Valuing bets and hedges: 
Implications for the construct of risk preference

Shane Frederick∗  Amanda Levis†  Steven Malliaris‡  Andrew Meyer§

Abstract

Risk attitudes implied by valuations of risk-increasing assets depart markedly from those implied by valuations of risk-reducing assets. For instance, many are unwilling to pay the expected value for a risky asset or for its perfect hedge. Although nearly every theory of risk preference (and logic) demands a negative correlation between valuations of bets and hedges, we observe positive correlations. This inconsistency is difficult to expunge.

Keywords: bets, hedges, risk attitude

A fair coin is about to be flipped. 
A Heads voucher pays $10 if that coin lands heads.
A Tails voucher pays $10 if that coin lands tails.
What is the most you would pay for a Heads voucher? $______
If you owned a Heads voucher, what is the most you would pay for a Tails voucher? $______

1 Introduction

Analyses of choice under uncertainty typically treat risk aversion as a primitive and stylized fact. As frequently-cited evidence, the certainty equivalent of a gamble is nearly always below its expected value. For instance, a typical person finds a sure $3 about as attractive as a coin flip for $10.

From the perspective of Expected Utility Theory, the level of risk aversion implied by such small-stakes choices exceeds the level exhibited at larger stakes. For example, someone who’d always prefer a sure $3 over a coin flip for $10 must also prefer a sure $100 over a coin flip for a billion dollars (Rabin 2000; Rabin & Thaler, 2001). Prospect Theory (Kahneman & Tversky, 1979) was developed, in part, to accommodate such discrepancies.

In this project, we investigate a different sort of inconsistency in risk attitudes by comparing the valuation of a bet (e.g., a voucher which pays $10 if a coin lands heads) with the valuation of its perfect hedge (e.g., a second voucher which pays $10 if tails obtains). Logic requires that these two valuations sum to the amount their joint possession guarantees (e.g., $10) and, accordingly, correlate −1.0.1 We find, instead, that they correlate positively. Furthermore, risk aversion implies that hedges should be worth more than their expected value – an implication many find counterintuitive.

Because lower bet valuations imply more risk aversion, whereas lower hedge valuations imply more risk tolerance, a positive correlation between them means that those who appear more risk-averse by one measure appear more risk-tolerant by the other. Although previous research has questioned the generalizability2 and predictive power3 of risk attitudes,

Note that someone who would only pay $3 for a $10 Heads voucher should pay up to $7 for the Tails voucher – simply because that is the difference between the $10 the pair of vouchers guarantees and their stated valuation for the Heads voucher ($3). This logic assumes that person has purchased the bet at the highest price they would pay (so that owners of a bet are not wealthier than non-owners) or that the stakes are small (so that wealth effects can be neglected). If neither of these is satisfied, this theoretical requirement no longer holds – though risk aversion still implies that hedges be worth more than their expected value. We discuss wealth effects in Appendix A.

For example, when examining behaviors that increase or reduce health risks (drinking, smoking, exercising, taking medications, mammograms), Cutler and Glaeser (2005) find that only smoking and drinking correlate appreciably (0.16). Similarly, the risk attitudes implied by insurance choices and 401(k) portfolio allocations are essentially uncorrelated (Einav et al., 2012; see also Barseghyan et al., 2011), as are behaviors like forgoing health insurance, choosing self-employment, or investing in stocks rather than Treasury bills (Barsky et al., 1997).

Since diversification is a nearly costless way to reduce variance, most models of investor behavior presume people will be highly diversified, yet they often are not. Besides constructing portfolios with too few stocks


our results are even more problematic, as these two measures come from the same domain (small stakes gambles involving money) and not only fail to cohere, but strongly contradict one another. Though levels of risk aversion are known to vary across elicitation procedures, many nevertheless assume that such procedures at least serve to rank individuals by their risk attitudes (see, e.g., Charness, Gneezy & Imas, 2013, p. 50). However, our results cast doubt upon even this more modest claim. They also raise the question of how bet and hedge valuations would be altered by an appreciation of this logic, if that could somehow be instilled – which is not so easy, as we will show.

2 Studies 1a to 1g

Our first set of experiments (summarized in Table 1) document this curious phenomenon. Though designs varied slightly (see Appendix B for methodological details), each participant was essentially asked the most they would pay for a 50% chance of $10 and the most they would pay to convert that to a certain $10 (by acquiring a perfect hedge). Logic requires that one’s valuation of the hedge equal $10 minus their valuation of the bet; that all responses in Figure 1 lie along the southeast diagonal (y=10-x). But as can be seen, few actually do, in any of the studies.4

In study 1b, we separately analyzed results for 267 participants who answered $10 when asked: “What is the most you would pay for $10? In study 1g, we separately analyzed results for 240 participants who valued a pair of $5 Amazon gift certificates at $10 and scored perfectly on an eight-item test intended to assess their comprehension of bets and hedges (see Appendix B). In both subsets, results are consistent with the full sample. Since those who would pay $10 for $10 (or for two $5 gift certificates) were probably not understating their willingness to pay, this weighs against the idea that the positive relation between bet and hedge valuations was driven by heterogeneity in the degree to which participants shaded their true valuations downward.

The non-normative positive correlation between bets and hedges is found among everyone, though it is weaker among those scoring higher on the Cognitive Reflection Test (Frederick, 2005); the bet-hedge correlations among those scoring a 0, 1, 2, and 3 on the CRT are 0.67, 0.58, 0.50, and 0.28 respectively. (See Appendix C for further details.)
Valuing bets and their hedges

Studies 2a and 2b

Evaluating Heads and Tails vouchers separately and sequentially partially obscures their complementarity. Thus, in our next two studies, MTurk workers chose between the pair of vouchers {Heads & Tails} and the package {Heads & $5} (see Table 2). With this more transparent formulation, most did prefer the voucher pair – and thus, at least implicitly, valued the Tails hedge above its expected value, as risk aversion dictates. However, even here, the explanatory power of risk preference remains in doubt, since those who preferred {Heads & Tails} to {Heads & $5} should also have preferred...
Suppose a fair coin is going to be flipped once. A “HEADS Voucher” pays out $10 if the coin lands heads. A “TAILS Voucher” pays out $10 if the coin lands tails.

Which would you rather have?
A) A HEADS voucher and a TAILS voucher 66%
B) A HEADS voucher and a five dollar bill 34%

Which would you rather have?
A) A HEADS voucher 30%
B) A five dollar bill 70%

A sure $5 over a single Heads voucher, and vice versa. Yet we observed little relation between those two choices.8

Study 3

When considered separately, the Heads and Tails vouchers are symmetric: equivalent and interchangeable. Thus, it is easy to understand why many respondents value them similarly, even though the first voucher adds risk and the second removes it. To test whether respondents would explicitly endorse the symmetry argument over the normative argument, we presented them side by side (Table 3), with order counterbalanced, and simply asked respondents to choose one.

Only one in ten respondents (53/505) chose the correct argument ($7) over the symmetry argument ($3). Moreover, this small minority might actually have been even more confused, as those choosing $7 were less likely to pass an attention check and did significantly worse on a numeracy test appended to subsequent demographics questions (see Appendix D).9

In 2a, those choosing the risk-free bundle of vouchers were more likely to choose the risky single voucher in the simple choice (r = 0.18). In 2b, the relation was still weak, but at least correctly signed (r = -0.14).

8We included an attention check following this question to help identify insincere, careless, or random responding. We excluded twenty-six participants for indicating that they had already suffered a fatal heart attack or been bitten by a great white shark. Among those who chose $7, nearly one in five failed this subsequent attention check (12/65), compared to fewer than one in thirty among those who answered $3 (14/466).

Study 4

To slightly generalize our basic paradigm, we next removed the symmetry between the bet and the hedge. We asked 182 MTurkers how much they would pay for four different bets: a 20% chance of $100, a 40% chance of $100, a 60% chance of $100, and an 80% chance of $100 as well as how much they would pay for the four corresponding hedges (which pay off in the remaining states). All four bet-hedge pairs were presented to each participant in random order. For each pair, respondents indicated their valuations of the bet, and then their valuations of the corresponding hedge. As before, possession of both guarantees the prize, and thus

### Table 2: Materials for Studies 2a and 2b.

<table>
<thead>
<tr>
<th>Study 2a (N=297)</th>
<th>Study 2b (N=299)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Suppose a fair coin is going to be flipped once.</strong> A “HEADS Voucher” pays out $10 if the coin lands heads. A “TAILS Voucher” pays out $10 if the coin lands tails.</td>
<td><strong>Suppose a fair coin is going to be flipped once.</strong> A “HEADS Voucher” pays out $10 if the coin lands heads. A “TAILS Voucher” pays out $10 if the coin lands tails.</td>
</tr>
<tr>
<td>Which would you rather have?</td>
<td>Which would you rather have?</td>
</tr>
<tr>
<td>A) A HEADS voucher and a TAILS voucher 66%</td>
<td>A) A HEADS voucher and a TAILS voucher, so that you make $10 if the coin lands heads and $10 if the coin lands tails 74%</td>
</tr>
<tr>
<td>B) A HEADS voucher and a five dollar bill 34%</td>
<td>B) A HEADS voucher and a five dollar bill, so that you make $15 if the coin lands heads and $5 if the coin lands tails 26%</td>
</tr>
<tr>
<td>Which would you rather have?</td>
<td>Which would you rather have?</td>
</tr>
<tr>
<td>A) A HEADS voucher 30%</td>
<td>A) A HEADS voucher, so that you make $10 if the coin lands heads and $0 if the coin lands tails 11%</td>
</tr>
<tr>
<td>B) A five dollar bill 70%</td>
<td>B) A five dollar bill, so that you make $5 if the coin lands heads and $5 if the coin lands tails 89%</td>
</tr>
</tbody>
</table>

### Table 3: Materials for Study 3

A fair coin will be flipped once. A HEADS voucher pays $10 if the coin lands HEADS. A TAILS voucher pays $10 if the coin lands TAILS.

Bob was willing to pay up to $3 for a HEADS voucher and purchased one at that price.

How much should he be willing to pay for a TAILS voucher? (So that he would own both vouchers before the coin is flipped.)
- $7 (He should value the pair of vouchers at $10 because owning both guarantees him $10.)
- $3 (He should value each voucher the same because each offers the same chance of $10.)
However, unlike before, the bet and hedge are not symmetric, as they have different probabilities of delivering the prize. The results are summarized in Figure 2.

With symmetry removed, valuations of the bet and the hedge correlate much more weakly, though still positively. Again, almost no data lie along the southeast diagonal, as the two valuations sum to well below $100 in all four cases. The mean valuations of the bet and hedge are reported in Table 4, with the subscript representing the asset’s valuation relative to its expected value.

A comparison of the subscripts reveals that the bets and hedges generally deviate in the same direction from expected value (below it), rather than diverging to reveal a coherent attitude toward risk. However, though the two valuations do not cohere, hedges are, at least, valued at a higher fraction of their expected value. Moreover, respondents are willing to pay more than expected value to hedge the small residual risk of the 80% bet – providing at least one instance where a perfect hedge is priced above its expected value ($20), as should be universally expected for respondents who are risk averse.

The considerable difference between bet and hedge subscripts in Table 4 suggests that respondents are not simply ignoring possession of the bet when evaluating the hedge. For instance, they are willing to pay $6 for a 20% chance of $100 alone, but $38 for the same asset once they already own an 80% chance of $100. Although respondents clearly fail to fully appreciate the covariance between bets and hedges, the pattern remains distinct from complete covariance neglect, in which hedges and bets are treated as independent.10

Nor are respondents in earlier “Heads and Tails” studies merely referencing their valuation of the bet to construct an equivalent valuation of the hedge. Although a substantial minority of hedge prices do equal bet prices in those studies, most don’t. And of the minority that do, many come from respondents who “simply” report the expected value for each asset ($5, $5). Though those are certainly reasonable valuations (and demanded by EUT), we strongly suspect that many of these respondents are treating these questions as math problems rather than an elicitation of their preferences (and would continue to just perform the math even if the numbers referenced millions of dollars). This suspicion draws some support from an analysis in Appendix C, which reports the data broken down by CRT score: those who answer ($5, $5) are no more reflective than off diagonal respondents and less reflective than the small minority of responses that lie elsewhere on the southeast diagonal.

In opposition to various “narrow framing” theories (e.g., Tversky & Kahneman, 1986) that assume respondents ignore possession of the bet when evaluating the hedge stand “multiple reference point” theories (e.g., Koszegi & Rabin, 2006, 2007) in which evaluation of hedge entails full consideration of both possible outcomes of the bet: the one in which the bet pays off (in which case money spent on the hedge is a waste and a loss) and the one in which the bet does not (but the hedge does, minus acquisition costs). By this multiple reference point formulation, acquisition of the hedge constitutes both a loss and a gain, which tends to make an unattractive combination to the extent that losses loom larger than gains. (See Appendix E for an examination of whether the multiple

10In general, evaluating the riskiness of a portfolio is challenging (Cornil & Bart 2013, Reinholz et al., 2018), as it depends on both the volatility of the components and their covariance. However, evaluating Heads and Tails vouchers should be comparatively easy, since the covariance structure is unusually stark and possession of the pair eliminates volatility.

![Figure 2: WTP for a P% chance at $100 and for its perfect hedge. The figure plots bet valuations (x-axis) against hedge valuations (y-axis) for each probability of $100 in Study 4. Dot area is proportional to number of participants at each coordinate.](image-url)
reference point perspective can make sense of our results. We conclude it cannot.11

3 Discussion

Valuations of bets and hedges are theoretically equivalent measures of risk attitudes, yet they often compel opposite conclusions. For instance, many who place a low value on risk-creating bets also place a low value on risk-reducing hedges. This marked departure from theoretical expectations seemingly impugns the construct validity of risk attitudes – even within the narrow domain of stylized monetary gambles. Of course, identifying discrepancies between a subset of measurement techniques isn’t usually regarded as sufficient cause to jettison a theoretically cherished construct – at least not within the social sciences. Nevertheless, to the extent that the two measures depart, it certainly raises the question about which, if either, better captures whatever we think we mean by risk aversion.

Adjudicating between potential measures of a putative construct requires the very thing that is lacking for constructs whose validity is still in question – agreement about the other thing(s) with which we should expect them to correlate. For instance, suppose Holt and Laury’s (2002) measure of risk preferences had corresponded much more highly with hedge valuations than with bet valuations. The conclusions drawn from this will still be conditioned by your prior faith in those measures. If you are confident that Holt and Laury’s measure captures the construct you care about, it would affirm hedge valuations over bet valuations, but if you are confident that bet valuations best capture risk attitudes, the lack of relation with that other method would impugn that method.

In the experimental paradigm we use most commonly, the hedges are perfect, such that possession of one renders the coin irrelevant. This evokes the image of a very unusual transaction in which an owner of the bet pays $7 for the hedge and is then immediately handed $10. Since this is obviously equivalent to simply receiving $3, buying the hedge is equivalent to selling the bet. But the value of a perfect hedge also determines the aggregate value of the partial hedges from which it might be constructed. Consider an experiment in which chances to win a $100 prize can be purchased in cumulative increments of one percentage point each: a single point entitles its owner to a 1% chance of $100; fifteen points yields a 15% chance of $100, and so on. The expected value of each 1% chance is, of course, $1. Now consider a respondent for whom a 50% chance of winning a $100 prize is worth $30. That person values the first 50 points at 60 cents each, on average. However, since the next 50 points must be worth $70 in total, their average value must be $1.40. Moreover, if any of the incremental percentage points beyond 50% are also valued below $1.00, achieving that $1.40 average requires that valuation of later increments exceed $1.40. In other words, continuity demands that these partial hedges must eventually be worth much more than their expected value – and this point will typically come well before one has acquired a 100% chance of winning.

Note further that once a prize becomes more likely than not, additional increases in the chances of winning reduce variance at an accelerating rate. Since aversion to variance dominates every formal definition of risk aversion, those who are risk averse should often treat partial hedges like perfect hedges – as variance reducing assets that are worth even more than their expected value. Thus, assets that eliminate risk, like Tails vouchers, are an illustrative case, but not a special case.

The construct of risk aversion seemingly draws support from the popularity of insurance contracts, on which U.S. customers alone spend over a trillion dollars a year.12 However, while people typically insure against their house catching fire, they rarely insure against a decline in house prices, though home equity comprises most of a household’s net worth at retirement.13 Moreover, analogous contracts that extract a premium to reduce the variability of uncertain gains are also rare.14 Thus, as Friedman et al. (2014) point out, while insurance contracts are typically invoked as evidence of risk aversion, customer behavior actually departs from textbook risk aversion; customers appear motivated to reduce the possibility of some types of harm, rather than reduce variance, per se. This distinction between downside risk and upside risk raises further questions about how subjects in our experiments interpret an actuarially unfair hedge: as an attractive premium to limit losses from unfavorable re-

11Although much auto and home insurance is legally or contractually required, this still reflects a massive social demand for this form of insurance contract.

12See, e.g., Poterba (2014). This calculation does not include the present value of future Social Security payments, which in most cases is even more valuable than accumulated home equity. While the opportunity for households to hedge home price risk became available with the introduction of S&P/Case-Shiller home price futures on the Chicago Mercantile Exchange in 2006, trading volumes and open interest were “not impressive” during the housing crisis; by 2014, volume had fallen even further, dropping below one-twentieth of its 2007 level (Choi and Lee 2015).

13For example, annuities remain available but unpopular, despite allowing households to substitute a steady consumption stream for a volatile one. Quoting Franco Modigliani’s Nobel speech, Brown (2009) writes: “It is a well-known fact that annuity contracts, other than in the form of group insurance through pension systems, are extremely rare. Why this should be so is a subject of considerable current interest. It is still ill-understood.”
eralizations of their risky asset (e.g., spending $700,000 on insurance to guard against the 50% chance their million dollar house will burn down) or as an unattractive censoring of the upside of their risky asset (e.g., as guaranteeing a mere $300,000 when they know their house will be worth a million if it does not burn down).

When we’ve presented this research, the most common objection is that respondents are just confused. We “concede” that, at some level, they are. For instance, consider the common response of someone who indicates they’d pay up to $3 for the bet and up to $3 for a hedge (if they had purchased or were endowed with the bet). This, in turn implies they’d pay up to $6 for both, but not, say, $7.50. But since the bet and hedge are worth $10 in combination, their responses imply they would decline receiving $2.50 if you attempted to hand it to them. This is obviously false and so their answer is, in an important sense, a mistake. And this mistake persists even when participants are explicitly given the normative explanation, as in Study 3. However, we see these mistakes as the phenomenon of interest. We don’t doubt that after a sufficiently intense and prolonged training session respondents could generate a normative pair of valuations, much as they could be taught to use Venn Diagrams or apply Bayes’ Rule, or produce normative responses in many other contexts. But this doesn’t vitiate the phenomenon nor remove the challenges it poses to conceptions of risk attitudes.

References


