them together (*contrast effect*) whereas objects (brands) perceived to be similar a priori will be seen as more similar by consumers after being exposed to them together (*assimilation effect*). (See Hovland et al. 1957 and Mussweiler 2003 for more on assimilation and contrast effects.) In our above example with Sony and Haier, it appears that consumers exhibited a contrast effect; i.e., Sony benefits when its advertisement is seen with Haier's relative to no Haier's advertisement, and the converse was found for Haier.

1.2. Overview of Results

The above leads us to develop a model of two firms selling horizontally and vertically differentiated products where the firms decide not only what price to charge but also whether to bid on (and purchase) either or both of the branded keywords in the keyword search advertising auction. Given the exposure and context effects associated with brand advertisements, each firm's incentive to buy the keyword is determined by expectations about whether or not the other firm will advertise. This leads to some interesting findings:

- Even for the low-quality firm, it is not always optimal to buy the competitor's brand name as a keyword. Firms need to consider the consequence of the vis-à-vis comparison with competitors.
- In some cases, a firm would prefer not to buy its own brand name as a keyword if the competitor is not expected to buy. However, if the competitor is expected to buy the keyword, the brand owner would find it optimal, for defense purposes, to also buy the keyword.
- With a very small exposure effect, firms never purchase generic keywords. However, the same small exposure effect may induce them to buy branded keywords because of the context effect.
- The practice of bidding on the competitor's brand name creates a prisoner's dilemma situation, and this helps the search engine earn more profits.

These results also provide insights as to (1) why brand owners might find it best to pay money to advertise under their own brand name in the sponsored section of the results page even when organic search results would prominently display that brand's website and (2) why some firms forgo the opportunity to advertise itself under well-known competitor's brand names.

We augment our model development and results with an empirical study that provides qualitative support for our model. In this study, we find that the quality difference between the brand owner and the competitor moderates the advertising decision of both firms in a way that is consistent with the prediction of our theory.

1.3. Relationship with the Existing Literature

Our paper builds on and extends two sets of literature. The first is the burgeoning analytic literature on keyword search advertising exemplified by the pioneering work of Edelman et al. (2007) and Varian (2007) and includes work on advertiser competition (Katona and Sarvary 2010), position auction mechanisms (Balachander et al. 2009, Zhu and Wilbur 2011, Jerath and Sayedi 2012, Amaldoss et al. 2014), consumer search (Athey and Ellison 2011, Chen and He 2011), position paradox (Jerath et al. 2011), and click fraud (Wilbur and Zhu 2009). The second set of literature focuses on advertisers' problems such as the optimal bidding strategy (Ghose and Yang 2009), keyword selection problem (Rutz and Bucklin 2011), and performance measurement and evaluation problem (Yang and Ghose 2010, Rutz et al. 2012, Joo et al. 2014). Our work also centers on the advertiser's problem—more specifically, the branded keyword choice problem. Although Rutz and Bucklin (2011) have investigated the spillover effect

from generic to branded keywords, suggesting the need for adjusting the effectiveness of generic keywords, there has been no research on when or if a firm should buy its own or a competitor's branded keyword when launching a search advertising campaign. The current research fills this gap.

We modify and extend these literatures in four ways. First, we deviate from the above-mentioned theoretical research, which almost always assumes the effectiveness of a sponsored link is independent of competitors' actions. We do this by allowing the advertising effect to depend in part on the presence or absence of the competitive advertisement. Jerath et al. (2011) also considered the interdependent advertising effect but in terms of the number of clicks. Second, instead of focusing on the click-generating role of search advertising, we center our attention on its role as a brand advertisement and the objective of enhancing the perceived value of the brand, a topic of significant recent interest among practitioners (Enquiro Research and Google 2007, Google and Media-Screen 2008). Third, as pointed out earlier, our model links what happens to consumer perceptions during the information search process with what happens when these consumers enter into the product market. We specifically consider the impact of the firms' advertising decisions on product market outcomes such as equilibrium prices, sales, and profits by jointly modeling the firm interactions in these two markets. Much of the search advertising literature has focused only on the search advertising market without considering this cross-market interaction.[2] Finally, we highlight the unique issues associated with the choice of branded keywords. This problem becomes more important as the connection between the branded keyword search and other media advertising gets uncovered (Online Publishers Association Europe 2009, Joo et al. 2014). In addition, recent lawsuits between Google and advertisers on the use of trademarked keyword reflect an increasing interest in the issue (Helft 2009, Orey 2009). By analyzing the effect of using one's own as well as the competitor's branded keywords, we offer insights to a practical problem in the field. (See also Chiou and Tucker 2012.)

Our investigation also extends the advertising literature to a media environment where two firms can intentionally choose to advertise in one space. Prior research on comparative advertising has highlighted the challenger's incentive to assimilate itself with the incumbent firm (Gorn and Weinberg 1984), but our model allows for both contrast and assimilation. Thus, it allows us to study both firms' incentives to strategically make use of the opportunity to be compared with the competitor.

The rest of the paper is organized as follows. In §2, we describe our model and its subcomponents. We analyze our model in §3 and provide some empirical support for the model's predictions in §4. We then conclude in §5 with the summary of findings and opportunities for future research.

2. Model

In this section, we develop a model of keyword search advertising that illustrates the relationship between the search advertising market and the product market. The basic assumption underlying our model is that a firm's advertisement can result in subtle changes in consumers' perception, either conscious or unconscious, and these changes can have a nontrivial impact on various stages of their buying process, including consideration set formation and evaluation of alternatives. As a result, the firm's demand is shifted, and thus, firms earn more or less profits. Consequently, in our model, we assume the benefit a firm seeks from search advertising is not only to generate clicks but also to increase its brand equity via increases in the perceived quality associated with its brand name (Keller 1993). This increased brand equity allows the firm to charge a higher price and also to possibly garner increased sales. This discussion is summarized in Figure 3.

In the next sections, we first describe the consumers' search process and how being exposed to different advertisements alters their quality perceptions of the two brands. We then discuss how these changed perceptions incrementally affect the two firms' demand functions. Noting that this determines the value to the firm of placing an advertisement and thus the amount they are willing to bid for the keyword in the keyword auction, we finally describe our model of the search advertising market.

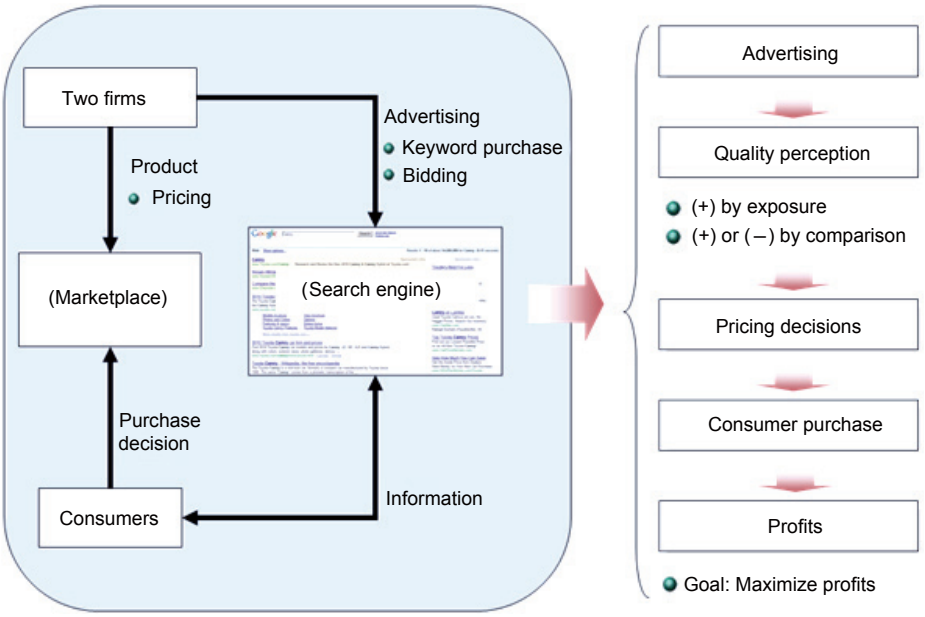
2.1. Consumer Information Search

We consider a consumer who is interested in buying a product from one of two brands, one marketed by a high-quality firm (Firm H) and the other by a low-quality firm (Firm L). She wants to conduct a search using a search engine to obtain product-related information, including product features, product availability, and store hours/locations.[3] In this search, she will

[2] Xu et al. (2011) is one exception. They explicitly investigated the interaction of the pricing decision and the bidding decision. A few other papers also consider both markets but do not explicitly link these two decisions. For example, Chen and He (2011) considered the product market competition but showed that, in equilibrium, firms set monopoly prices, following the logic of Diamond (1971). Thus, in their model, search advertising decisions have no bearing on the equilibrium price in the product market. Similarly, models of consumer search and search advertising (Athey and Ellison 2011, Jerath et al. 2011) assumed exogenous profits of firms and thus did not explicitly model the firm's reaction (in pricing) in the product market to the advertising market outcome.

[3] We thank the associate editor for suggesting this rationale. We also acknowledge that the consumer might search to obtain price

Figure 3 Overview of the Model



use a brand name of the product that she currently is interested in buying as the keyword. As discussed in more detail in the next section, the identity of this brand name is based on the relative attractiveness of the two products, which in turn is affected by her status quo perception on both products' quality: q_i ($i = \text{H, L}$), the consumer's tastes for the particular offerings, and the prices of the two products. After this information search, she might revise the quality perception and thus her choice of the product to buy based on what she observes in the process.[4]

Specifically, if she begins her search with Firm H's branded keyword, she will always find the organic link of Firm H in the search engine results page. At the same time, she may also observe Firm H's sponsored link listed on the right-hand side of the page. If this is the case, the perceived quality of Firm H's product may increase because the consumer has been exposed to Firm H's advertising message as well as its brand name. We call this increase in quality perception the exposure effect and denote its magnitude by E_e.

If Firm L buys Firm H's brand name as a keyword, then this consumer will also be exposed to Firm L's advertising message. In this case, as above, the perceived quality of Firm L's product increases by E_e (i.e., the exposure effect). In addition, because this observation is made in the context of considering Firm H's product, the consumer is naturally induced to compare the two brands. This can result in either contrast or assimilation. If the consumer perceives the quality of the two brands to be significantly different prior to her search, the two products are contrasted and thus the qualities are perceived to be more different than they are viewed in isolation. With a smaller difference in perceived quality, the two brands are assimilated, and thus, the quality difference is perceived to be even smaller. We call this change in the difference of the perceived quality the context effect and denote its magnitude by E_c. Here, positive values of E_c represent contrast while the negative values correspond to assimilation. Thus, whenever the two brands are listed together, E_c is added to the difference of the perceived quality. Finally, note that the context effect comes into play whenever Firm L buys Firm H's branded keyword regardless of whether the consumer observes Firm H's sponsored link because the organic link of Firm H always provides the comparison.

We capture the stylized fact that consumers pay less attention to the advertisement at the second slot than the first slot (Google 2009, Lew 2009) by reducing the magnitude of the exposure effect at the second slot by ε. This will become important when we determine the order of the sponsored advertisements in our model of the keyword auction. Finally, when the consumer observes no advertisement in the search results page, her current quality perception remains unchanged.

information as well. Although this case adds some complexity to the model, we find that the qualitative results do not change.

[4] We show in the next section how consumers choose the brand to search with and to buy, based on the utility of each product. Although our primary focus is on the consumer who, toward the end of her search process, looks for more specific information about a certain brand(s) and thus searches with branded keyword(s), we also consider the case of searching with generic keywords (category names such as car, e-book, etc.) in our later analysis.

In cases where the consumer conducts a search with Firm L's branded keyword, she alters her quality perception in the same way. In particular, with any advertisement, the exposure effect increases the perceived quality of the advertised brand by either E_e or $(1-\varepsilon)E_e$ (depending on the position of the advertisement), whereas the context effect increases the quality difference by E_c (either positive or negative) assuming Firm H buys Firm L's branded keyword.

Next, we acknowledge that there is variance in the number of searches across consumers. It is well documented that some consumers seek little information, limiting their search to one store or outlet, whereas others seek information from multiple outlets (Newman and Staelin 1971, Johnson et al. 2004). We capture this stylized fact by assuming that after conducting an information search starting with one firm's branded keyword, some consumers may make a purchase decision, and others may continue their information search with another keyword—in our case, the branded keyword of the other firm. We call this latter type of consumers "comparison shoppers" and assume the size of this segment to be α. Because comparison shoppers are exposed to the search result pages of both branded keywords, they will be influenced by advertisements under both keywords.[5] However, noncomparison shoppers, which represent a $(1-\alpha)$ portion of the population, search with only one firm's branded keyword and thus will be affected only by the advertisements under their interested brand's keyword.

2.2. Consumer Behavior in the Product Market

Based on the revised quality perception from their search, consumers derive utility from buying the product in the market. Because of different information search experiences, different segments derive different utilities. First, we assume that in each segment, consumers are uniformly distributed along a unit line with the two products located at the ends of this line segment (Hotelling 1929). Then in segment S ($S=C$: comparison shoppers or $S=N$: noncomparison shoppers), a consumer located at point x obtains utility from each product at the time of purchase decision as follows:

$$U_{\mathrm{H}}^{S} = \tilde{q}_{\mathrm{H}}^{S} - p_{\mathrm{H}} - tx, \tag{1}$$

$$U_{\mathrm{L}}^{S} = \tilde{q}_{\mathrm{L}}^{S} - p_{\mathrm{L}} - t(1-x), \tag{2}$$

where t is the transportation cost, p_i is the price of the product, and $\tilde{q}_i^S$ refers to the perceived quality of each product after the information search ($i=$ H, L). We additionally assume $q_{\mathrm{H}} + q_{\mathrm{L}} > 3t + |E_c|$, which guarantees that the market is fully covered regardless of the advertising outcomes. Thus we can easily derive each firm's demand in each segment as follows:

$$D_{\mathrm{H}}^{S} = \frac{1}{2} + \frac{\Delta\tilde{q}^{S} - p_{\mathrm{H}} + p_{\mathrm{L}}}{2t}, \tag{3}$$

$$D_{\mathrm{L}}^{S} = \frac{1}{2} - \frac{\Delta\tilde{q}^{S} - p_{\mathrm{H}} + p_{\mathrm{L}}}{2t}, \tag{4}$$

where $\Delta\tilde{q}^S$ is the difference in the perceived quality in segment S (i.e., $\Delta\tilde{q}^S \equiv \tilde{q}_{\mathrm{H}}^S - \tilde{q}_{\mathrm{L}}^S$), which we derive below. Before doing so, we introduce additional notation to describe various advertising scenarios; we denote outcomes in the two keyword auctions by $K[A_{\mathrm{H}}A_{\mathrm{L}}]$, where $K(=$ H, L) represents the keyword being auctioned and A_i shows the existence (Y) or nonexistence (N) of Firm i's advertisement.[6] Thus, for instance, H[YN] implies that in the auction for Keyword H, Firm H buys the keyword and Firm L does not.

In the noncomparison shopper segment, prior to search, $U_{\mathrm{H}}^N \geq U_{\mathrm{L}}^N$ is equivalent to $x \leq x^0 \equiv \frac{1}{2} + (\Delta q - (p_{\mathrm{H}} - p_{\mathrm{L}}))/(2t)$, where $\Delta q \equiv q_{\mathrm{H}} - q_{\mathrm{L}}$, because $\tilde{q}_i^{\mathrm{N}}$ is given by q_i prior to search. Thus, in this segment, consumers located to the left of x^0 (i.e., $x \leq x^0$) choose Firm H's branded keyword in their search, and those to the right of x^0 (i.e., $x > x^0$) search with Firm L's brand name. Recall that search advertising can affect the difference in the perceived qualities as discussed in the previous section. Table 2 summarizes how it changes by advertising scenario. For those searching with Keyword H, based on the consumer search process discussed in the previous section, the difference in the perceived quality is given by

$$\Delta\tilde{q}^{N|\mathrm{H}} = \begin{cases} \Delta q & \text{without any advertisement, H[NN];} \\ \Delta q + E_e & \text{with an advertisement of only Firm H, H[YN];} \\ \Delta q - E_e + E_c & \text{with an advertisement of only Firm L, H[NY];} \\ \Delta q + \varepsilon E_e + E_c & \text{with both firms listed by order of H} \rightarrow \text{L, H[Y}_1\text{Y}_2\text{];} \\ \Delta q - \varepsilon E_e + E_c & \text{with both firms listed by order of L} \rightarrow \text{H, H[Y}_2\text{Y}_1\text{].} \end{cases} \tag{5}$$

[5] We clarify that the consumers who search for two keywords are not engaged in a sequential search process as in Weitzman (1979). Because they are interested in searching for information about both products, their decision to search for the second keyword does not depend on the information they obtain about the first keyword.

[6] When both firms advertise together under a keyword, we indicate the slot each firm takes by a subscript. For example, if Firm H takes the first slot and Firm L takes the second slot under Keyword H, we denote this case by H[Y_1Y_2]. Since, as we prove in Lemma A2 in the appendix, Firm H always takes the first slot in equilibrium, we drop the subscript in this notation when we discuss the equilibrium results.

Table 2 Effects of Firms' Advertising Decisions on Consumers' Perceptions of Product Qualities

Brand owner's decision about buying its keyword	Competitor's decision about buying the brand owner's keyword	
	Yes	No
Yes	Exposure effects for both brands Assimilation or contrast effects	Exposure effect for brand owner No assimilation or contrast effects
No	Exposure effect for competitor's brand Assimilation or contrast effects	None

For the others searching with Keyword L,

$$\Delta\tilde{q}^{N|\mathrm{L}} = \begin{cases} \Delta q & \text{without any advertisement, L[NN];} \\ \Delta q + E_e + E_c & \text{with an advertisement of only Firm H, L[YN];} \\ \Delta q - E_e & \text{with an advertisement of only Firm L, L[NY];} \\ \Delta q + \varepsilon E_e + E_c & \text{with both firms listed by order of H} \to \text{L, L[Y}_1\text{Y}_2\text{];} \\ \Delta q - \varepsilon E_e + E_c & \text{with both firms listed by order of L} \to \text{H, L[Y}_2\text{Y}_1\text{].} \end{cases} \quad (6)$$

Now define $x^K \equiv \frac{1}{2} + (\Delta\tilde{q}^{N|\mathrm{K}} - p_\mathrm{H} + p_\mathrm{L})/(2t)$ $(K = \mathrm{H}, \mathrm{L})$. Then the demand from the noncomparison shopper segment is given as $\min\{x^0, x^\mathrm{H}\} + \max\{0, x^\mathrm{L} - x^0\}$ for Firm H, where the first term captures consumers who use Keyword H and the second term captures those who use Keyword L. Similarly, the demand for Firm L is $\min\{1 - x^0, 1 - x^\mathrm{L}\} + \max\{0, x^0 - x^\mathrm{H}\}$.

In the comparison shopper segment, every consumer uses both keywords in his or her search and hence is affected by advertising decisions made for both keywords. For this segment, the total change in quality perceptions is determined by addition of the effects arising from each keyword search. More precisely,

$$\Delta\tilde{q}^{C} = \Delta q + (\Delta\tilde{q}^{\mathrm{C}|\mathrm{H}} - \Delta q) + (\Delta\tilde{q}^{\mathrm{C}|\mathrm{L}} - \Delta q), \quad (7)$$

where $\Delta\tilde{q}^{\mathrm{C}|\mathrm{H}}$ is defined in the same way as $\Delta\tilde{q}^{N|\mathrm{H}}$ in Equation (5), and $\Delta\tilde{q}^{\mathrm{C}|\mathrm{L}}$ is defined in the same way as $\Delta\tilde{q}^{N|\mathrm{L}}$ in Equation (6). To consider the effect of search advertising in a reasonable parameter space, we assume that both $\Delta\tilde{q}^{C} \geq 0$ and $\Delta\tilde{q}^{N|K} \geq 0$ hold; i.e., the advertising cannot reverse the overall quality order.

Then we can easily derive each firm's demand by the weighted average of demands from both segments, and based on this, the profits of both firms are as follows:

$$\Pi_\mathrm{H} = p_\mathrm{H}\left\{\alpha\left(\frac{1}{2} + \frac{\Delta\tilde{q}^{C} - p_\mathrm{H} + p_\mathrm{L}}{2t}\right) + (1-\alpha)(\min\{x^0, x^\mathrm{H}\} + \max\{0, x^\mathrm{L} - x^0\})\right\} - C_\mathrm{H}, \quad (8)$$

$$\Pi_\mathrm{L} = p_\mathrm{L}\left\{\alpha\left(\frac{1}{2} - \frac{\Delta\tilde{q}^{C} - p_\mathrm{H} + p_\mathrm{L}}{2t}\right) + (1-\alpha)(\min\{1 - x^0, 1 - x^\mathrm{L}\} + \max\{0, x^0 - x^\mathrm{H}\})\right\} - C_\mathrm{L}, \quad (9)$$

where C_i refers to the total advertising cost of firm i $(i = \mathrm{H}, \mathrm{L})$. As we show subsequently, C_i depends on who else is bidding for the keywords of their choice. Note that Firm H's profits (weakly) increase with the difference in the quality perception between the two firms, i.e., $\Delta\tilde{q}^{C}$ and $\Delta\tilde{q}^{N|K}$, whereas Firm L's profits (weakly) decrease with it. Thus, it is in the best interests of Firm H to increase this difference in perceived quality, but Firm L wants to decrease this difference. Advertising is one way of altering this perceived difference. We discuss this next.

2.3. Firm Behavior in the Search Advertising Market

In the advertising market, firms decide whether to buy the two branded keywords: Keyword H and Keyword L. There are four possible strategies for each firm: buying both keywords, buying only Keyword H, buying only Keyword L, and buying neither. Based on this buying decision, consumers searching with that keyword will (or will not) be exposed to their advertising messages.

As is true in practice, we assume the search engine holds a separate auction for each keyword. Thus, if any firm decides to buy any keyword, it makes a bid in the auction for that keyword. In our model of the keyword search auction, there are three slots available for three potential participants: Firm H, Firm L, and an additional advertiser Firm X. This last advertiser is not a product market competitor but still values a slot under a branded keyword. To focus on the interaction between Firm H and Firm L, we assume that Firm X has very low relevance and/or valuation and thus always remains in the last slot.[7] The three

[7] The sufficient and necessary condition for this assumption is that the additional profit from moving up by one slot is higher for Firm H or Firm L than Firm X. We derive this condition in the appendix. However, the model does not require Firm X; Firm X with an exogenous bid can be replaced by an exogenous minimum bid of the auction.

slots in the auction have different click-through rates: s_j, with $s_1 > s_2 > s_3$. In addition, Advertiser i differs in terms of its relevance to Keyword K: r_i^K ($i =$ H, L, X; $K =$ H, L), which is also called the quality score by Google. For simplicity, the relevance is assumed to be the same as the advertiser-specific click-through rate, and thus, if Advertiser i takes Slot j, it can collect $s_j r_i^K$ clicks. In the bidding game for Keyword K, each advertiser submits its bid b_i^K based on its valuation of the advertisement in terms of eventual incremental sales. Then the search engine ranks the advertisers by the product of relevance r_i^K and bid amount b_i^K and assigns the slots according to this rank. Under the generalized second-price auction rule, each advertiser's per-click payment is the minimum price that guarantees that the advertiser would be awarded its slot. In particular, if Advertiser i takes Slot j, it has to pay $(r^K_{(j+1)} b^K_{(j+1)})/r_i^K$ per click, where $(j+1)$ refers to the advertiser at Slot $j+1$. Therefore, the total advertising cost under each keyword, C_i^K ($K =$ H, L), is given as follows, depending on the realized scenarios:

- When advertising alone, Firm i collects at the first slot $s_1 r_i^K$ clicks and pays per click $(r_X^K b_X^K)/r_i$, and thus the total cost is $C_0^K \equiv s_1 r_i^K ((r_X^K b_X^K)/r_i^K) = s_1 r_X^K b_X^K$.
- When both firms advertise, if Firm i wins the first slot, Firm i collects at the first slot $s_1 r_i^K$ clicks and pays per click $(r_{i'}^K b_{i'}^K)/r_i^K$, resulting in $C_{i1}^K \equiv s_1 r_i^K ((r_{i'}^K b_{i'}^K)/r_i^K) = s_1 r_{i'}^K b_{i'}^K$, where i' refers to the other firm.
- When both firms advertise, if Firm i loses to the other firm and thus gets the second slot, Firm i collects $s_2 r_i^K$ clicks and pays $(r_X^K b_X^K)/r_i^K$ per click, and thus the total cost of advertising is $C_2^K \equiv s_2 r_i^K ((r_X^K b_X^K)/r_i^K) = s_2 r_X^K b_X^K$.

Note that $C_{i1}^K > C_0^K > C_2^K$ and that both C_0^K and C_2^K remain the same across the two firms. Finally, the total advertising cost across the two keywords is given as the sum, $C_i \equiv C_i^{\mathrm{H}} + C_i^{\mathrm{L}}$ ($i =$ H, L). This is the aggregate cost found in Equations (8) and (9). Finally, the click-through rate in our model is not directly connected with product demands. The current structure is an acknowledgment of the fact that context effects can affect consumers' purchase decisions independent of consumers' clicking behavior. For instance, it is possible that consumers are affected by context effects created by sponsored links, but they may click on organic links. In addition, consumers may click on two sponsored links but end up buying only one of the two products.

2.4. Order of Events

There are two stages of firms' decisions in our model. In the first stage, the two firms simultaneously set prices for their own products in the product market, with an anticipation of the advertising outcome. In the second stage, they decide whether to buy each of the branded keywords and, if they buy, simultaneously submit their bids in the keyword auction for the chosen keyword(s). This order is based on the notion that keyword search advertising decisions can be made in real time. However, our results are robust to the alternative order. As a result of firms' advertising decisions, whenever consumers search with branded keywords, participating firms' advertisements are shown to consumers, together with the organic search results. This alters their quality perceptions and, potentially, their choice of the brand to buy.

In every stage, firms have complete information. Thus, we derive a Nash equilibrium in every subgame and a subgame perfect Nash equilibrium in the full game. Also note that in the bidding subgame, we consider a more stringent set of Nash equilibria, a symmetric Nash equilibrium, where no firm has an incentive to switch its position with anyone else (Varian 2007). Even in this setup, the equilibrium bid is not unique, so we further refine the equilibrium by considering the upper bound of the solution. However, our main results are robust to this choice. Finally, Table 3 summarizes the notation used in our model.

Table 3 Summary of Notation

q_i, Δq	Status quo perceived quality of Firm i ($i =$ H, L) and their difference
$\tilde{q}_i^S$, $\Delta\tilde{q}^S$	Perceived quality of Firm i ($i =$ H, L) and their difference after advertising in segment S
E_e	Magnitude of exposure effect
E_c	Magnitude of context effect
ε	Difference in exposure effect between the first and the second slot
t	Transportation cost
p_i	Price of product i
D_i^S	Demand for product i in segment S ($S = C, N$)
Π_i	Profit of Firm i
C_i, C_i^K	Total advertising cost to Firm i and cost under keyword K ($K =$ H, L)
s_j	Slot-specific click-through rate in slot j
r_i^K	Relevance (or advertiser-specific click-through rate) of Firm i for Keyword K
b_i^K	Bid amount of Firm i in an auction for Keyword K
$C_0^K(C_0)$	Advertising cost of Firm i when advertising alone
C_{H1}^K	Advertising cost of Firm i when taking the first slot under the K[YY] scenario ($K =$ H, L)
$C_2^K(C_2)$	Advertising cost of Firm i when taking the second slot under the K[YY] scenario ($K =$ H, L)

3. Analysis

We solve the game using backward induction. Our primary interest is in examining the choice of specific keywords, so we focus on each firm's decision to purchase two branded keywords in this section. However, it is worthwhile to note that, in the bidding equilibrium, Firm H always takes the first slot under every keyword. Intuitively, this is because Firm H can generate greater profits from the same increase in the exposure benefit when moving up by one slot because of its higher price. We prove this claim in Lemma A2 in the appendix and provide additional details about bid amounts and other outcomes also in the appendix.

3.1. Branded Keyword Choice

3.1.1. Equilibrium. In the second stage of the game, firms decide whether to bid on a specific keyword. We consider brand names of the two competitors as potential keywords that can be purchased by each firm. Because each keyword's auction is held separately by the search engine, the outcome in each auction is independent of that in the other auction.[8] In each of the two auctions, the advertising outcomes can be characterized in terms of only brand owner advertising, only the competitor advertising, and both firms advertising. In this section, we present the conditions under which each of these outcomes would be observed in equilibrium. At the advertising stage, because prices had already been chosen, the conditions for each keyword choice equilibrium are based on the predetermined prices.[9] We also note that, unlike most traditional advertising media models, the costs and the benefits of search advertising are affected by the other firm's advertising decision because of the bidding process and the close proximity of the two advertisements. Therefore, our results describe how the advertising effectiveness is affected by the competitor's strategies as well as their own, especially through the exposure and context effects. (For ease of exposition, all proofs are provided in the appendix.)

Proposition 1 (Only Brand Owner Advertising). *When the exposure effect is moderately large but the context effect is small in either direction, only the brand owner's advertisement appears under each keyword. More formally, when* $E_e \geq \frac{2tC_0}{\alpha P_L}$ $(\geq \frac{2tC_0}{\alpha P_H})$ *and* $\max\{(1-\varepsilon)E_e - \frac{2tC_2}{\alpha P_L}, (\alpha-\varepsilon)E_e - \frac{2tC_2}{P_L}\} \leq E_c \leq \min\{-(1-\varepsilon)E_e + \frac{2tC_2}{\alpha P_H}, -(\alpha-\varepsilon)E_e + \frac{2tC_2}{P_H}\}$ *hold, L[NY] and H[YN] are obtained as the keyword choice equilibrium.*[10]

The above proposition shows that each branded keyword is purchased only by the brand owner when the exposure effect (E_e) is large but the context effect (E_c) is small in both assimilation and contrast forms. Given that the size of both types of context effects is determined by the difference of the perceived quality, we can observe this equilibrium when the status quo quality difference is neither very large nor very small. To see the intuition for these conditions, note that with moderately large exposure effect, both firms have strong incentives to buy their own keyword unilaterally. However, a weak context effect also reduces each advertiser's incentives to buy each other's brand names. The reason is that if advertisers buy their competitor's keyword, the exposure benefit will be canceled out by the brand owner's purchase of the same keyword, but the context effect will not provide large enough gain to compensate for this loss.

The conditions in Proposition 1 can also be decomposed into the condition for each firm's decisions. In particular, the conditions involving p_L are for Firm L's decisions, whereas those including p_H are for Firm H. A simple inspection of these conditions reveals that at the same level of the exposure effect, Firm H is more likely to purchase its own brand name than Firm L. This is because the same amount of the advertising effect can be translated into larger profits for Firm H as a result of the higher price. For the same reason, at the same level of the favorable context effect (i.e., contrast for Firm H and assimilation for Firm L), Firm L is more likely to give up buying the competitor's brand name. Together, these imply that the equilibrium for Keyword H (i.e., H[YN]) is more likely to be sustained than that for Keyword L (i.e., L[NY]). Next we discuss the case in which the decisions of both firms are flipped from the first proposition.

Proposition 2 (Only Competitor Advertising). (a) *When the exposure effect is small but the contrast effect is relatively large, only Firm H's advertisement appears under Keyword L. More formally, when* $E_e \leq \frac{2tC_2}{(1-\varepsilon)P_L}$ *and* $E_c \geq -E_e + \frac{2tC_0}{P_H}$ *hold, L[YN] is obtained as the keyword choice equilibrium.* (b) *When the exposure effect is small but the assimilation effect is relatively large, only Firm L's advertisement appears under Keyword H. More formally, when* $E_e \leq \frac{2tC_2}{(1-\varepsilon)P_H}$ *and* $E_c \leq E_e - \frac{2tC_0}{P_L}$ *hold, H[NY] is obtained as the keyword choice equilibrium.*

The conditions in Proposition 2 require that the exposure effect is small but the relevant context effect is relatively large—a large contrast for Keyword L ($E_c \geq -E_e + \frac{2tC_0}{P_H}$) and large assimilation for Keyword H ($E_c \leq E_e - \frac{2tC_0}{P_L}$).[11] The context effect conditions further suggest that the result is likely to occur when the status quo quality difference between the two advertisers is sufficiently large or sufficiently small. Interestingly, without the context effect, neither firm would buy any keyword at such a low level of exposure effect. Thus, the existence of this equilibrium shows that firms can be motivated to advertise mainly by the context effect. However, each firm cannot create the context effect by itself and, thus, will never buy its own brand name under the conditions of the proposition. In contrast, if a firm buys the brand name of its competitor, this would create the context effect even without the brand owner's advertisement. This is because consumers

[8] The situation is different for advertisers' pricing decisions, which are based on outcomes in both auctions.

[9] In the next subsection, we also provide the conditions based on relevant prices in each advertising scenario.

[10] Given that C_0^K and C_2^K remain the same across the two keywords, we use C_0 and C_2 for notational convenience.

[11] Recall that E_c represents the contrast effect when positive but the assimilation effect when negative.

searching for a branded keyword will have top-of-the-mind awareness of the brand owner. In addition, the brand owner's link will also be present in the organic results provided by the search engine. Thus, if the favorable context effect exists, each firm will consider buying the competitor's keyword.

Another interesting aspect of the equilibrium in Proposition 2 is that exposure and context effects work together for the advertiser whose advertisement appears under each keyword. More specifically, in this case, the advertising firm gets benefits of both the exposure effect and the context effect. As a result, as the exposure effect grows stronger, they do not need much benefit from the context effect. This is evident from the observation that the threshold of the context effect decreases as the exposure effect increases: $E_c \geq -E_e + \frac{2tC_0}{P_H}$ for the contrast effect in the L[YN] equilibrium and $(-E_c) \geq -E_e + \frac{2tC_0}{P_L}$ for the assimilation effect in the H[NY] equilibrium. We next consider the equilibrium in which both advertisers buy both keywords.

Proposition 3 (Both Firms Advertising). (a) *When the exposure effect is large and the contrast effect is large, both firms' advertisements appear under Keyword L. More formally, when* $E_e \geq \frac{2tC_2}{(1-\varepsilon)P_L}$ *and* $E_c \geq \min\{-(1-\varepsilon)E_e + \frac{2tC_2}{\alpha P_H}, -(\alpha-\varepsilon)E_e + \frac{2tC_2}{P_H}\}$ *hold, L[YY] is obtained as the keyword choice equilibrium.* (b) *When the exposure effect is large and the assimilation effect is large, both firms' advertisements appear under Keyword H. More formally, when* $E_e \geq \frac{2tC_2}{(1-\varepsilon)P_H}$ *and* $E_c \leq \max\{(1-\varepsilon)E_e - \frac{2tC_2}{\alpha P_L}, (\alpha-\varepsilon)E_e - \frac{2tC_2}{P_L}\}$ *hold, H[YY] is obtained as the keyword choice equilibrium.*

Proposition 3 considers the case when consumers can observe both firms' advertisements under each keyword. The intuition for the conditions can be understood by considering each advertiser's incentive to buy a given keyword. For instance, the proposition shows that the L[YY] equilibrium is possible when both the exposure effect and the contrast effect are sufficiently large. For Firm H, buying Keyword L can bring benefits of contrast as well as exposure effects. As these effects become stronger, Firm H will have more incentive to buy Keyword L. However, anticipating this incentive, Firm L also has an incentive to purchase its own brand name so that it can offset some of the potential loss resulting from Firm H's purchase of Keyword L through the exposure benefit. The argument is reversed for the equilibrium under Keyword H, but the intuition remains the same.

The above discussion suggests that the brand owner derives some defensive benefits from purchasing its own brand name. It can be shown that, under some conditions, the brand owner buys the keyword solely for defensive reasons. Specifically, if $\frac{2tC_2}{(1-\varepsilon)P_L} \leq E_e \leq \frac{2tC_0}{\alpha P_L}$ and $E_c \geq \min\{-(1-\varepsilon)E_e + \frac{2tC_2}{\alpha P_H}, -(\alpha-\varepsilon)E_e + \frac{2tC_2}{P_H}\}$ hold, Firm L prefers not to buy its own brand name by itself because the exposure benefit is too small. However, at this level of the exposure effect, Proposition 3(a) suggests that Firm L chooses to advertise under its own brand name in equilibrium together with Firm H. Therefore, we can deduce that the only motivation for this strategy is to defend its own consumers against Firm H's encroachment. We call this case a defensive purchase of the brand owner. Similarly, if $\frac{2tC_2}{(1-\varepsilon)P_H} \leq E_e \leq \frac{2tC_0}{\alpha P_H}$ and $E_c \leq \max\{(1-\varepsilon)E_e - \frac{2tC_2}{\alpha P_L}, (\alpha-\varepsilon)E_e - \frac{2tC_2}{P_L}\}$ hold, Firm H does not buy its own brand name on its own but only buys together with the competitor. Thus, both firms can engage in purely defensive purchase of their own brand names. Our discussion of defensive purchases raises the question if such purchases can lead to prisoner's dilemma situations. We examine this issue next.

3.1.2. Prisoner's Dilemma. As suggested above, firms can be trapped into the situation where they are forced to advertise because of a competitive threat. Proposition 4 below shows that this not only leads to a prisoner's dilemma outcome but also provides interesting strategic benefits to the search engine.

Proposition 4 (Prisoner's Dilemma 1). *The ability to bid on the competitors' brand names as keywords can lead to prisoner's dilemma outcomes for the advertisers. In such cases, the search engine benefits by enjoying higher profits.*

Proposition 4 first confirms that both firms' profits may be lower when both advertise under the same branded keyword than when neither advertises. In particular, we find the prisoner's dilemma case when $-\varepsilon E_e - \frac{2tC_0}{\alpha p_L} \leq E_c \leq \varepsilon E_e + \frac{2tC_0}{p_H}$ in the L[YY] equilibrium and when $-\varepsilon E_e - \frac{2tC_0}{p_L} \leq E_c \leq \varepsilon E_e + \frac{2tC_0}{\alpha p_H}$ in the H[YY] equilibrium—that is, when the context effect is not large in either direction. This implies that both firms cannot benefit enough from the mild context effects but are still forced to advertise by competitive pressure and, thus, become worse off by doing so. Interestingly, the conditions for the "mild" context effect are different across the two keywords. In particular, each firm is more likely to be worse off with advertising when it bids on its own brand name than the competitor's. This is because the advertisement under its own brand name is in general less effective in increasing demand than that under the competitor's brand name because of its inability to reach to the noncomparison shoppers who are interested in buying from the competitor. Thus, when firms advertise under their own brand name, they are less likely to recover the cost and thus more likely to be worse off with advertising.

More important, Proposition 4 also indicates that the prisoner's dilemma is a consequence of the search engine's policy to allow advertisers to bid on the brand

names of their competitors. Without such a policy, both firms will buy their own branded keyword only when they can improve their profits from doing so. However, under this policy, because of the possibility of competitors buying the branded keyword of the focal firm, the focal firm may be induced to buy the keyword even at a loss. The lost profit in this case is captured as the search engine profit. Thus, although the search engine provides an opportunity for highly effective advertising, here we see how it can create opportunities for itself to earn more profits. In the current context, by allowing competing firms to bid on each other's brand names as keywords, the search engine can engender a prisoner's dilemma among the competitors and thus increase its own profits. It is this intentional mechanism that differentiates the prisoner's dilemma situation in search advertising from those in other advertising media.

Yet another, perhaps more interesting, case of the prisoner's dilemma from the L[NY] equilibrium exists as well. Recall from Proposition 1 that this equilibrium exists when $E_e \geq \frac{2tC_0}{\alpha P_L}$ and $E_c \leq -(1-\varepsilon)E_e + \frac{2tC_2}{\alpha P_H}$ hold. Then we have the following proposition.

Proposition 5 (Prisoner's Dilemma 2). *When only Firm L buys Keyword L in equilibrium, both firms' profits may be lower than if it were the competitor (i.e., Firm H) that solely purchases Keyword L. This happens if* $\frac{2tC_0}{p_H} \leq (1+\alpha)E_e + E_c \leq \frac{2tC_0}{p_L}$ *holds.*

Recall that Proposition 4 describes a case where merely being able to bid on the competitor's brand name makes both firms worse off. Proposition 5 shows a somewhat different scenario: bidding on the competitor's keyword could make firms better off, but in equilibrium, this does not happen. In this case, it is worthwhile to note that the competitor's purchase could benefit both firms more than the brand owner's purchase does. Under the conditions of the proposition and the L[NY] equilibrium, if the competitor (i.e., Firm H) buys the keyword, the brand owner (i.e., Firm L) can be better off because the two firms' qualities can be assimilated. However, in the L[NY] equilibrium, there is no such assimilation. At the same time, this can also make Firm H better off because the exposure effect it gets from all the consumers searching for Firm L's brand name is large enough to compensate for the loss from the assimilation. Together, the above proposition suggests that even though in equilibrium only the brand owner (Firm L) advertises, it might be better for both firms if only the competitor (Firm H) advertises.

However, the same argument does not hold for the other keyword (i.e., Keyword H). In the H[YN] equilibrium, only the brand owner advertises. If, however, the competitor was the only buyer of the keyword, this does not result in both firms being better off at the same time, compared with the equilibrium. To see the reason, we need to compare profits associated with the equilibrium (i.e., H[YN]) and the alternative scenario (i.e., H[NY]) for both firms. For Firm H to obtain greater benefit in the latter situation than the former, the contrast effect needs to be bigger than the exposure effect. However, the level of the contrast effect that makes Firm H better off with the competitor's purchase is so large that Firm L experiences a loss that cannot be compensated with the exposure effect under the condition of the H[YN] equilibrium.[12] Thus, it is impossible for both firms to be better off under Firm L's purchase of Keyword H. Therefore, we observe this type of prisoner's dilemma only under Keyword L.

3.1.3. Branded vs. Generic Keywords. So far, we have focused our attention only on branded keywords. However, another important type of keyword exists: generic keywords. In this section, we examine both firms' advertising incentives under generic keywords to highlight the difference between generic and branded keywords. To focus on this issue, in our analysis, we ignore other distinct roles of generic keywords, such as increasing brand awareness and inducing subsequent branded keyword search (Rutz and Bucklin 2011). We also assume that, under a generic keyword, neither brand will appear prominently in the organic search section, which is indeed the case for most generic keywords. Also note that, with generic keywords, there is no ex ante segmentation of consumers based on their search behavior. Thus, we consider one group of consumers behaving similarly to comparison shoppers. The following proposition highlights divergent equilibrium results between the two types of keywords.

Proposition 6 (Generic Keywords). (a) *When the exposure effect is very small, neither firm purchases any generic keyword in equilibrium, but both can buy the branded keyword of the competitor.* (b) *When the exposure effect is large and the context effect is large in either direction (i.e., large contrast or large assimilation), both firms will not purchase the same generic keyword at the same time in equilibrium, but they can buy the same branded keyword.*

According to Proposition 2, both firms will, under certain conditions, buy each other's branded keyword, despite the small exposure effect. However, part (a) of Proposition 6 suggests that this is not the case with generic keywords. This is because consumers searching with generic keywords are unlikely to have any brand in mind, and thus any single brand's advertisement shown in the search results cannot create the context effect. Thus, the advertising decision under generic

[12] Because in equilibrium Firm H takes a higher slot than Firm L, Firm L's exposure benefit is less than that of Firm H.

keywords solely depends on the direct benefit from exposure. With small exposure effect, neither firm initiates advertising without the other's decision to advertise.

Part (b) of Proposition 6 shows another comparison when the exposure effect is large. In this case, recall from Proposition 3 that the large contrast effect leads both firms to buy Keyword L, and the large assimilation effect induces both to advertise under Keyword H. In these equilibria, both firms' incentive to buy the brand name of the competitor is enhanced by the favorable context effect. However, the brand owner's advertising decision is not affected by the context effect because it occurs regardless of its decision. Thus, the brand owner may buy even at the unfavorable context effect. In contrast, with generic keywords, the unfavorable context effect can be easily avoided by not advertising together with the competitor. Thus, both firms can never advertise together under generic keywords if the context effect is large in either direction.

3.2. Pricing Equilibrium

Thus far, we have derived and examined each equilibrium of branded keyword choice while taking prices as given. To complete the equilibrium analysis, we finally derive the pricing equilibrium. Since firms choose prices with anticipation of advertising outcomes, equilibrium prices vary depending on the advertising scenario. In the appendix, we present the equilibrium prices for all possible advertising scenarios.

Based on the equilibrium prices, we can also rewrite the conditions for each proposition of the previous subsection. Because equilibrium prices are functions of advertising outcomes under both keywords, note that the equilibrium conditions for one keyword are affected by what equilibrium is expected to be played for the other keyword. However, for simplicity, we present the conditions for each keyword by keeping the decisions on the other keyword at the most interesting and likely equilibrium. Table 4 shows these conditions, where the first column presents the original conditions with endogenous prices plugged in and the second column shows the no-deviation condition at the pricing stage. Note that rewriting the conditions does not alter any insights from the propositions.

On further examining these conditions, we find an interesting role of the comparison shopper segment, which we summarize below.

Proposition 7 (Comparison Shoppers). *As the size of the comparison shopper segment increases*, (a) *both firms have greater incentive to bid on their own brand name but only when the competitor is not expected to bid on the same keyword*; (b) *they also have greater incentive to bid on the competitor's brand name but only when the competitor is expected to bid on the same keyword*.

This proposition discusses what happens to the firms' incentives to advertise when the product market becomes more competitive as a result of the increase in the number of comparison shoppers. At first blush, it may seem that in such a situation each advertiser will have a greater incentive to advertise in order to try to influence these consumers. However, Proposition 7 shows that this is not always the case; the advertising incentives can indeed be positively affected by the size of the comparison shopper segment (i.e., α), but the occurrence of this depends on the competitor's advertising decision.

To understand part (a) of Proposition 7, note that just by advertising under one's own brand name, neither firm can take any consumer away from the competitor in the noncomparison shopper segment. This is because noncomparison shoppers initially interested in buying from the competitor will never encounter the focal firm's brand in their search in this situation. However, in the comparison shopper segment, it is possible to encroach on the competitor's market because consumers use both keywords in their search. Therefore, when the competitor does not bid on the focal firm's brand name and thus the focal firm can indeed increase the market share in the comparison shopper segment by buying its own brand name as

Table 4 Conditions for Equilibria in Propositions 1–3

Equilibrium	Conditions with endogenous prices	No-deviation conditions
H[YN] (with L[NY])	• $6tC_1 \le (3t+\Delta q)\alpha E_e$	• $18tC_1 \le (6t+2\Delta q-\alpha E_e)\alpha E_e$
	• $6tC_2 \ge \alpha(3t-\Delta q)\{(1-\varepsilon)E_e-E_c\}$	• $18tC_2 \ge \{6t+(\alpha-\varepsilon-2\alpha\varepsilon)E_e-(2\alpha+1)E_c\}\{-(\alpha-\varepsilon-2\alpha\varepsilon)E_e+(2\alpha-1)E_c\}$
L[NY] (with H[YN])	• $6tC_1 \le (3t-\Delta q)\alpha E_e$	• $18tC_1 \le (6t-2\Delta q-\alpha E_e)\alpha E_e$
	• $6tC_2 \ge \alpha(3t+\Delta q)\{(1-\varepsilon)E_e+E_c\}$	• $18tC_2 \ge \{6t+(\alpha-\varepsilon-2\alpha\varepsilon)E_e+(2\alpha+1)E_c\}\{-(\alpha-\varepsilon-2\alpha\varepsilon)E_e-(2\alpha-1)E_c\}$
H[NY] (with L[NN])	• $6tC_1 \le (E_e-E_c)(3t-\Delta q-E_e-E_c)$	• $18tC_1 \le (E_e-E_c)\{6t-2\Delta q+E_e-E_c\}$
	• $6tC_2 \ge (1-\varepsilon)E_e(3t+\Delta q-E_e+E_c)$	• $18tC_2 \ge (3\varepsilon-1)E_e\{6t+2\Delta q-(1+3\varepsilon)E_e+2E_c\}$
L[YN] (with H[NN])	• $6tC_1 \le (E_e+E_c)(3t+\Delta q+E_e+E_c)$	• $18tC_1 \le (E_e-E_c)\{6t+2\Delta q+E_e+E_c\}$
	• $6tC_2 \ge (1-\varepsilon)E_e(3t-\Delta q-E_e-E_c)$	• $18tC_2 \ge (3\varepsilon-1)E_e\{6t-2\Delta q-(1+3\varepsilon)E_e-2E_c\}$
H[YY] (with L[NN])	• $6tC_2 \le (1-\varepsilon)E_e(3t+\Delta q-3\varepsilon E_e+E_c)$	• $18tC_2 \le (1-3\varepsilon)E_e\{6t-2\Delta q-(1+3\varepsilon)E_e-2E_c\}$
	• $6tC_2 \le \{(\alpha-\varepsilon)E_e-E_c\}(3t-\Delta q-3\varepsilon E_e-E_c)$	• $18tC_2 \le \{(\alpha-3\varepsilon)E_e+E_c\}\{6t+2\Delta q-(\alpha+3\varepsilon)E_e+E_c\}$
L[YY] (with H[NN])	• $6tC_2 \le (1-\varepsilon)E_e(3t-\Delta q+3\varepsilon E_e-E_c)$	• $18tC_2 \le (1-3\varepsilon)E_e\{6t+2\Delta q-(1+3\varepsilon)E_e+2E_c\}$
	• $6tC_2 \le \{(\alpha-\varepsilon)E_e+E_c\}(3t+\Delta q-3\varepsilon E_e+E_c)$	• $18tC_2 \le \{(\alpha-3\varepsilon)E_e-E_c\}\{6t-2\Delta q-(\alpha+3\varepsilon)E_e-E_c\}$

a keyword, the focal firm's incentive to buy its own branded keyword will increase with the size of the comparison shopper segment. However, when the competitor does bid on the focal firm's brand name and thus the focal firm cannot increase its share even in the comparison shopper segment, the segment size does not affect the advertising incentive of the focal firm to purchase its own brand name. As a result, with a larger α, the equilibria in which only the brand owner buys (i.e., H[YN] and L[NY]) become more likely while those in which none of them buys (i.e., H[NN] and L[NN]) becomes less likely. However, the other equilibria (i.e., H[YY] versus H[NY] and L[YY] versus L[YN]) are not affected by α.

Part (b) of Proposition 7 can be similarly understood. When the competitor advertises under its own brand name, it can potentially take some consumers away from the focal firm in the comparison shopper segment but not in the noncomparison shopper segment. Given that the negative consequence of such encroachment in the comparison shopper segment can be lessened if the focal firm also advertises under the competitor's brand name, the focal firm has more incentive to do so as the comparison shopper segment grows larger. However, when the competitor does not bid on its own branded keyword, the focal firm's purchase of the competitor's branded keyword is no longer defensive, but it is aimed at increasing its market share beyond its current share, which is possible in both segments. Therefore, such an advertising incentive is not affected by the segment size. This discussion implies that increases in α make the equilibria in which both firms buy the same keyword (i.e., H[YY] and L[YY]) more likely and the equilibria in which only the brand owner buys its keyword (i.e., H[YN] and L[NY]) less likely. However, it also implies no changes in the likelihood of equilibria: H[NY] versus H[NN] and L[YN] versus L[NN].

Taken together, Proposition 7 shows that as α increases, the equilibria in which both firms buy the same keyword (i.e., H[YY] and L[YY]) become more likely, whereas the equilibria in which neither buys the keyword (i.e., H[NN] and L[NN]) become less likely. However, the likelihood of the equilibria where only the competitor buys the branded keyword (i.e., H[NY] and L[YN]) is not affected and the likelihood of the equilibria where only the brand owner buys (i.e., H[YN] and L[NY]) ambiguously changes with α because in these latter cases part (a) of the proposition results in an increase whereas part (b) implies a decrease in the likelihood. The proposition also suggests that the comparison shopper segment promotes the offensive purchase of own branded keyword as well as the defensive purchase of the competitor's branded keyword. However, all these purchases crucially depend on the competitive context of the market.

4. Empirical Support

Our theoretical development has a number of implications concerning the branded keyword choice in search advertising. In this section, we use empirical observations from Google search results to assess how often the model predictions are observed for a given product class. In particular, we examine whether the equilibrium predictions, as summarized in Table 5, are consistent with empirical observations. Note that, based on the discussion in §2.1, the context effect can be operationalized by the quality difference. Because of the lack of a proper measure for the exposure effect, we test the simplified hypotheses given as follows, assuming that the exposure effect does not change much across keywords in this product class:

Hypothesis 1 (H1). *Under Keyword L, a pair of firms exhibiting the pattern L[YY] has a greater quality difference than a pair displaying the pattern L[NY].*

Hypothesis 2 (H2). *Under Keyword L, a pair of firms exhibiting the pattern L[YN] has a greater quality difference than a pair displaying the pattern L[NN].*

Hypothesis 3 (H3). *Under Keyword H, a pair of firms exhibiting the pattern H[YN] has a greater quality difference than a pair displaying the pattern H[YY].*

Hypothesis 4 (H4). *Under Keyword H, a pair of firms exhibiting the pattern H[NN] has greater quality difference than a pair displaying the pattern H[NY].*

4.1. Data

We consider the product category of e-book readers and specifically choose the following five brands: iPad, Kindle, Sony, Nook, and Cybook. We used two different sources of data in this study: quality ratings data and the keyword purchase patterns data. The former were obtained from a survey of 297 online subjects who were asked to rate the quality of the five brands using a seven-point scale anchored by the words "lowest possible quality" and "best possible quality."[13] The mean quality ratings obtained for each brand are as follows: $q_{\text{iPad}} = 5.30$, $q_{\text{Kindle}} = 5.14$, $q_{\text{Sony}} = 4.57$, $q_{\text{Nook}} = 4.33$, and $q_{\text{Cybook}} = 3.42$. We also collected the keyword purchase pattern for each keyword from Google search results page. These patterns were consistently observed two months before and after the period the survey was taken—that is, February 2011.[14] Table 6 presents all the possible pairs of the brand owner and the competitor as well as their purchase decisions.

[13] We indicated that these are e-book reader brands by inserting "e-book reader" following the brand names.

[14] Although the purchase patterns were obtained as a long-term equilibrium outcome, they can vary by some outside shocks (e.g., when the book retailer Borders closed down in March 2011). Thus, they do not necessarily match the current purchase pattern. However, the pattern was stably observed during the period of the study.

Table 5 Summary of Keyword Choice Equilibrium

Context effect	Small exposure effect	Large exposure effect
	Keyword L	
Contrast	L[YN]	L[YY]
Assimilation	L[NN]	L[NY]
	Keyword H	
Contrast	H[NN]	H[YN]
Assimilation	H[NY]	H[YY]

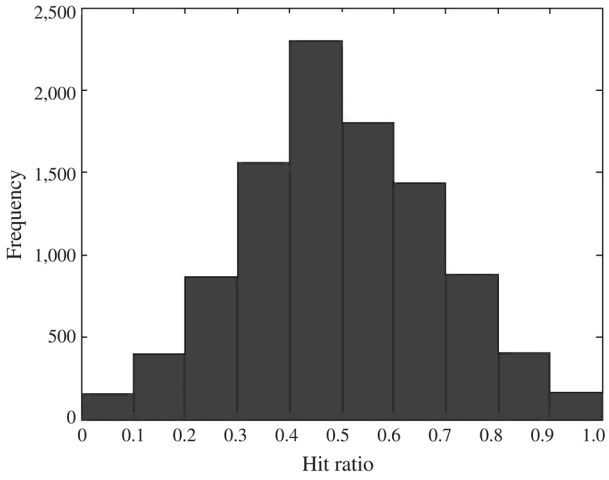
Table 6 Keyword Purchase Patterns in E-book Reader Category

Brand owner	Competitor	Purchase pattern	Quality difference
		Keyword H	
iPad	Kindle	H[YN]	0.16
iPad	Sony	H[YN]	0.73
iPad	Nook	H[YN]	0.97
iPad	Cybook	H[YN]	1.88
Kindle	Sony	H[YY]	0.57
Kindle	Nook	H[YN]	0.81
Kindle	Cybook	H[YN]	1.72
Sony	Nook	H[YY]	0.24
Sony	Cybook	H[YN]	1.15
Nook	Cybook	H[YN]	0.91
		Keyword L	
Kindle	iPad	L[NY]	0.16
Sony	iPad	L[YY]	0.73
Sony	Kindle	L[NY]	0.57
Nook	iPad	L[NY]	0.97
Nook	Kindle	L[YY]	0.81
Nook	Sony	L[NY]	0.24
Cybook	iPad	L[YN]	1.88
Cybook	Kindle	L[NN]	1.72
Cybook	Sony	L[NN]	1.15
Cybook	Nook	L[NN]	0.91

4.2. Results

These five brands constitute 10 pairs of brand owner and competitor for Keyword L and another 10 pairs for Keyword H. For Keyword L, there were four pairs exhibiting L[NY] and two pairs showing L[YY]. Of all eight ($= 2 \times 4$) combinations, only two are not consistent with the predicted pattern in H1, resulting in a hit ratio of 0.75. Next, for the same keyword, we had three pairs of L[NN] but only one pair of L[YN]. In all three ($= 3 \times 1$) possible combinations of L[NN] and L[YN] cases, the quality difference showed the pattern consistent with H2. For Keyword H, eight pairs display H[YN] and the other two exhibit H[YY]. Only 2 out of 16 combinations have the opposite pattern to H3, whereas the remaining 14 are consistent with its prediction. Thus, the hit ratio in this case is 0.875. However, because there are no cases falling in the category of H[NY] or H[NN] in our data, we cannot test H4. In sum, we find from our data, that the theoretical predictions in H1, H2, and H3 match with empirical observation in 23 of the 27 cases. Although this does not guarantee the universal applicability of the theory, our exercise provides one example where the purchase pattern generated by our model also generally holds true in the field.

Figure 4 Distribution of Hit Ratios



To complement the previous analysis, we also examine how likely the overall hit ratio from our analysis: 0.85 (23 out of 27 combinations) can be obtained from randomly generated purchase patterns. In this analysis, we use the same quality ratings for the five brands given in our data but randomly generated keyword purchase decisions from the binomial distribution based on the purchase probability of each keyword obtained from the sample (i.e., the probability of own keyword purchase is 0.8 ($= 4/5$), and the probability of the competitor's keyword purchase is 0.25 ($= 5/20$)). Then we followed the same procedure to calculate the hit ratio for the four hypotheses, which we repeated for 10,000 sets of randomly generated keyword purchase patterns. We find that only 353 cases (out of 10,000) exhibited the hit ratio of 0.85 or higher, and the rest had a hit ratio lower than that. (See also Figure 4 for the distribution of the hit ratios.) This shows that our result would be rarely observed if the purchase pattern is randomly generated. Therefore, we cannot disprove our theory about the keyword purchase pattern.

5. Conclusion

This paper examines strategic benefits and costs of buying a firm's own and a competitor's brand names as keywords in situations where consumers conduct information search using these branded keywords. We start our investigation with the empirical observation that consumers' quality perceptions of a brand are affected by not only the focal brand's advertisement but also another brand's advertisement in close proximity. Our model uses the parsimonious, but realistic setting, where two firms, offering horizontally and vertically differentiated products, not only set prices but also engage in brand advertising within a search engine. Our analysis helps us provide deeper understanding to the following questions.

1. *When do firms buy their own and a competitor's branded keyword*? We find that when the exposure effect is large, firms always buy their own brand name as a keyword. In this case, however, they buy the competitor's brand name only under the favorable context effect (i.e., contrast for the high-quality firm and assimilation for the low-quality firm) because it compensates for the advertising cost as well as the lost exposure benefit resulting from the competitor's purchase. When the exposure effect is small, neither firm buys its own brand name but both firms can buy each other's brand name. This is because buying the competitor's brand name triggers the context effect. Because there is no brand ownership in generic keywords, this pattern is unique to branded keywords.

2. *Why do brand owners buy their own brand name as a keyword? Why do some lower-quality firms forgo the opportunity to advertise under a well-known competitor's brand name*? We find that even when the exposure effect is weak and the context effect is unfavorable, the brand owner may buy its own brand name as a keyword, but only for a defensive purpose. In other words, it may not want to advertise but does so only because its competitor advertises. In situations where the exposure effect is strong and the context effect is favorable, each firm is able to preclude its competitor from buying its own brand name even though in isolation the competitor would find it best to buy this keyword. In this case, the mere presence of the brand owner's advertisement may prevent the competitor from purchasing the same keyword and thus protect the brand from being abused by the competitor.

3. *Who benefits from a policy of letting advertisers bid on the competitor's brand name*? Our analysis finds that this keyword bidding policy could induce prisoner's dilemma situations. Thus, it is a potentially welfare-reducing policy for advertisers. Interestingly, the search engine captures the lost profit from prisoner's dilemma and thus becomes better off.

4. *How does the consumer search influence the keyword choice equilibrium*? Consumers can search for either one or both branded keywords. We find that as more consumers search for both keywords, firms become more likely to buy their own keyword without the competitor's purchase because they can convert these comparison shoppers by doing so. At the same time, they become more likely to buy the competitor's keyword together with the competitor because they now have to defend their market from the competitor's encroachment. However, there are also situations where increases in the size of the comparison shopper segment have no effect on the advertising incentives of the two firms.

In addition to generating these insights, we also provide strong empirical evidence based on the advertising behavior of five e-book manufacturers that shows our model predictions are reasonably well aligned with reality.

Our work contributes to the search advertising literature in that this is the first attempt to model the impact of search advertising on brand value. We build on the work of Edelman et al. (2007) and Varian (2007) that recognizes the effect of search advertising depends on the characteristics of the advertiser (relevance) and the slot the advertiser takes (slot-specific effect). We extend this framework by capturing the empirical observation that the effect of search advertising is also affected by other advertisers who appear in the same results page. In addition, by considering the effects of the advertising on the product market, we offer insights on how firms behave optimally.

This paper is not without limitation. First, we consider a duopoly; in reality, in most cases, more than two firms are competing in the market, and more important, keywords are usually bought by more than three firms. Second, in our analysis, we consider the perceived quality of the product as the important driver of the keyword purchase decision. However, other variables, such as website traffic from the search advertising, can affect the keyword purchase decision. Considering other variables will be interesting future research. Third, in our model of the product market, we consider horizontally heterogeneous consumers. Alternatively, the vertical differentiation model might be considered. Although we do not expect any qualitative result to change, it might add some interesting insights that result from the asymmetric effect of advertising among consumers with different sensitivity to quality. Fourth, as noted earlier, we do not explicitly consider the potential link between the click-through rate and the incremental consumer demand. Again, qualitative results are not expected to change, but establishing the link might require a slightly different model of click-through rate. Finally, the impact of other advertisers that are not product market competitors, such as retailers, is assumed away in our model. This impact may be explicitly investigated. See Chiou and Tucker (2012) for example.

With these limitations noted, we still see this work contributing to the theoretical advertising literature in that it makes clear the firm's incentive in a situation where multiple firms simultaneously advertise in the same space. We hope that others will be able to build on our initial efforts to provide new insights.

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Appendix

Profit Derivation

We first state and prove Lemma A1, and based on this, we derive the profits in each advertising scenario.

Lemma A1. *When the advertising decisions are made after the pricing decisions, the advertising effect of each keyword is separable in the profit of both firms.*

Proof. First note that in the advertising stage, the prices of both firms have already been chosen and thus do not change with advertising outcome. Given this, the profit of Firm i ($i = \mathrm{H}, \mathrm{L}$) can be written as

$$\begin{aligned}\Pi_i &= p_i(d_i^0 + d_i^{\mathrm{H}} + d_i^{\mathrm{L}}) - C_i^{\mathrm{H}} - C_i^{\mathrm{L}} \\ &= p_i d_i^0 + (p_i d_i^{\mathrm{H}} - C_i^{\mathrm{H}}) + (p_i d_i^{\mathrm{L}} - C_i^{\mathrm{L}}),\end{aligned} \tag{10}$$

where d_i^0 is the baseline demand without advertising and d_i^K is the additional demand from advertising under Keyword K ($K = \mathrm{H}, \mathrm{L}$). Because $(p_i d_i^K - C_i^K)$ represents the additional profit of Firm i from advertising under Keyword K, the advertising effect of each keyword is separable in the profit of both firms. □

Now, based on Lemma A1, we derive each firm's profit. First note that in the noncomparison shopper segment, each firm's demand can neither increase by an advertisement under own keyword nor decrease by an advertisement under the competitor's keyword. Then, depending on the net effect of advertising, the additional demand from advertising changes as follows: for Keyword H, with $\Delta\tilde{q}^{N|\mathrm{H}}$ given in (5),

$$\begin{aligned}d_{\mathrm{H}}^{\mathrm{H}} &= \min\left\{0, \frac{1}{2} + \frac{\Delta\tilde{q}^{N|\mathrm{H}} - p_{\mathrm{H}} + p_{\mathrm{L}}}{2t} - d_{\mathrm{H}}^0\right\} \quad \text{and} \\ d_{\mathrm{L}}^{\mathrm{H}} &= \max\left\{0, \frac{1}{2} - \frac{\Delta\tilde{q}^{N|\mathrm{H}} - p_{\mathrm{H}} + p_{\mathrm{L}}}{2t} - d_{\mathrm{L}}^0\right\}.\end{aligned} \tag{11}$$

And for Keyword L, with $\Delta\tilde{q}^{N|\mathrm{L}}$ given in (6),

$$\begin{aligned}d_H^{\mathrm{L}} &= \max\left\{0, \frac{1}{2} + \frac{\Delta\tilde{q}^{N|\mathrm{L}} - p_{\mathrm{H}} + p_{\mathrm{L}}}{2t} - d_{\mathrm{H}}^0\right\} \quad \text{and} \\ d_L^{\mathrm{L}} &= \min\left\{0, \frac{1}{2} - \frac{\Delta\tilde{q}^{N|\mathrm{L}} - p_{\mathrm{H}} + p_{\mathrm{L}}}{2t} - d_{\mathrm{L}}^0\right\}.\end{aligned} \tag{12}$$

In contrast, in the comparison shopper segment, the net effect of advertising is fully reflected in the demand. Thus, for both keywords,

$$\begin{aligned}d_{\mathrm{H}}^{\mathrm{H}} &= \frac{1}{2} + \frac{\Delta\tilde{q}^{C} - p_{\mathrm{H}} + p_{\mathrm{L}}}{2t} - d_{\mathrm{H}}^0 \quad \text{and} \\ d_{\mathrm{L}}^{\mathrm{H}} &= \frac{1}{2} - \frac{\Delta\tilde{q}^{C} - p_{\mathrm{H}} + p_{\mathrm{L}}}{2t} - d_{\mathrm{L}}^0,\end{aligned} \tag{13}$$

where $\Delta\tilde{q}^{C}$ is given in (7). Based on this, we derive the profit of Firm i from advertising *only* under Keyword K: $\Pi_i^K \equiv p_i d_i^0 + p_i d_i^K - C_i^K$, in each advertising scenario as follows. For Keyword H,

$$\Pi_{\mathrm{H}}^{\mathrm{H[NN]}} = p_{\mathrm{H}}\left\{\frac{1}{2} + \frac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}}}{2t}\right\}, \tag{14}$$

$$\Pi_{\mathrm{H}}^{\mathrm{H[YN]}} = p_{\mathrm{H}}\left\{\frac{1}{2} + \frac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} + \alpha E_e}{2t}\right\} - C_0, \tag{15}$$

$$\Pi_{\mathrm{H}}^{\mathrm{H[NY]}} = \begin{cases} p_{\mathrm{H}}\left\{\dfrac{1}{2} + \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} - \alpha(E_e - E_c)}{2t}\right\} \\ \qquad\qquad \text{if } E_c \geq E_e, \\ p_H\left\{\dfrac{1}{2} + \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} - E_e + E_c}{2t}\right\} \\ \qquad\qquad \text{if } E_c < E_e, \end{cases} \tag{16}$$

$$\Pi_{\mathrm{H}}^{\mathrm{H[Y_1Y_2]}} = \begin{cases} p_H\left\{\dfrac{1}{2} + \dfrac{\Delta q - p_H + p_L + \alpha(\varepsilon E_e + E_c)}{2t}\right\} - C_{\mathrm{H1}}^{\mathrm{H}} \\ \qquad\qquad \text{if } E_c \geq -\varepsilon E_e, \\ p_H\left\{\dfrac{1}{2} + \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} + \varepsilon E_e + E_c}{2t}\right\} - C_{\mathrm{H1}}^{\mathrm{H}} \\ \qquad\qquad \text{if } E_c < -\varepsilon E_e, \end{cases} \tag{17}$$

$$\Pi_{\mathrm{H}}^{\mathrm{H[Y_2Y_1]}} = \begin{cases} p_{\mathrm{H}}\left\{\dfrac{1}{2} + \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} - \alpha(\varepsilon E_e - E_c)}{2t}\right\} - C_2 \\ \qquad\qquad \text{if } E_c \geq \varepsilon E_e, \\ p_{\mathrm{H}}\left\{\dfrac{1}{2} + \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} - \varepsilon E_e + E_c}{2t}\right\} - C_2 \\ \qquad\qquad \text{if } E_c < \varepsilon E_e, \end{cases} \tag{18}$$

$$\Pi_{\mathrm{L}}^{\mathrm{H[NN]}} = \alpha p_{\mathrm{L}}\left\{\frac{1}{2} - \frac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}}}{2t}\right\}, \tag{19}$$

$$\Pi_{\mathrm{L}}^{\mathrm{H[YN]}} = \alpha p_{\mathrm{L}}\left\{\frac{1}{2} - \frac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} + \alpha E_e}{2t}\right\}, \tag{20}$$

$$\Pi_{\mathrm{L}}^{\mathrm{H[NY]}} = \begin{cases} \alpha p_{\mathrm{L}}\left\{\dfrac{1}{2} - \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} - (E_e - E_c)}{2t}\right\} - C_0 \\ \qquad\qquad \text{if } E_c \geq E_e, \\ p_{\mathrm{L}}\left\{\dfrac{\alpha}{2} - \dfrac{\alpha(\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}}) - E_e + E_c}{2t}\right\} - C_0 \\ \qquad\qquad \text{if } E_c < E_e, \end{cases} \tag{21}$$

$$\Pi_{L}^{\mathrm{H[Y_1Y_2]}} = \begin{cases} \alpha p_{\mathrm{L}}\left\{\dfrac{1}{2} - \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} - \alpha(\varepsilon E_e - E_c)}{2t}\right\} - C_2 \\ \qquad\qquad \text{if } E_c \geq -\varepsilon E_e, \\ p_{\mathrm{L}}\left\{\dfrac{\alpha}{2} - \dfrac{\alpha(\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}}) - \varepsilon E_e + E_c}{2t}\right\} - C_2 \\ \qquad\qquad \text{if } E_c < -\varepsilon E_e, \end{cases} \tag{22}$$

$$\Pi_{\mathrm{L}}^{\mathrm{H[Y_2Y_1]}} = \begin{cases} \alpha p_{\mathrm{L}}\left\{\dfrac{1}{2} - \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} - \varepsilon E_e + E_c}{2t}\right\} - C_{\mathrm{L1}}^{\mathrm{H}} \\ \qquad\qquad \text{if } E_c \geq \varepsilon E_e, \\ p_{\mathrm{L}}\left\{\dfrac{\alpha}{2} - \dfrac{\alpha(\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}}) - \varepsilon E_e + E_c}{2t}\right\} - C_{\mathrm{L1}}^{\mathrm{H}} \\ \qquad\qquad \text{if } E_c < \varepsilon E_e. \end{cases} \tag{23}$$

And for Keyword L,

$$\Pi_{\mathrm{H}}^{\mathrm{L[NN]}} = \alpha p_{\mathrm{H}} \left\{ \frac{1}{2} + \frac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}}}{2t} \right\}, \tag{24}$$

$$\Pi_{\mathrm{H}}^{\mathrm{L[YN]}} = \begin{cases} p_{\mathrm{H}} \left\{ \dfrac{\alpha}{2} + \dfrac{\alpha(\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}}) + E_e + E_c}{2t} \right\} - C_0 \\ \qquad\qquad \text{if } E_c \geq -E_e, \\ \alpha p_{\mathrm{H}} \left\{ \dfrac{1}{2} + \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} + E_e + E_c}{2t} \right\} - C_0 \\ \qquad\qquad \text{if } E_c < -E_e, \end{cases} \tag{25}$$

$$\Pi_{\mathrm{H}}^{\mathrm{L[NY]}} = \alpha p_{\mathrm{H}} \left\{ \frac{1}{2} + \frac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} - E_e}{2t} \right\}, \tag{26}$$

$$\Pi_{\mathrm{H}}^{\mathrm{L[Y_1Y_2]}} = \begin{cases} p_{\mathrm{H}} \left\{ \dfrac{\alpha}{2} + \dfrac{\alpha(\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}}) + \varepsilon E_e + E_c}{2t} \right\} - C_{\mathrm{H1}}^{\mathrm{L}} \\ \qquad\qquad \text{if } E_c \geq -\varepsilon E_e, \\ \alpha p_{\mathrm{H}} \left\{ \dfrac{1}{2} + \dfrac{\Delta q - p_{H} + p_{\mathrm{L}} + \varepsilon E_e + E_c}{2t} \right\} - C_{\mathrm{H1}}^{\mathrm{L}} \\ \qquad\qquad \text{if } E_c < -\varepsilon E_e, \end{cases} \tag{27}$$

$$\Pi_{\mathrm{H}}^{\mathrm{L[Y_2Y_1]}} = \begin{cases} p_{\mathrm{H}} \left\{ \dfrac{\alpha}{2} + \dfrac{\alpha(\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}}) - \varepsilon E_e + E_c}{2t} \right\} - C_2 \\ \qquad\qquad \text{if } E_c \geq \varepsilon E_e, \\ \alpha p_{\mathrm{H}} \left\{ \dfrac{1}{2} + \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} - \varepsilon E_e + E_c}{2t} \right\} - C_2 \\ \qquad\qquad \text{if } E_c < \varepsilon E_e, \end{cases} \tag{28}$$

$$\Pi_{\mathrm{L}}^{\mathrm{L[NN]}} = p_{\mathrm{L}} \left\{ \frac{1}{2} - \frac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}}}{2t} \right\}, \tag{29}$$

$$\Pi_{\mathrm{L}}^{\mathrm{L[YN]}} = \begin{cases} p_{\mathrm{L}} \left\{ \dfrac{1}{2} - \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} + E_e + E_c}{2t} \right\} \\ \qquad\qquad \text{if } E_c \geq -E_e, \\ p_{\mathrm{L}} \left\{ \dfrac{1}{2} - \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} + \alpha(E_e + E_c)}{2t} \right\} \\ \qquad\qquad \text{if } E_c < -E_e, \end{cases} \tag{30}$$

$$\Pi_{\mathrm{L}}^{\mathrm{L[NY]}} = p_{\mathrm{L}} \left\{ \frac{1}{2} - \frac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} - \alpha E_e}{2t} \right\} - C_0, \tag{31}$$

$$\Pi_{\mathrm{L}}^{\mathrm{L[Y_1Y_2]}} = \begin{cases} p_{\mathrm{L}} \left\{ \dfrac{1}{2} - \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} + \varepsilon E_e + E_c}{2t} \right\} - C_2 \\ \qquad\qquad \text{if } E_c \geq -\varepsilon E_e, \\ p_{\mathrm{L}} \left\{ \dfrac{1}{2} - \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} + \alpha(\varepsilon E_e + E_c)}{2t} \right\} - C_2 \\ \qquad\qquad \text{if } E_c < -\varepsilon E_e, \end{cases} \tag{32}$$

$$\Pi_{\mathrm{L}}^{\mathrm{L[Y_2Y_1]}} = \begin{cases} p_{\mathrm{L}} \left\{ \dfrac{1}{2} - \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} - \varepsilon E_e + E_c}{2t} \right\} - C_{\mathrm{L1}}^{\mathrm{L}} \\ \qquad\qquad \text{if } E_c \geq \varepsilon E_e, \\ p_{\mathrm{L}} \left\{ \dfrac{1}{2} - \dfrac{\Delta q - p_{\mathrm{H}} + p_{\mathrm{L}} - \alpha(\varepsilon E_e - E_c)}{2t} \right\} - C_{\mathrm{L1}}^{\mathrm{L}}, \\ \qquad\qquad \text{if } E_c < \varepsilon E_e. \end{cases} \tag{33}$$

Bidding Equilibrium

Here, we derive the bidding equilibrium and, based on this, advertising costs for the two firms. First note that among the product market competitors, when only one firm advertises under a certain keyword, the advertising cost for that keyword is determined by Firm X and thus is exogenously given by C_0^K. Similarly, even when both firms advertise together, the advertising cost at the second slot is exogenously fixed at C_2^K. Now, we derive the advertising cost of the first slot winner when both firms advertise together. Suppose Firm H takes the first slot and Firm L takes the second slot in equilibrium. Then Firm H should not deviate to the second slot. By letting π_{ij} denote the profits Firm i earns from the product market when advertising at the jth slot of the sponsored links section, this condition is translated to

$$\pi_{\mathrm{H1}} - C_{\mathrm{H1}}^K \geq \pi_{\mathrm{H2}} - C_2^K. \tag{34}$$

In addition, Firm L should have no incentive to deviate to the first slot:

$$\pi_{\mathrm{L2}} - C_2^K \geq \pi_{\mathrm{L1}} - C_{\mathrm{H1}}^K. \tag{35}$$

Summing these two inequalities side by side and rearranging order, we have

$$\pi_{\mathrm{H1}} - \pi_{\mathrm{H2}} \geq \pi_{\mathrm{L1}} - \pi_{\mathrm{L2}}. \tag{36}$$

Now, based on (36), Lemma A2 derives the equilibrium listing order.

Lemma A2. *In the bidding equilibrium for every keyword, Firm H always takes the first slot.*

Proof. Note that π_{ij} is the profit prior to subtracting the advertising cost. Then, for Keyword H, based on (17), (18), (22), and (23), we have $\pi_{\mathrm{H1}} = \Pi_{\mathrm{H}}^{\mathrm{H[Y_1Y_2]}} + C_{\mathrm{H1}}^{\mathrm{H}}$, $\pi_{\mathrm{H2}} = \Pi_{\mathrm{H}}^{\mathrm{H[Y_2Y_1]}} + C_2$, $\pi_{\mathrm{L1}} = \Pi_{\mathrm{L}}^{\mathrm{H[Y_2Y_1]}} + C_{\mathrm{L1}}^{\mathrm{H}}$, and $\pi_{\mathrm{L2}} = \Pi_{\mathrm{L}}^{\mathrm{H[Y_1Y_2]}} + C_2$. Thus, $\pi_{\mathrm{H1}} - \pi_{\mathrm{H2}} \geq \pi_{\mathrm{L1}} - \pi_{\mathrm{L2}}$ is equivalent to

$$\alpha\varepsilon(p_{\mathrm{H}} - p_{\mathrm{L}})E_e \geq 0 \quad \text{when } E_c \geq \varepsilon E_e,$$

$$\{(1+\alpha)\varepsilon E_e - (1-\alpha)E_c\}(p_{\mathrm{H}} - p_{\mathrm{L}}) \geq 0 \text{ when } -\varepsilon E_e \leq E_c < \varepsilon E_e,$$

and

$$\varepsilon(p_{\mathrm{H}} - p_{\mathrm{L}})E_e \geq 0 \quad \text{when } E_c < -\varepsilon E_e.$$

For Keyword L, from (27), (28), (32), and (33), we have $\pi_{\mathrm{H1}} = \Pi_{\mathrm{H}}^{\mathrm{L[Y_1Y_2]}} + C_{\mathrm{H1}}^{\mathrm{L}}$, $\pi_{\mathrm{H2}} = \Pi_{\mathrm{H}}^{\mathrm{L[Y_2Y_1]}} + C_2$, $\pi_{\mathrm{L1}} = \Pi_{\mathrm{L}}^{\mathrm{L[Y_2Y_1]}} + C_{\mathrm{L1}}^{\mathrm{L}}$, and $\pi_{\mathrm{L2}} = \Pi_{\mathrm{L}}^{\mathrm{L[Y_1Y_2]}} + C_2$. Thus, $\pi_{\mathrm{H1}} - \pi_{\mathrm{H2}} \geq \pi_{\mathrm{L1}} - \pi_{\mathrm{L2}}$ is equivalent to

$$\varepsilon(p_{\mathrm{H}} - p_{\mathrm{L}})E_e \geq 0 \quad \text{when } E_c \geq \varepsilon E_e,$$

$$\{(1+\alpha)\varepsilon E_e + (1-\alpha)E_c\}(p_{\mathrm{H}} - p_{\mathrm{L}}) \geq 0 \text{ when } -\varepsilon E_e \leq E_c < \varepsilon E_e,$$

and

$$\alpha\varepsilon(p_{\mathrm{H}} - p_{\mathrm{L}})E_e \geq 0 \quad \text{when } E_c < -\varepsilon E_e.$$

In all cases, this implies that the condition is equivalent to $p_H \geq p_L$, which is indeed the case, by the assumption that Firm H's perceived quality is higher than that of Firm L even with advertising; i.e., $\Delta\tilde{q}^S \geq 0$ $(S = C, N)$. Therefore, Firm H always takes the first slot. □

Intuitively, Firm H takes the first slot because the additional exposure from the first slot (versus the second slot) can be more effective when the price and thus the profit margin is higher. Also, note that the condition for Firm L's taking the first slot is the flip side of the inequality given in (36) (because both inequalities in (34) and (35) are also flipped). This implies that no such occasion exists.

Given this equilibrium listing order, we now derive the range for Firm L's equilibrium bid b_L^K, or equivalently, Firm H's equilibrium cost $C_2^K (= s_2 r_L^K b_L^K)$, from inequalities in (34) and (35):

$$\pi_{L1} - \pi_{L2} + C_2^K \leq C_{H1}^K \leq \pi_{H1} - \pi_{H2} + C_2^K, \tag{37}$$

where the profits (π_{ij}) are given as in the proof of Lemma A2. Although we use the upper bound of this solution in our later analysis, note that at any cost in this range, no firm has any incentive to deviate from the equilibrium. Finally, because both C_0^K and C_2^K do not depend on the keyword, we instead use C_0 and C_2 for notational convenience.

Advertising Cost in H[YY] and L[YY] Scenario

Based on Lemma A2, we derive the equilibrium bid and thus the equilibrium advertising cost. From the condition in (37), we choose the upper bound solution, which is given as follows:

$$C_{H1}^H = \begin{cases} C_2 + \dfrac{\alpha\varepsilon E_e p_H}{t} & \text{when } E_c \geq \varepsilon E_e, \\ C_2 + \dfrac{\{(1+\alpha)\varepsilon E_e - (1-\alpha)E_c\}p_H}{t} & \\ \qquad\qquad \text{when } -\varepsilon E_e \leq E_c < \varepsilon E_e, & \\ C_2 + \dfrac{\varepsilon E_e p_H}{t} & \text{when } E_c < -\varepsilon E_e, \end{cases} \tag{38}$$

$$C_{H1}^L = \begin{cases} C_2 + \dfrac{\varepsilon E_e p_H}{t} & \text{when } E_c \geq \varepsilon E_e, \\ C_2 + \dfrac{\{(1+\alpha)\varepsilon E_e + (1-\alpha)E_c\}p_H}{t} & \\ \qquad\qquad \text{when } -\varepsilon E_e \leq E_c < \varepsilon E_e, & \\ C_2 + \dfrac{\alpha\varepsilon E_e p_H}{t} & \text{when } E_c < -\varepsilon E_e. \end{cases} \tag{39}$$

Note that when these solutions are plugged in, $\Pi_H^{H[Y_1Y_2]} = \Pi_H^{H[Y_2Y_1]}$ and $\Pi_H^{L[Y_1Y_2]} = \Pi_H^{L[Y_2Y_1]}$ hold. Thus, we use the profits given in (18) and (28) as $\Pi_H^{H[YY]}$ and $\Pi_H^{L[YY]}$, respectively, in our later analysis. (Note that we drop the subscript in H[YY] and L[YY] because Firm H always takes the first slot by Lemma A2.)

Conditions for Firm X Taking the Last Slot

For Firm X to take the third slot in equilibrium, (1) Firm L should prefer the second slot to the third, $\pi_{L2} - C_2^K \geq \pi_{L3} - C_3^K$; and (2) Firm X should prefer the third to the second slot, $\pi_{X3} - C_3^K \geq \pi_{X2} - C_2^K$. Summing the two inequalities side by side, we obtain $\pi_{L2} - \pi_{L3} \geq \pi_{X2} - \pi_{X3}$, which was verbally described in Footnote 7. Because we do not assume anything about how Firm X operates in the market, we take $\Delta_{23} \equiv \pi_{X2} - \pi_{X3}$ as exogenous and derive the condition assuming that the exposure effect also decreases by ε from the second to the third slot. Then, the condition becomes $\Delta_{23} \leq \{1 + (\alpha - 1)I_{[E_c \geq -\varepsilon E_e]}\}((\varepsilon E_e p_L)/(2t))$ for Keyword H and $\Delta_{23} \leq \{1 + (\alpha - 1)I_{[E_c \leq \varepsilon E_e]}\}((\varepsilon E_e p_L)/(2t))$ for Keyword L, where $I_{[*]}$ is an indicator function.

When Firm H does not buy the keyword, the condition for Firm X to take the second slot is similarly derived: $\pi_{L1} - \pi_{L2} \geq \pi_{X1} - \pi_{X2}$. Letting $\Delta_{12} \equiv \pi_{X1} - \pi_{X2}$, it is easy to see that the condition is equivalent to $\Delta_{12} \leq \{1 + (\alpha - 1) \cdot I_{[E_c \geq -\varepsilon E_e]}\}((\varepsilon E_e p_L)/(2t))$ for Keyword H and $\Delta_{12} \leq \{1 + (\alpha - 1) \cdot I_{[E_c \leq \varepsilon E_e]}\}((\varepsilon E_e p_L)/(2t))$ for Keyword L. Finally, when Firm L does not buy the keyword, the condition is $\pi_{H1} - \pi_{H2} \geq \pi_{X1} - \pi_{X2}$, but if the above condition holds, this condition also holds because (36) always holds by Lemma A2.

Keyword Choice Equilibrium

Based on the bidding equilibrium, we derive the keyword choice equilibrium. We first prove Lemma A3, which shows that the keyword choice equilibrium for each keyword can be separately considered.

Lemma A3. *When the advertising effect of each keyword is separable in the profit of both firms, the equilibrium of the keyword choice game (across the two keywords) is equivalent to the equilibrium of each keyword.*

Proof. First denote the strategy of Firm i by $(S_i^H S_i^L)$, where S_i^K is the decision on Keyword K and takes the value of Y (purchase) and N (no purchase). Suppose that $\{(S_H^{H*} S_H^{L*}), (S_L^{H*} S_L^{L*})\}$ is an equilibrium of the keyword choice game. Then by definition, we have

$$\Pi_H(S_H^{H*} S_H^{L*}, S_L^{H*} S_L^{L*}) \geq \Pi_H(S_H^H S_H^{L*}, S_L^{H*} S_L^{L*}) \quad \forall S_H^H, \tag{40}$$

$$\Pi_H(S_H^{H*} S_H^{L*}, S_L^{H*} S_L^{L*}) \geq \Pi_H(S_H^{H*} S_H^L, S_L^{H*} S_L^{L*}) \quad \forall S_H^L, \tag{41}$$

$$\Pi_L(S_H^{H*} S_H^{L*}, S_L^{H*} S_L^{L*}) \geq \Pi_L(S_H^{H*} S_H^{L*}, S_L^H S_L^{L*}) \quad \forall S_L^H, \tag{42}$$

$$\Pi_L(S_H^{H*} S_H^{L*}, S_L^{H*} S_L^{L*}) \geq \Pi_L(S_H^{H*} S_H^{L*}, S_L^{H*} S_L^L) \quad \forall S_L^L. \tag{43}$$

Now define $\Pi_i(S_H^K, S_L^K) \equiv p_i d_i^K - C_i^K$ as the additional profit of Firm i from advertising under Keyword K. Since the advertising effect of each keyword is separable in the profits by Lemma A1, we can restate the profits as $\Pi_i(S_H^H S_H^L, S_L^H S_L^L) = p_i d_i^0 + \Pi_H(S_H^H, S_L^H) + \Pi_H(S_H^L, S_L^L)$, just as was shown in (10). Thus, the above inequalities can be restated as

$$\Pi_H(S_H^{H*}, S_L^{H*}) + \Pi_H(S_H^{L*}, S_L^{L*}) \geq \Pi_H(S_H^H, S_L^{H*}) + \Pi_H(S_H^{L*}, S_L^{L*}) \quad \forall S_H^H, \tag{44}$$

$$\Pi_H(S_H^{H*}, S_L^{H*}) + \Pi_H(S_H^{L*}, S_L^{L*}) \geq \Pi_H(S_H^{H*}, S_L^{H*}) + \Pi_H(S_H^L, S_L^{L*}) \quad \forall S_H^L, \tag{45}$$

$$\Pi_L(S_H^{H*}, S_L^{H*}) + \Pi_L(S_H^{L*}, S_L^{L*}) \geq \Pi_L(S_H^{H*}, S_L^H) + \Pi_L(S_H^{L*}, S_L^{L*}) \quad \forall S_L^H, \tag{46}$$

$$\Pi_L(S_H^{H*}, S_L^{H*}) + \Pi(S_H^{L*}, S_L^{L*}) \geq \Pi_L(S_H^{H*}, S_L^{H*}) + \Pi_L(S_H^{L*}, S_L^L) \quad \forall S_L^L, \tag{47}$$

which respectively reduce to

$$\Pi_H(S_H^{H*}, S_L^{H*}) \geq \Pi_H(S_H^H, S_L^{H*}) \quad \forall S_H^H, \tag{48}$$

$$\Pi_H(S_H^{L*}, S_L^{L*}) \geq \Pi_H(S_H^L, S_L^{L*}) \quad \forall S_H^L, \tag{49}$$

$$\Pi_L(S_H^{H*}, S_L^{H*}) \geq \Pi_L(S_H^{H*}, S_L^H) \quad \forall S_L^H, \tag{50}$$

$$\Pi_L(S_H^{L*}, S_L^{L*}) \geq \Pi_L(S_H^{L*}, S_L^L) \quad \forall S_L^L. \tag{51}$$

Then by (48) and (50), (S_H^{H*}, S_L^{H*}) constitutes an equilibrium for Keyword H, whereas by (49) and (51), (S_H^{L*}, S_L^{L*}) constitutes an equilibrium for Keyword L.

Next, suppose (S_H^{H*}, S_L^{H*}) is an equilibrium of the game for Keyword H and (S_H^{L*}, S_L^{L*}) is an equilibrium of the game for Keyword L. Then, by definition, the inequalities in (48)–(51) hold. Adding the same quantity to both sides of each inequality, we obtain (44)–(47). In addition, by adding (48) and (49) side by side, we have

$$\begin{aligned}&\Pi_H(S_H^{H*}, S_L^{H*}) + \Pi_H(S_H^{L*}, S_L^{L*})\\ &\quad \geq \Pi_H(S_H^H, S_L^{H*}) + \Pi_H(S_H^L, S_L^{L*}) \quad \forall S_H^H, \forall S_H^L. \end{aligned}\tag{52}$$

Similarly, by adding (50) and (51) side by side, we have

$$\begin{aligned}&\Pi_L(S_H^{H*}, S_L^{H*}) + \Pi_L(S_H^{L*}, S_L^{L*})\\ &\quad \geq \Pi_L(S_H^{H*}, S_L^H) + \Pi_L(S_H^{L*}, S_L^L) \quad \forall S_L^H, \forall S_L^L. \end{aligned}\tag{53}$$

Finally, the inequalities in (44)–(47) are equivalent to (40)–(43), and those in (52) and (53) are equivalent to

$$\Pi_H(S_H^{H*}S_H^{L*}, S_L^{H*}S_L^{L*}) \geq \Pi_H(S_H^H S_H^L, S_L^{H*}S_L^{L*}) \quad \forall S_H^H \forall S_H^L, \tag{54}$$

$$\Pi_H(S_H^{H*}S_H^{L*}, S_L^{H*}S_L^{L*}) \geq \Pi_H(S_H^{H*}S_H^{L*}, S_L^H S_L^L) \quad \forall S_L^H, \forall S_L^L. \tag{55}$$

By (40)–(43), together with (54) and (55), $\{(S_H^{H*}S_H^{L*}), (S_L^{H*}S_L^{L*})\}$ is an equilibrium of the keyword choice game. □

Based on Lemma A3, we now prove the propositions of the paper as well as the claims made in the text below these propositions.

Proof of Proposition 1. By Lemma A3, we can separately derive the equilibrium for each keyword. First, for Keyword L, the following two inequalities define the L[NY] equilibrium: $\Pi_L^{L[NY]} \geq \Pi_L^{L[NN]}$ and $\Pi_H^{L[NY]} \geq \Pi_H^{L[YY]}$. Based on profits given in (14)–(33), it is easy to see that the first inequality is equivalent to $E_e \geq (2tC_0)/(\alpha P_L)$ and the second one is equivalent to

$$E_c \leq \begin{cases} -(1-\varepsilon)E_e + \dfrac{2tC_2}{\alpha P_H} & \text{when } E_c < \varepsilon E_e, \\ -(\alpha-\varepsilon)E_e + \dfrac{2tC_2}{P_H} & \text{when } E_c \geq \varepsilon E_e. \end{cases} \tag{56}$$

Next, for Keyword H, we obtain the H[YN] equilibrium when both $\Pi_L^{H[YN]} \geq \Pi_L^{H[YY]}$ and $\Pi_H^{H[YN]} \geq \Pi_H^{H[NN]}$ hold. The latter can be seen as the same as $E_e \geq (2tC_0)/(\alpha P_H)$, and the former is equivalent to

$$E_c \geq \begin{cases} (1-\varepsilon)E_e - \dfrac{2tC_2}{\alpha P_L} & \text{when } E_c \geq -\varepsilon E_e, \\ (\alpha-\varepsilon)E_e - \dfrac{2tC_2}{P_L} & \text{when } E_c < -\varepsilon E_e. \end{cases} \tag{57}$$

Now, note that $(2tC_0)/(\alpha P_L) \geq (2tC_0)/(\alpha P_H)$ holds since $P_L < P_H$. Also, (56) is equivalent to $E_c \leq \min\{-(1-\varepsilon)E_e + (2tC_2)/(\alpha P_H), -(\alpha-\varepsilon)E_e + (2tC_2)/P_H\}$, and (57) can be rewritten as $E_c \geq \max\{(1-\varepsilon)E_e - (2tC_2)/(\alpha P_L), (\alpha-\varepsilon)E_e - (2tC_2)/P_L\}$. Hence, the conditions for L[NY] and H[YN] are given as in the proposition. □

Claim A1. *Under the condition of Proposition* 1, (a) *at the same level of the exposure effect, Firm H is more likely to purchase its own keyword than Firm L, and* (b) *at the same level of the favorable context effect, Firm L is more likely to give up buying the competitor's keyword than Firm H.*

Proof. Part (a) can be easily shown: since $P_L < P_H$, we have $(2tC_0)/(\alpha P_L) \geq (2tC_0)/(\alpha P_H)$. Similarly, note that both $(2tC_2)/(\alpha P_L) \geq (2tC_2)/(\alpha P_H)$ and $(2tC_2)/P_L \geq (2tC_2)/P_H$ hold. Thus, we have $-(1-\varepsilon)E_e + (2tC_2)/(\alpha P_H) \leq -\{(1-\varepsilon)E_e - (2tC_2)/(\alpha P_L)\}$ and $-(\alpha-\varepsilon)E_e + (2tC_2)/P_H \leq -\{(\alpha-\varepsilon)E_e - (2tC_2)/P_L\}$ for any E_e. This implies that the condition for Firm H's not buying the competitor's keyword is harder to satisfy than that for Firm L. Hence, the second part follows. □

Proof of Proposition 2. For H[NY] to be an equilibrium under Keyword H, the following conditions should be satisfied: $\Pi_H^{H[NY]} \geq \Pi_H^{H[YY]}$ and $\Pi_L^{H[NY]} \geq \Pi_L^{H[NN]}$, which (based on profits given in (14)–(33)) are respectively equivalent to $E_e \leq (2tC_2)/((1-\varepsilon)P_H)$ and $E_c \leq E_e - (2tC_0)/P_L$. Similarly, L[YN] is an equilibrium under Keyword L, if $\Pi_L^{L[YN]} \geq \Pi_L^{L[YY]}$ and $\Pi_H^{L[YN]} \geq \Pi_H^{L[NN]}$, which are respectively equivalent to $E_e \leq (2tC_2)/((1-\varepsilon)P_L)$ and $E_c \geq -E_e + (2tC_0)/P_H$. □

Proof of Proposition 3. H[YY] is an equilibrium under Keyword H, when $\Pi_H^{H[YY]} \geq \Pi_H^{H[NY]}$ and $\Pi_L^{H[YY]} \geq \Pi_L^{H[YN]}$ hold. Based on profits given in (14)–(33), these conditions are equivalent to $E_e \geq (2tC_2)/((1-\varepsilon)P_H)$ and $E_c \leq \max\{(1-\varepsilon)E_e - (2tC_2)/(\alpha P_L), (\alpha-\varepsilon)E_e - (2tC_2)/P_L\}$, respectively. Next, L[YY] is an equilibrium under Keyword L if $\Pi_L^{L[YY]} \geq \Pi_L^{L[YN]}$ and $\Pi_H^{L[YY]} \geq \Pi_H^{L[NY]}$, which are equivalent to $E_e \geq (2tC_2)/((1-\varepsilon)P_L)$ and $E_c \geq \min\{-(1-\varepsilon)E_e + (2tC_2)/(\alpha P_H), -(\alpha-\varepsilon)E_e + (2tC_2)/P_H\}$, respectively. □

Claim A2. *Under the condition of Proposition* 3, (*a*) *Firm H buys its own keyword only for a defensive purpose if* $E_e \leq (2tC_0)/(\alpha P_L)$, *and* (*b*) *Firm L also does so if* $E_e \leq (2tC_0)/(\alpha P_H)$.

Proof. The defensive purchase is defined by $\Pi_H^{H[NN]} \geq \Pi_H^{H[YN]}$ but $\Pi_H^{H[YY]} \geq \Pi_H^{H[NY]}$, since $\Pi_L^{H[YY]} \geq \Pi_L^{H[YN]}$ for Firm H, and $\Pi_L^{L[NN]} \geq \Pi_L^{L[NY]}$ but $\Pi_L^{L[YY]} \geq \Pi_L^{L[YN]}$, since $\Pi_H^{L[YY]} \geq \Pi_H^{L[NY]}$ for Firm L. In both cases, the last two are the conditions for Proposition 3. Then, based on profits given in (14)–(33), it is easy to see that $\Pi_H^{H[NN]} \geq \Pi_H^{H[YN]}$ is equivalent to $E_e \leq (2tC_0)/(\alpha P_L)$, and $\Pi_L^{L[NN]} \geq \Pi_L^{L[NY]}$ is equivalent to $E_e \leq (2tC_0)/(\alpha P_H)$. □

Proof of Proposition 4. In this proof, we first show that the prisoner's dilemma within a single keyword can exist if and only if advertisers are allowed to bid on the competitor's keyword. First, when advertisers are not allowed to do so, it can never be worse off by advertising on its own keyword because each firm buys its own keyword if and only if $\Pi_H^{H[YN]} \geq \Pi_H^{H[NN]}$ (for Firm H) and $\Pi_L^{L[NY]} \geq \Pi_L^{L[NN]}$ (for Firm L). Thus, no prisoner's dilemma exists under each keyword.

Next, suppose advertisers are allowed to bid on the competitor's keyword. First, under Keyword H, a prisoner's dilemma exists if and only if $\Pi_H^{H[YY]} \le \Pi_H^{H[NN]}$ and $\Pi_L^{H[YY]} \le \Pi_L^{H[NN]}$. Based on profits given in (14)–(33), it is easy to see the conditions are equivalent to $-\varepsilon E_e - (2tC_0)/p_L \le E_c \le \varepsilon E_e + (2tC_0)/(\alpha p_H)$. Under Keyword L, we observe the prisoner's dilemma if and only if $\Pi_H^{L[YY]} \le \Pi_H^{L[NN]}$ and $\Pi_L^{L[YY]} \le \Pi_L^{L[NN]}$, which are reduced to $-\varepsilon E_e - (2tC_0)/(\alpha p_L) \le E_c \le \varepsilon E_e + (2tC_0)/p_H$. Note that both conditions constitute nonempty space because all of E_e, C_0, p_H, p_L, α, ε, and t take positive values.

Finally, the search engine's profit in the prisoner's dilemma case is $C_{H1}^H + C_2$ (in the H[YY] equilibrium) and $C_{H1}^L + C_2$ (in the L[YY] equilibrium), but it is C_0 or zero if advertisers are not allowed to bid on the competitor's keyword. Since $C_{H1}^K \ge C_0$ ($K = $ H, L), it is easy to see that the search engine's profit is higher as a result of the prisoner's dilemma when advertisers are allowed to bid on the competitor's keyword. □

Proof of Proposition 5. The conditions for this type of prisoner's dilemma are given as $\Pi_H^{L[NY]} \le \Pi_H^{L[YN]}$ and $\Pi_L^{L[NY]} \le \Pi_L^{L[YN]}$. Based on profits given in (14)–(33), it is easy to see these conditions are equivalent to $(2tC_0)/p_H \le (1+\alpha)E_e + E_c \le (2tC_0)/p_L$. Since $P_L < P_H$, this constitutes a nonempty space. □

Claim A3. *In the H[YN] equilibrium, there exists no case where both firms are better off by having the competitor (i.e., Firm L) solely purchase Keyword H.*

Proof. For both firms to be better off with H[NY], both $\Pi_H^{H[YN]} \le \Pi_H^{H[NY]}$ and $\Pi_L^{H[YN]} \le \Pi_L^{H[NY]}$ need to be satisfied. These are equivalent to $(1+\alpha)E_e - E_c \le (2tC_0)/p_H$ and $(1+\alpha)E_e - E_c \ge (2tC_0)/p_L$, respectively. Since $P_L < P_H$, both inequalities cannot be satisfied at the same time. □

Proof of Proposition 6. Because the claims about branded keywords have been proved above (see the proofs of Propositions 2 and 3), here we only show the cases for the generic keyword. First note that, by definition of the generic keyword, noncomparison shoppers cannot exist. In addition, under a generic keyword, the context effect comes into play only when the keyword was bought by both firms. Based on this, we derive the profits of each firm when advertising under a generic keyword by plugging in $\alpha = 1$ into (14)–(23) and $E_c = 0$ into (16) and (21). To avoid confusion, we use E_e^G and E_c^G to denote the effects associated with generic keywords. The profits thus derived are given as follows:

$$\Pi_H^{G[NN]} = p_H\left\{\frac{1}{2} + \frac{\Delta q - p_H + p_L}{2t}\right\}, \tag{58}$$

$$\Pi_H^{G[YN]} = p_H\left\{\frac{1}{2} + \frac{\Delta q - p_H + p_L + E_e^G}{2t}\right\} - C_0, \tag{59}$$

$$\Pi_H^{G[NY]} = p_H\left\{\frac{1}{2} + \frac{\Delta q - p_H + p_L - E_e^G}{2t}\right\}, \tag{60}$$

$$\Pi_H^{G[YY]} = p_H\left\{\frac{1}{2} + \frac{\Delta q - p_H + p_L - \varepsilon E_e^G + E_c^G}{2t}\right\} - C_2, \tag{61}$$

$$\Pi_L^{G[NN]} = p_L\left\{\frac{1}{2} - \frac{\Delta q - p_H + p_L}{2t}\right\}, \tag{62}$$

$$\Pi_L^{G[YN]} = p_L\left\{\frac{1}{2} - \frac{\Delta q - p_H + p_L + E_e^G}{2t}\right\}, \tag{63}$$

$$\Pi_L^{G[NY]} = p_L\left\{\frac{1}{2} - \frac{\Delta q - p_H + p_L - E_e^G}{2t}\right\} - C_0, \tag{64}$$

$$\Pi_L^{G[YY]} = p_L\left\{\frac{1}{2} - \frac{\Delta q - p_H + p_L - \varepsilon E_e^G + E_c^G}{2t}\right\} - C_2. \tag{65}$$

Note that $\Pi_H^{G[YY]}$ has already incorporated the bidding equilibrium results (i.e., the advertising cost). Given this, $\Pi_L^{G[NY]} \ge \Pi_L^{G[NN]}$ and $\Pi_H^{G[YN]} \ge \Pi_H^{G[NN]}$ are respectively equivalent to $E_e^G \ge (2tC_0)/P_L$ and $E_e^G \ge (2tC_0)/P_H$. In addition, $\Pi_L^{G[YY]} \ge \Pi_L^{G[YN]}$ and $\Pi_H^{G[YY]} \ge \Pi_H^{G[NY]}$ are respectively equivalent to $E_c^G \le (1-\varepsilon)E_e^G - (2tC_2)/P_L$ and $E_c^G \ge -(1-\varepsilon)E_e^G + (2tC_2)/P_H$. Therefore, if $E_e \le \min\{(2tC_0)/P_H, (2tC_2(P_H + P_L))/((1-\varepsilon)P_H P_L)\}$, none of NY, YN, or YY can be an equilibrium. This proves part (a). The above also implies that YY is an equilibrium if and only if $-(1-\varepsilon)E_e^G + (2tC_2)/P_H \le E_c^G \le (1-\varepsilon)E_e^G - (2tC_2)/P_L$. This proves part (b). □

Pricing Equilibrium

Based on advertising equilibrium, both firms set prices. Thus, for all possible advertising scenarios, we derive the equilibrium prices as $p_H^* = t + X_H/3$ and $p_L^* = t - X_L/3$, where X_H and X_L are given as shown in Table A.1.

Keyword Choice Equilibrium Conditions

Given the equilibrium prices, it is straightforward to rewrite the conditions for each advertising scenario (given in Propositions 1–3) with exogenous parameters only. We first plug in the equilibrium prices into the given conditions. Then, we also check for deviation at the pricing stage by comparing the equilibrium profits at the pricing stage with the deviation profits based on prices of other advertising scenarios. We report these conditions for several cases in Table 3, while omitting the rest because of complexity.

Proof of Proposition 7. First, define

$$\varphi_H^{K[S_H S_L]} \equiv \Pi_H^{K[S_H S_L]}(p_H^{K[S_H S_L]}, p_L^{K[S_H S_L]}) - \Pi_H^{K[S'_H S_L]}(p_H^{K[S_H S_L]}, p_L^{K[S_H S_L]}), \tag{66}$$

$$\varphi_L^{K[S_H S_L]} \equiv \Pi_L^{K[S_H S_L]}(p_H^{K[S_H S_L]}, p_L^{K[S_H S_L]}) - \Pi_L^{K[S_H S'_L]}(p_H^{K[S_H S_L]}, p_L^{K[S_H S_L]}), \tag{67}$$

$$\phi_H^{K[S_H S_L]} \equiv \Pi_H^{K[S_H S_L]}(p_H^{K[S_H S_L]}, p_L^{K[S_H S_L]}) - \Pi_H^{K[S'_H S_L]}(p_H^{K[S'_H S_L]}, p_L^{K[S'_H S_L]}), \tag{68}$$

$$\phi_L^{K[S_H S_L]} \equiv \Pi_L^{K[S_H S_L]}(p_H^{K[S_H S_L]}, p_L^{K[S_H S_L]}) - \Pi_L^{K[S_H S'_L]}(p_H^{K[S_H S'_L]}, p_L^{K[S_H S'_L]}), \tag{69}$$

where S_i is the equilibrium keyword choice strategy, S'_i is the deviation strategy of Firm i ($S_i =$ Y, N and $S_i \ne S'_i$), and $p_i^{K[S_H S_L]}$ is the equilibrium price under the advertising scenario $K[S_H S_L]$. Recall that $\Pi_i^{K[S_H S_L]}$ is given in (14)–(33) and $p_i^{K[S_H S_L]}$ is given in Table A.1. Note that the pricing equilibrium is defined in relation to the equilibrium for the other keyword as well. However, to control for the illegitimate impact of the other keyword through equilibrium prices, we keep the

Table A.1. Pricing Equilibrium by Advertising Scenario

Advertising scenario	X_H	X_L
H[NN] and L[NN]	Δq	Δq
H[NN] and L[YN]	$\Delta q + E_e + E_c$	$\Delta q + E_e + E_c$
H[NN] and L[NY]	$\Delta q - \alpha E_e$	$\Delta q - \alpha E_e$
H[NN] and L[YY(1)][a]	$\Delta q - 3\varepsilon E_e + E_c$	$\Delta q - 3\varepsilon E_e + E_c$
H[NN] and L[YY(2)]	$\Delta q - (1+2\alpha)\varepsilon E_e - (1-2\alpha)E_c$	$\Delta q + (2+\alpha)\varepsilon E_e + (2-\alpha)E_c$
H[NN] and L[YY(3)]	$\Delta q - 3\alpha\varepsilon E_e + \alpha E_c$	$\Delta q + 3\alpha\varepsilon E_e + \alpha E_c$
H[YN] and L[NN]	$\Delta q + \alpha E_e$	$\Delta q + \alpha E_e$
H[YN] and L[YN]	$\Delta q + (1+\alpha)E_e + E_c$	$\Delta q + (1+\alpha)E_e + E_c$
H[YN] and L[NY]	Δq	Δq
H[YN] and L[YY(1)]	$2\Delta q + (\alpha - 3\varepsilon)E_e + E_c$	$2\Delta q + (\alpha + 3\varepsilon)E_e + E_c$
H[YN] and L[YY(2)]	$\Delta q + (\alpha - \varepsilon - 2\alpha\varepsilon)E_e - (1-2\alpha)E_c$	$\Delta q + (\alpha + 2\varepsilon + \alpha\varepsilon)E_e + (2-\alpha)E_c$
H[YN] and L[YY(3)]	$\Delta q + (\alpha - 3\varepsilon)E_e + \alpha E_c$	$\Delta q + (\alpha - 3\varepsilon)E_e + \alpha E_c$
H[NY] and L[NN]	$\Delta q - E_e + E_c$	$\Delta q - E_e + E_c$
H[NY] and L[YN]	$\Delta q + 2E_e$	$\Delta q + 2E_e$
H[NY] and L[NY]	$\Delta q - (1+\alpha)E_e + E_c$	$\Delta q - (1+\alpha)E_e + E_c$
H[YY(1)] and L[NN]	$\Delta q - 3\alpha\varepsilon E_e + \alpha E_c$	$\Delta q + 3\alpha\varepsilon E_e + \alpha E_c$
H[YY(2)] and L[NN]	$\Delta q - (2+\alpha)\varepsilon E_e + (2-\alpha)E_c$	$\Delta q + (1+2\alpha)\varepsilon E_e - (1-2\alpha)E_c$
H[YY(3)] and L[NN]	$\Delta q - 3\varepsilon E_e + E_c$	$\Delta q + 3\varepsilon E_e + E_c$
H[YY(1)] and L[YN]	$\Delta q + (1-3\alpha\varepsilon)E_e + (1+\alpha)E_c$	$\Delta q + (1+3\alpha\varepsilon)E_e + (1+\alpha)E_c$
H[YY(2)] and L[YN]	$\Delta q + (1-2\varepsilon-\alpha\varepsilon)E_e + (3-\alpha)E_c$	$\Delta q + (1+\varepsilon+2\alpha\varepsilon)E_e + 2\alpha E_c$
H[YY(3)] and L[YN]	$\Delta q + (1-3\varepsilon)E_e + 2E_c$	$\Delta q + (1+3\varepsilon)E_e + 2E_c$
H[YY(1)] and L[NY]	$\Delta q - \alpha(1+3\varepsilon)E_e + \alpha E_c$	$\Delta q - \alpha(1-3\varepsilon)E_e + \alpha E_c$
H[YY(2)] and L[NY]	$\Delta q - (\alpha + 2\varepsilon + \alpha\varepsilon)E_e + (2-\alpha)E_c$	$\Delta q - (\alpha - \varepsilon - 2\alpha\varepsilon)E_e - (1-2\alpha)E_c$
H[YY(3)] and L[NY]	$\Delta q - (\alpha + 3\varepsilon)E_e + E_c$	$\Delta q - (\alpha - 3\varepsilon)E_e + E_c$
H[YY] and L[YY(1)]	$\Delta q - 3(1+\alpha)\varepsilon E_e + (1+\alpha)E_c$	$\Delta q + 3(1+\alpha)\varepsilon E_e + (1+\alpha)E_c$
H[YY] and L[YY(2)]	$\Delta q - 3(1+\alpha)\varepsilon E_e + (1+\alpha)E_c$	$\Delta q + 3(1+\alpha)\varepsilon E_e + (1+\alpha)E_c$
H[YY] and L[YY(3)]	$\Delta q - 3(1+\alpha)\varepsilon E_e + (1+\alpha)E_c$	$\Delta q + 3(1+\alpha)\varepsilon E_e + (1+\alpha)E_c$

[a]Recall from (14)–(33) that when both firms buy the same keyword (i.e., H[YY] or L[YY]), the profits are given differently depending on values of (E_e, E_c). Thus, (1) corresponds to the case of $E_e \geq \varepsilon E_c$, (2) to $-\varepsilon E_c \leq E_e \leq \varepsilon E_c$, and (3) to $E_e \leq -\varepsilon E_c$. However, even though under H[NY] and L[YN] profits also depend on the values of (E_e, E_c), because in equilibrium H[NY] never happens when $E_e \geq E_c$ and L[YN] never happens when $E_e \leq -E_c$, we have only one set of price equilibrium for each of them. Finally, we observe no case of H[NY] and L[YY] in equilibrium because one condition for H[NY] ($E_e \leq (2tC_2)/((1-\varepsilon)p_H)$) contradicts one condition for L[YY] ($E_e \geq (2tC_2)/((1-\varepsilon)p_L)$). Thus, this case is omitted.

equilibrium of the other keyword such that α does not affect the equilibrium prices. Thus, in examining the impact of the segment size, we focus on the following equilibria: H[YN] paired with L[NY], L[NY] paired with H[YN], H[NY] paired with L[NN], L[YN] paired with H[NN], H[YY] paired with L[NN], and L[YY] paired with H[NN].

Then, the proposition can be proved by showing the following inequalities:

For part (a), $\partial\varphi_H^{H[YN]}/\partial\alpha \geq 0$ and $\partial\phi_H^{H[YN]}/\partial\alpha \geq 0$, but $\partial\varphi_H^{H[YY]}/\partial\alpha = 0$ and $\partial\phi_H^{H[YY]}/\partial\alpha = 0$ for Firm H, while $\partial\varphi_L^{L[NY]}/\partial\alpha \geq 0$ and $\partial\phi_L^{L[NY]}/\partial\alpha \geq 0$, but $\partial\varphi_L^{L[YY]}/\partial\alpha = 0$ and $\partial\phi_L^{L[YY]}/\partial\alpha = 0$ for Firm L.

For part (b), $\partial\varphi_H^{L[YY]}/\partial\alpha \geq 0$ and $\partial\phi_H^{L[YY]}/\partial\alpha \geq 0$, but $\partial\varphi_H^{L[YN]}/\partial\alpha = 0$ and $\partial\phi_H^{L[YN]}/\partial\alpha = 0$ for Firm H, while $\partial\varphi_L^{H[YY]}/\partial\alpha \geq 0$ and $\partial\phi_L^{H[YY]}/\partial\alpha \geq 0$, but $\partial\varphi_L^{H[NY]}/\partial\alpha = 0$ and $\partial\phi_L^{H[NY]}/\partial\alpha = 0$ for Firm L.

We show these inequalities in order:

$$\frac{\partial\varphi_H^{H[YN]}}{\partial\alpha} = \frac{(3t+\Delta q)E_e}{6t} \geq 0.$$

$$\frac{\partial\phi_H^{H[YN]}}{\partial\alpha} = \frac{(3t+\Delta q-\alpha E_e)E_e}{9t} \geq 0, \text{ noting that}$$

$3t + \Delta q - \alpha E_e \geq$, since $p_H^{H[NN]/L[NY]} \geq 0$.

$$\frac{\partial\varphi_H^{H[YY]}}{\partial\alpha} = 0,$$

since $\varphi_H^{H[YY]} = \dfrac{(1-\varepsilon)(3t+\Delta q-3\varepsilon E_e+E_c)E_e - 6tC_2}{6t}$

does not depend on α.

$$\frac{\partial\phi_H^{H[YY]}}{\partial\alpha} = 0, \text{ since}$$

$\phi_H^{H[YY]} = \dfrac{(1-3\varepsilon)(6t+2\Delta q-(1+3\varepsilon)E_e+2E_c)E_e - 18tC_2}{18t}$

does not depend on α.

$$\frac{\partial\varphi_L^{L[NY]}}{\partial\alpha} = \frac{(3t-\Delta q)E_e}{6t} \geq 0, \text{ noting that } 3t-\Delta q \geq 0,$$

since $p_L^{H[NN]/L[NN]} \geq 0$.

$$\frac{\partial\phi_L^{L[NY]}}{\partial\alpha} = \frac{(3t-\Delta q-\alpha E_e)E_e}{9t} \geq 0, \text{ noting that}$$

$3t - \Delta q - \alpha E_e \geq$, since $p_L^{H[YN]/L[NN]} \geq 0$.

$$\frac{\partial\varphi_L^{L[YY]}}{\partial\alpha} = 0,$$

since $\varphi_L^{L[YY]} = \dfrac{(1-\varepsilon)(3t-\Delta q+3\varepsilon E_e-E_c)E_e - 6tC_2}{6t}$

does not depend on α.

$\dfrac{\partial \phi_{\mathrm{L}}^{\mathrm{L[YY]}}}{\partial \alpha} = 0$, since

$\phi_{\mathrm{H}}^{\mathrm{H[YY]}} = \dfrac{(1-3\varepsilon)(6t-2\Delta q-(1+3\varepsilon)E_e-2E_c)E_e-18tC_2}{18t}$

does not depend on α.

$\dfrac{\partial \varphi_{\mathrm{H}}^{\mathrm{L[YY]}}}{\partial \alpha} = \dfrac{(3t+\Delta q-3\varepsilon E_e+E_c)E_e}{6t} \geq 0$, noting that

$3t+\Delta q-3\varepsilon E_e+E_c \geq 0$, since $p_{\mathrm{H}}^{\mathrm{H[NN]/L[YY(1)]}} \geq 0$.

$\dfrac{\partial \phi_{\mathrm{H}}^{\mathrm{L[YY]}}}{\partial \alpha} = \dfrac{(3t+\Delta q-\alpha E_e)E_e}{9t} \geq 0$, noting that

$3t+\Delta q-\alpha E_e \geq 0$, since $p_{\mathrm{H}}^{\mathrm{H[NN]/L[NY]}} \geq 0$.

$\dfrac{\partial \varphi_{\mathrm{H}}^{\mathrm{L[YN]}}}{\partial \alpha} = 0$,

since $\varphi_{\mathrm{H}}^{\mathrm{L[YN]}} = \dfrac{(3t+\Delta q+E_e+E_c)(E_e+E_e)-6tC_1}{6t}$

does not depend on α.

$\dfrac{\partial \phi_{\mathrm{H}}^{\mathrm{L[YN]}}}{\partial \alpha} = 0$,

since $\phi_{\mathrm{H}}^{\mathrm{L[YN]}} = \dfrac{(6t+2\Delta q+E_e+E_c)(E_e+E_e)-18tC_1}{18t}$

does not depend on α.

$\dfrac{\partial \varphi_{\mathrm{L}}^{\mathrm{H[YY]}}}{\partial \alpha} = \dfrac{(3t-\Delta q-3\varepsilon E_e-E_c)E_e}{6t} \geq 0$, noting that

$3t-\Delta q-3\varepsilon E_e-E_c \geq 0$, since $p_{\mathrm{L}}^{\mathrm{H[YY(3)]/L[NN]}} \geq 0$.

$\dfrac{\partial \phi_{\mathrm{L}}^{\mathrm{H[YY]}}}{\partial \alpha} = \dfrac{(3t-\Delta q-\alpha E_e)E_e}{9t} \geq 0$, noting that

$3t-\Delta q-\alpha E_e \geq 0$, since $p_{\mathrm{L}}^{\mathrm{H[YN]/L[NN]}} \geq 0$.

$\dfrac{\partial \varphi_{\mathrm{L}}^{\mathrm{H[NY]}}}{\partial \alpha} = 0$,

since $\varphi_{\mathrm{L}}^{\mathrm{H[NY]}} = \dfrac{(3t-\Delta q+E_e-E_c)(E_e-E_e)-6tC_1}{6t}$

does not depend on α.

$\dfrac{\partial \phi_{\mathrm{L}}^{\mathrm{H[NY]}}}{\partial \alpha} = 0$,

since $\phi_{\mathrm{L}}^{\mathrm{H[NY]}} = \dfrac{(6t-2\Delta q+E_e-E_c)(E_e-E_e)-18tC_1}{9t}$

does not depend on α. □

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