

# **An Empirical Bargaining Model with Numbers Biases**

## **– A Study on Auto Loan Monthly Payments**

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### *Abstract*

This paper studies price bargaining when both parties are subject to perception biases with numbers. The empirical analysis focuses on the auto finance market in the U.S., using a large data set of 35 million auto loans. I observe that the scheduled monthly payments of auto loans bunch at \$9- and \$0-ending digits, especially over \$100 marks. The number of loans also increases from \$1- to \$8-ending digits. It is especially intriguing that \$9-ending loans carry a higher interest rate and \$0-ending loans have a lower interest rate than loans ended at other digits. Motivated by these observations, I develop and estimate a Nash bargaining model that allows for number biases from both consumers and finance managers of auto dealers. Results suggest that both parties perceive a discontinuity between payments ending at \$99 and \$00, and a steeper slope for larger ending digits, in their payoff functions. Low income and minority consumers have a lower bargaining power than the others. This model can explain the phenomena of payments bunching and differential interest rates for loans with different ending digits. I use counterfactual to show that, counter-intuitively, having number biases is beneficial in a bargaining setting. Consumers' payments are reduced by \$203 million in total and the aggregate payments of finance managers increased by \$102 million because of own number biases. Another counterfactual quantifies the economic impact of imposing non-discretionary markup compensation policies. I find that the payments of African American consumers will be lowered by \$452-473 million and that of Hispanic consumers by \$275-300 million in total.

*Key words:* Bargaining, Number Biases, Auto Finance, Minority Consumers, Dealer Compensation

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# 1. Introduction

Bargaining is a commonly used price-setting mechanism in many markets such as automobiles and B-to-B transactions. In bargaining, final prices vary across transactions instead of set by one side as fixed posted prices. The two parties in negotiations evaluate the key variable of interest (e.g., price) and reach a bargaining outcome depending on their relative bargaining power. Most of the empirical bargaining literature characterizes the perceived value of the bargaining outcome with a fully rational model and focuses on evaluating the key determinants of bargaining power that lead to the observed bargaining outcomes (e.g., Draganska et al. 2010). However, people often use simple cognitive shortcuts when processing information, which makes accounting for bounded rationality important in describing economic behaviors (see Conlisk 1996 for a review). In a bargaining setting, decision-makers on both sides are human beings. Behavioral decision researchers have long recognized psychological influence in negotiation, such as status quo bias and reciprocity heuristic (Malhotra and Bazerman 2008). Decision-makers may also be subject to perception biases when evaluating numbers. For example, people have the tendency to focus on the leftmost digit of a number while partially ignoring other digits (Poltrock and Schwartz 1984, Lacetera et al. 2012). With such a bias, a number with 99-ending (e.g., \$299) may be perceived to be significantly lower than the next round number (e.g., \$300). One consequence of such biases in the marketplace is the ubiquitous 99-cents pricing (Thomas and Morwitz 2005, Basu 2006).

In this paper, I empirically study a bargaining setting where the bargaining outcomes are affected by number biases in addition to bargaining power. When *both parties* are influenced by number biases, they will try to push the price toward their favorite side. For example, while buyers prefer a price with 99-ending digit, sellers perceive a price a bit higher with 00-ending digit to be a better deal. This makes the bargaining outcome different from when prices are only set by one party. In this study, I use a large data set with 35 million auto loans in the U.S. over a period of four years, and discover several intriguing data patterns. First, the scheduled monthly payments of auto loans bunch at both \$9- and \$0-endings. This bunching pattern is stronger over \$100 marks, with more than twice as many loans with \$99-ending and 1.5 times as many loans with \$00-ending, as loans with \$01-ending. Furthermore, the number of loans is systematically higher for larger ending digits (from \$1 to \$8). Second, while the interest rate for \$9-ending loans is 0.6% higher than the average, the rate for \$0-ending loans is 0.5% lower, after controlling for all consumer characteristics such as credit scores. Finally, I find that consumers with a minority origin (African American or Hispanic) and low income are more likely to have \$9-ending loans, and pay a higher interest rate, than other consumers with a similar credit profile and loan attributes. These data patterns are difficult to explain by a standard economic model. I therefore develop a bargaining model that allows for number biases from

both parties in the bargaining, which can explain the phenomena of payments bunching and differential interest rates across consumer loan payments in the data.

The auto finance market provides a perfect setting for studying price bargaining. The dealer markup compensation policy in the indirect auto lending market leads to negotiations that cause loan payments to vary across transactions. In a standard loan arrangement, banks quote a risk-adjusted interest rate, called bank buy rate, based on the consumers' risk profile (e.g., credit score). On top of the bank buy rate, auto dealers charge consumers a markup, which represents their compensation for arranging the loan. Unlike the bank buy rate, the markup reflects the relative bargaining power between consumers and finance managers of auto dealers. Thus, loan payments are the outcome of price negotiations instead of fixed prices. Studying how consumer loan payments are determined has important policy implications. The markup compensation policy has attracted much debate and legal actions. Opponents to this policy alleged that minority consumers end up paying interest rates higher than similarly situated Caucasian borrowers (e.g., Munro et al. 2005). Auto loans represent an expensive purchase for consumers with large impact on their financial situations. With 107 million Americans carrying an auto loan<sup>2</sup>, the size of the auto finance market makes this study economically important.

I seek to address two main research questions in this study. The first question is to understand how individual number biases affect bargaining outcomes. I show that, counter-intuitively, having number biases is beneficial for the party with the biases in bargaining. This is achieved by building a bargaining model that incorporates number biases from both sides, estimating the model with auto loan data, and exploring the effect of biases on bargaining outcomes through a counterfactual analysis. The second question is to quantify the change in loan payments for minority consumers if the discretionary markup compensation is banned through regulatory policy. This is obtained by evaluating the change in payment outcomes among different consumer groups through another counterfactual analysis, in which banks offer dealers two alternative non-discretionary compensation policies.

### **1.1. Research Strategy and Main Findings**

Given the nature of the dealer compensation policy, I propose a bargaining model involving individual consumers and finance managers with loan payments as the equilibrium outcome of a Nash bargaining game. The model allows both parties to have potential perception biases toward numbers in their payoff functions. Guided by reduced-form data patterns, I assume the payoff functions can have two types of biases. First, payoff functions can have a discontinuity between payments ending at \$99 and \$00. Moreover,

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<sup>2</sup> Federal Reserve Bank of New York, Quarterly Report on Household Debt and Credit, May 2017 Q1. [https://www.newyorkfed.org/medialibrary/interactives/householdcredit/data/pdf/HHDC\\_2017Q1.pdf](https://www.newyorkfed.org/medialibrary/interactives/householdcredit/data/pdf/HHDC_2017Q1.pdf)

the perceived difference from a \$1 change in payment can depend on its ending digit, i.e., the perceived change from \$8 to \$9 can be different from that from \$7 to \$8. Note that neither of the biases is imposed in payoff functions. Depending on the model parameters estimated from data, the payoff functions of both parties can reflect the biases or reduce to a standard bargaining model without biases.

I estimate the model on the data that consist of realized loan outcomes and consumer characteristics. I use the simulated method of moments, with the first and second moment conditions, in the estimation. To pin down the model parameters associated with the number biases of consumers and finance managers, I impose a set of linear equality constraints, under which the proportion of simulated loan payments ending at each digit is the same as observed from data, in the criterion function.

Estimation results suggest that number biases exist not only for consumers but also for finance managers. For consumers, the perceived difference between \$99- and \$00-ending payments is \$2.05 instead of \$1. Their sensitivity toward a \$1 change is higher with larger ending digits (e.g., the perceived gap between \$9- and \$0-ending is \$1.27, which is higher than the gap between \$0- and \$1-ending at \$0.9). The estimated biases for finance managers are similar. Standard economic studies usually assume that companies are fully rational entities in making business decisions. In this setting, however, finance managers who represent auto dealers are also human beings. They can be subject to the same human tendency with numbers when negotiating with consumers. This study thus adds to the existing literature that documents psychological biases among professionals, such as lawyers in legal disputes (Birke and Fox 1999), professional traders in trading activities (Coval and Shumway 2000), and managers in a multinational corporation regarding strategic initiatives (Workman 2012). I show that incorporating number biases from both parties is important for two reasons. First, it is essential to explain the observed data patterns. In particular, the biases drive payments bunching at \$0-ending digits with a lower interest rate and at \$9-ending digits with a higher interest rate. Second, failure to account for the biases can lead to biased estimates for consumers' bargaining power.

With the model estimates, I explore the effects of the biases on bargaining outcomes using a counterfactual analysis. Under the scenario where consumers and/or finance managers are not subject to the biases, their payoff functions become linear and continuous. I compare the loan payments when consumers are subject to biases compared to when they are not. Behavioral biases are typically thought to make people worse off. Counter-intuitively, I find that having the biases actually benefits consumers in the bargaining, as they end up with lower monthly payments (\$203 million in total, or 0.025%). This is because the biases act as a psychological hurdle to stop finance managers to push the payments higher than \$9- or \$99-ending digits. The effect is more significant for low bargaining power consumers. Similarly, dealers will receive a higher markup profit (\$102 million in total, or 0.013%) when finance managers are subject

to the biases, as it is more difficult for consumers to push payments below \$0- or \$00-ending payments. When both parties have the biases, the total loan payments will be reduced by \$33 million compared to the scenario where neither party has the biases.

The estimated bargaining model allows me to quantify the economic impacts from alternative dealer compensation policies. In 2013, the Consumer Financial Protection Bureau (CFPB) issued a bulletin announcing that it would hold indirect auto lenders accountable for discriminatory pricing.<sup>3</sup> Since then, the CFPB has taken action against several large auto lenders with significant fines. Despite this, the discretionary markup practice is still commonly used by most indirect auto lenders. I use another counterfactual analysis to investigate the impacts from a non-discretionary markup policy. Fixing the total profit for dealers at the same level, I measure the subsequent change in loan payments for minority consumers. My calculation shows that consumers from predominantly African American and Hispanic geographical locations will pay \$374 and \$373 less for a loan, respectively. In total, the savings are \$452 million for African American consumers, and \$275 million for Hispanic consumers.

## **1.2. Related Literature**

This paper is related to the literature in bargaining, numerical cognition and 9-ending prices, as well as studies of the bunching phenomenon.

The prior bargaining literature has studied price negotiation in the contexts of automotive sales (Chen et al. 2008, Morton et al. 2011, Larsen 2018), B-to-B transactions (Draganska et al. 2010, Grennan 2014) and interactions between online sellers and buyers (Backus et al. 2018, Zhang et al. 2018). Most of the empirical bargaining literature assumes fully rational agents, and studies how bargaining power influences bargaining outcomes. This paper contributes to the empirical bargaining literature by studying how number biases from both sides also influence the bargaining outcomes. I show that considering number biases is essential in explaining the puzzling reduced-form data patterns in bargaining outcomes. In addition, failure to incorporate these biases could lead to biased estimates in the model. The insights could generalize to other settings where bargaining outcomes are numeric in nature.

This paper also draws on the literature on numerical cognition, and the marketing literature on 9-ending prices. The numerical cognition literature in psychology primarily focuses on the differences in behavioral perception between round and precise numbers. Past research has shown that buyers may underestimate the magnitude of precise prices (Thomas et al. 2010), and that precise numbers signal sellers' confidence (Jerez-Fernandez et al. 2014). Offers at round numbers, however, can symbolize completion (Yan et al. 2017) and willingness to cut prices (Backus et al., 2018). This paper also draws from the

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<sup>3</sup> [https://files.consumerfinance.gov/f/201303\\_cfpb\\_march\\_-Auto-Finance-Bulletin.pdf](https://files.consumerfinance.gov/f/201303_cfpb_march_-Auto-Finance-Bulletin.pdf)

marketing literature that studies the prevalence of 9-ending prices in retail sales (e.g., Monroe 1973, Schindler and Kibarian 1996, Stiving and Winer 1997, Anderson and Simester 2003, Thomas and Morwitz 2005). 9-ending prices are generally found to have positive impact on sales, because consumers round down the prices or the prices signal a low-price image. This phenomenon is not limited to prices only. Lacetera et al. (2012) find a discontinuous drop in the price of used cars when the odometer crosses 10,000 miles, driven by the left-digit bias of consumers when processing mileage. In this paper, I build on the numerical biases theory in several ways. I study the impact of such biases in a bargaining setting with an economically significant purchase. Bargaining involves two-sided interaction. I show that in a bargaining setting number biases exist not only among consumers but also among finance managers. Furthermore, beyond the effect of \$9- and \$0-endings, I examine the different sensitivity of payment changes as the ending digit increases from \$1 to \$9.

This paper is also related to the economic literature that studies of bunching phenomena. Bunching is commonly observed at the level where discontinuities in monetary incentives occur, such as income bunching at the level where tax rate changes (Saez 2010), and drug demand bunching at the level where insurance payment jumps (Einav et al. 2015). Bunching can also be driven by psychological incentives. For example, the finishing times of marathon races bunch before hour marks, because the hour marks serve as a reference point (Allen et al. 2016). In the above examples, bunching occurs because consumers make one-sided decisions that are driven by the incentive discontinuity. This paper studies the bunching phenomenon with consumers and finance managers bargaining on auto loan payments. It leads to payments bunching at both \$9- and \$0-ending digits with systematically different interest rates in the opposite direction.

The rest of the paper is organized as follows. Section 2 introduces the auto finance industry background and presents reduced-form data patterns. I describe the bargaining model incorporating biases with numbers in Section 3 and discuss the model estimation and identification issue in Section 4. Section 5 presents the estimation results and findings from counterfactual analyses. Finally, Section 6 concludes.

## **2. Industry Background and Data**

### **2.1. The Auto Finance Industry Background**

The auto finance market is of high economic significance. With a \$1.2 trillion balance in 2017,<sup>4</sup> auto loans represent the third largest consumer credit market in the United States. The auto finance market is crucial

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<sup>4</sup> Federal Reserve Bank of New York, Quarterly Report on Household Debt and Credit, May 2017 Q1. [https://www.newyorkfed.org/medialibrary/interactives/householdcredit/data/pdf/HHDC\\_2017Q1.pdf](https://www.newyorkfed.org/medialibrary/interactives/householdcredit/data/pdf/HHDC_2017Q1.pdf)

to the automotive industry as over 80% of new vehicles sold in the United States are financed.<sup>5</sup> In this market, consumers typically obtain financing through auto dealers (i.e., indirect auto loans). Cohen (2012) shows that about 80% of auto loans are originated at a dealer location following the purchase of a new or used vehicle. Indirect auto loans are a significant source of profit for dealers. Keenan (2000) estimates that 12.9% of dealership profit come from financing and insurance.

In this paper, I focus on cases where consumers get auto loans from a traditional bank through an auto dealer<sup>6</sup>. In a typical transaction at the auto dealer, the consumer first chooses a car and negotiates on the car price itself. After that, she will be brought to the finance manager's office to arrange auto financing. The focus of this paper is to study how the monthly payment number is determined after consumers have selected the loan amount, i.e., how much to borrow, and the loan length, i.e., how long to borrow<sup>7</sup>. Why is the monthly payment a bargained outcome? This is because auto dealers get compensated by arranging auto financing for consumers from a bank. Finance managers from auto dealers add a markup on top of the bank buy rate as part of the total consumer cost. The extra markup serves as the dealer compensation for arranging the loan. Unlike the bank buy rate, which is determined by the consumer's credit risk, the markup is at the dealer's discretion and depends on with whom the dealer arranges the loan.

Because of the markup policy, the finance manager has incentive to increase the loan payment so that the dealer will receive a higher markup. Yet the consumer can negotiate for a lower payment if she finds the payment too high. A report by the Center for Responsible Lending estimates that the average markup is \$714 per transaction using 2009 auto industry data and the markup varies across individual consumers. In my data, I find that the interest rate for auto loans varies a lot for consumers with the same credit profile and loan characteristics. This suggests that there is room for bargaining the loan payment in each transaction.

## **2.2. Data Description**

The empirical analysis of this paper leverages anonymized data on individual credit profiles provided by Equifax Inc., one of the three major credit bureaus in the United States. The data sample includes all non-subprime<sup>8</sup> auto loans originated from banks or credit unions in the United States during a four-year period

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<sup>5</sup> Consumer Reports, Consumers Rely on Car Financing More than Ever.

<http://www.consumerreports.org/cro/news/2013/09/car-financing-on-rise-loans-and-leases/index.htm>

<sup>6</sup> I do not consider auto loans from manufacturing financing (e.g., Toyota Financial) because these loans are often provided to promote the vehicle sales.

<sup>7</sup> After loan amount and length are determined, the monthly payment and interest rate are one-to-one, where a higher interest rate will imply a higher monthly payment and vice versa.

<sup>8</sup> Non-subprime consumers refer to those with at least 620 credit score at the time of auto loan origination. Subprime lending typically involves additional information required, such as verified employment and income through providing pay stubs or tax return documents, beyond the standard credit profile. This information can lead to additional variation in interest rates. As the required additional information is unobserved from my data, I exclude subprime consumers in

from 2011 to 2014. For each auto loan in the sample, I observe the origination date, loan amount, loan length, and scheduled monthly payment. The annual percentage rate (APR)<sup>9</sup> can be calculated from the loan amount, loan length, and the monthly payment (see Appendix A for detail). To remove potential outliers, I select auto loans with loan lengths from 2 to 8 years, loan amount between \$10k and \$60k, and APRs above 1.9%.<sup>10</sup> The selected data sample includes 35 million auto loans. Panel A of Table 1 shows some descriptive statistics for the loan characteristics. The average loan amount is about \$23,000, with a \$399 monthly payment for about five and half years, and the average APR is 4.3%.

**Table 1. Summary Statistics**

	Mean	25 <sup>th</sup> Percentile	Median	75 <sup>th</sup> Percentile
Panel A: Loan Characteristics				
Loan Amount (\$)	22,965	15,821	21,161	28,115
Loan Length (years)	5.4	5	5.8	6
Monthly Payment (\$)	399	294	370	475
APR	4.8%	3.0%	4.0%	5.5%
Panel B: Consumer Characteristics				
Credit Score (620-850)	726	674	725	778
Age	46	33	45	56
Income (\$)	83,749	56,578	74,659	101,062
House Value (\$)	207,185	121,200	168,300	248,100
Caucasian (%)	0.733	0.603	0.787	0.904
Hispanic (%)	0.097	0.025	0.056	0.131
African American (%)	0.089	0.014	0.040	0.104
Asian (%)	0.040	0.009	0.020	0.045
Other (%)	0.041	0.009	0.022	0.054

For consumer characteristics, I observe the credit score and age of each consumer as well as the 5-digit zip code of her living place. The credit score is measured at the month of auto loan origination. I further obtain the average household income, house value and racial composition at the zip code. The average house value comes from the American Community Survey. Household income and racial composition data comes from the Census. It measures the percentage of population that is Caucasian, African American, Hispanic, Asian, or others. I use these data to proxy for the household characteristics of individual consumers. Panel B of Table 1 shows some descriptive statistics for these variables. An average

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the analysis to avoid potential bias in the analysis (e.g. a high loan payment can be due to the consumer being unemployed and not because of her low bargaining power).

<sup>9</sup> I use APR and interest rate interchangeably in the paper.

<sup>10</sup> Loans with lower interest rates are very likely to be special promotional rates. They are commonly seen in manufacturer financing (e.g., Toyota Financial Service), with the goal of promoting vehicle sales.



consumer in the data sample is 46 years old, has 726 credit score, lives in an area with an average \$83.7k household income, \$207k house value, 73.3% Caucasians, 9.7% Hispanics, and 8.9% African Americans.

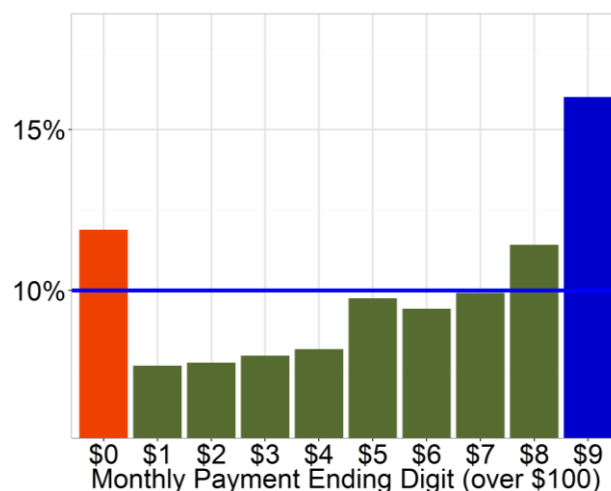
## 2.3. Reduced Form Data Analysis

### 2.3.1. The Bunching Phenomenon

I illustrate the bunching patterns in monthly loan payments. Scheduled monthly payments bunch at both \$9- and \$0-endings. Such bunching pattern is more significant at \$100 marks. Beyond \$9- and \$0-endings, the number of loans also increases with larger ending digits from \$1 to \$8. Moreover, the level of \$9-ending bunching varies systematically across different groups of consumers.

Figure 1 plots the frequency of the monthly payment ending digit when payments cross \$100. Each bar represents the percentage of loans with ending digit from \$0 to \$9. Instead of a uniform distribution of 10% probability for each number, there are more loans with \$9-ending payments as well as \$0-ending payments.<sup>11</sup> When payments cross \$100 marks, \$9-ending payments are more than twice as many, and \$0-ending payments are 1.5 times as many, as payments ending at \$01. Bunching pattern is similar, although less pronounced, at other \$10 marks, where \$9-ending payments are 30% more, and \$0-ending payments are 12% more, than \$1-ending payments. Beyond \$9- and \$0-endings, another interesting pattern is that the percentage of loans is higher for payments with a larger ending digit.<sup>12</sup> For example, payments ending at \$8 are 17% more than payments ending at \$1.

**Figure 1. Frequency of Monthly Payment Ending Digit**



<sup>11</sup> The data sample includes all auto loans from banks and credit unions. Some loans may be originated directly from banks or credit unions and are not subject to the typical markup process in indirect auto lending. I expect the bunching pattern to be more significant for loans originated at the dealer location.

<sup>12</sup> \$5-ending is an exception. The number of loans is especially high for payments ending at \$25 or \$75. This is likely driven by consumers and finance managers perceiving these payments as round numbers.

Consumers who pay monthly payments with \$9-ending digits and those who pay with \$0-ending digits are different across multiple consumer characteristics. Panel A in Table 2 shows on the ratio of the number of \$99-ending over the next \$01-ending loans (e.g., \$399/\$401). The \$9-ending bunching is higher among consumers with lower credit scores, older ages, and living in areas with lower incomes and larger minority populations. Panel B in Table 2 shows the ratio of the number of \$00-ending loans over the next \$01-ending loans (e.g., \$400/\$401). Opposite to \$9-ending, the \$0-ending bunching is higher among consumers with higher credit scores.

**Table 2. Heterogeneous Levels of Payment Bunching at \$99- and \$00-endings**

Panel A: The Ratio of \$99-ending Loans to \$01-ending Loans (Overall ratio: 2.08)					
Credit Score	620-660	661-700	701-740	741-780	781-850
	2.27	2.21	2.10	2.02	1.90
Age	<30	31-40	41-50	51-60	>60
	2.03	2.08	2.10	2.09	2.13
Income (zip-level)	< \$50k	\$50-70k	\$70-90k	\$90-120k	> \$120k
	2.23	2.07	2.08	2.07	1.97
Caucasian Proportion (zip-level)	<50%	50-70%	70-80%	80-90%	>90%
	2.31	2.14	2.08	2.00	1.98
Hispanic Proportion	<2%	2-5%	5-10%	10-20%	>20%
	2.04	2.04	2.05	2.12	2.24
African American Proportion	<2%	2-5%	5-10%	10-20%	>20%
	1.93	2.05	2.16	2.17	2.36
Panel B: The Ratio of \$00-ending Loans to \$01-ending Loans (Overall ratio: 1.55)					
Credit Score	620-660	661-700	701-740	741-780	781-850
	1.50	1.50	1.54	1.57	1.61
Age	<30	31-40	41-50	51-60	>60
	1.48	1.55	1.56	1.56	1.59
Income (zip-level)	< \$50k	\$50-70k	\$70-90k	\$90-120k	> \$120k
	1.57	1.56	1.54	1.54	1.52
Caucasian Proportion (zip-level)	<50%	50-70%	70-80%	80-90%	>90%
	1.58	1.54	1.52	1.55	1.55
Hispanic Proportion	<2%	2-5%	5-10%	10-20%	>20%
	1.56	1.54	1.53	1.54	1.59
African American Proportion	<2%	2-5%	5-10%	10-20%	>20%
	1.56	1.55	1.55	1.52	1.53

### 2.3.2. Interest Rates

This sub-section illustrates two points regarding loan interest rates. First, \$9-ending loans have a higher average interest rate, while \$0-ending loans have a lower interest rate, than loans with payments ending at other digits. Second, minority consumers pay a higher interest rate on average than Caucasian consumers.

Loans with \$9- and \$0-ending payments are systematically different. Table 3 compares the characteristics for loans with different ending digits. On average, \$9-ending loans have lower credit scores, larger loan amounts, longer loan lengths, and higher APRs compared with \$0-ending loans.

**Table 3. Characteristics for Loans with Different Ending Digits**

Ending Digit	Credit Score	Loan Amount (\$1000)	Loan Length (Years)	APR (%)
	(1)	(2)	(3)	(4)
\$5	725.52	22.97	5.45	4.785
\$6	725.90	22.90	5.44	4.778
\$7	725.66	22.96	5.45	4.791
\$8	725.46	23.04	5.46	4.804
\$9	724.20	23.34	5.54	4.847
\$0	726.23	22.82	5.42	4.754
\$1	726.29	22.84	5.41	4.761
\$2	726.18	22.84	5.41	4.770
\$3	726.10	22.88	5.42	4.776
\$4	725.86	22.93	5.44	4.787

To further investigate the difference in interest rates for loans with different ending digits, I use regression analysis to control for other factors that can affect the interest rate as follows:

$$int_i = \sum_{j=1}^9 \gamma_j \cdot I(d(payment_i) = j) + X_i\beta + \epsilon_i$$

where  $int_i$  is the interest rate of loan  $i$ , and  $I(d(payment_i) = j)$  an indicator variable that equals 1 if the ending digit of the monthly payment is  $j$  ( $j$  is from 1 to 9, with 0 as the normalized factor).  $X_i$  includes credit score, loan amount, and loan length. I also include date and state fixed effects for each loan. Results are reported in Table 4. To capture the potential non-linearity of the relationship between interest rates and covariates  $X_i$ , Column (1) and (3) use third order polynomial functions of these variables, while Column (2) and (4) categorize them into bins and use bin fixed effects. Column (3) and (4) also include consumer characteristics, including age, ethnicity, income, and average house value.

Across different specifications, \$9-ending loans consistently carry the highest interest rate, about 0.053% higher than \$0-ending loans.<sup>13</sup> To put the numbers in perspective, for a 5 year, \$25000 loan with 6% APR, this difference would result in a \$36 higher cost for consumers. As the coefficients for \$1-ending to \$9-ending are all significant positive, it implies that \$0-ending payments have the lowest interest rate. Figure 2 visually presents the regression results from Column (1). Beyond \$9- and \$0-ending, loans with large ending digits generally have a higher interest rate than loans with small ending digits.<sup>14</sup>

**Figure 2. Interest Rate for Loans with Different Ending Digits**

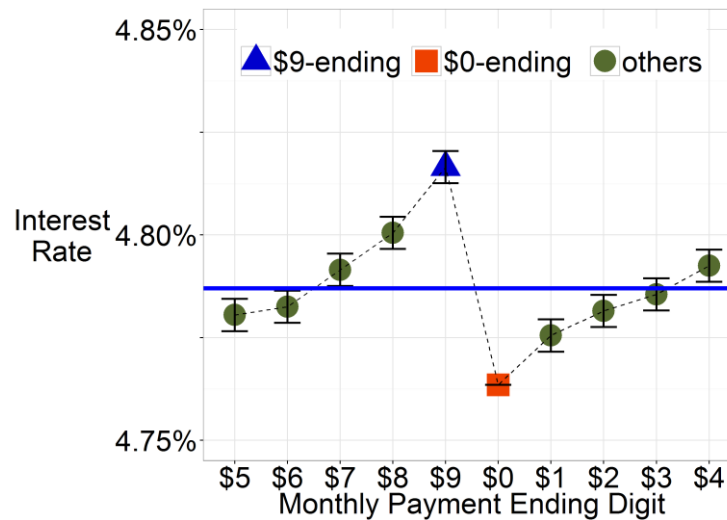


Table 4 also shows that minority consumers, as well as consumers with older age, lower income, and lower house value are more likely to have higher interest rates. Furthermore, consumers from geographical regions with high African American and Hispanic population are charged higher interest rates. Since banks do not use these characteristics when deciding the bank buy rate, the interest rate difference reflects the dealer markup. Put in the context of bargaining, the reduced-form analysis provides an evidence that these consumers have a lower bargaining power.

To summarize, there are more loans with \$9-ending payments, which carry a higher interest rate on average, and there are more loans with \$0-ending payments with a lower interest rate. In addition, the tendency to have \$9-ending loans is higher among consumers with a lower bargaining power, who receive a higher interest rate. I discuss how number biases from both consumers and finance managers in a bargaining setting can explain these data patterns after introducing the model.

<sup>13</sup> For robustness, I have also implemented a machine learning method, using boosted trees, to predict APR for loans with different ending digits, and the results are very similar (see Appendix B for details).

<sup>14</sup> The slightly higher interest rate of \$4-ending loans than that of \$5-ending payments is an exception. This is likely due to \$5-ending payments being perceived as round numbers, similar to \$0-ending payments.

**Table 4. Interest Rate Regression Results**

	<i>Dependent variable: APR</i>			
	(1)	(2)	(3)	(4)
\$1-ending	0.00012*** (0.00002)	0.00013*** (0.00002)	0.00014*** (0.00002)	0.00014*** (0.00002)
\$2-ending	0.00018*** (0.00002)	0.00019*** (0.00002)	0.00019*** (0.00002)	0.00020*** (0.00002)
\$3-ending	0.00022*** (0.00002)	0.00024*** (0.00002)	0.00023*** (0.00002)	0.00025*** (0.00002)
\$4-ending	0.00029*** (0.00002)	0.00031*** (0.00002)	0.00028*** (0.00002)	0.00031*** (0.00002)
\$5-ending	0.00017*** (0.00002)	0.00021*** (0.00002)	0.00016*** (0.00002)	0.00020*** (0.00002)
\$6-ending	0.00019*** (0.00002)	0.00022*** (0.00002)	0.00019*** (0.00002)	0.00022*** (0.00002)
\$7-ending	0.00028*** (0.00002)	0.00031*** (0.00002)	0.00027*** (0.00002)	0.00030*** (0.00002)
\$8-ending	0.00037*** (0.00002)	0.00040*** (0.00002)	0.00036*** (0.00002)	0.00039*** (0.00002)
\$9-ending	0.00053*** (0.00002)	0.00061*** (0.00002)	0.00049*** (0.00002)	0.00057*** (0.00002)
Age			0.00004*** (0.0000003)	0.00003*** (0.0000003)
Income (in \$1 million)			-0.03293*** (0.00015)	-0.03260*** (0.00015)
African American percentage			0.01045*** (0.00003)	0.00985*** (0.00003)
Hispanic percentage			0.01463*** (0.00006)	0.01388*** (0.00005)
Average house value (in \$1 million)			-0.00071*** (0.00005)	-0.00039*** (0.00005)
Covariates <i>X</i>	Polynomial	Categorical	Polynomial	Categorical
Date Opened Fixed Effects	Yes	Yes	Yes	Yes
State Fixed Effects	Yes	Yes	Yes	Yes
Observations	34,760,946	34,760,946	34,760,577	34,760,577
$R^2$	0.30016	0.31733	0.30829	0.32458

*Covariates X include credit score, loan length and loan amount. Results from two specifications are shown: Column (1) and (3) use third order polynomials, and Column (2) and (4) categorize each covariate into bins and use bin fixed effects.*

*Note:*

\* p<0.1; \*\* p<0.05; \*\*\* p<0.01

### 3. The Bargaining Model Incorporating Number Biases

In this section, I propose a bargaining model that involves consumers and finance managers of auto dealers. Importantly, it allows for number biases from both parties. The proposed model can explain the bunching phenomenon and the differential interest rates shown in data.

#### 3.1. Nash Bargaining

Because of the discretionary markup policy in indirect auto financing, I characterize the realized monthly payments as the bargaining outcome between consumers and finance managers. I assume that, by the time of discussing the loan arrangement, consumers have already chosen the car they want to buy and have agreed on the car price with the dealer. In addition, consumers have decided the loan amount and loan length. What is left for bargaining is the monthly payment, which will determine the loan interest rate.<sup>15</sup>

The bargaining outcome can be a result of back-and-forth counter-offers from both parties. Not observing these from data, I cannot model the potentially complicated negotiation process. Instead, I borrow the standard Nash solution concept and focus on the outcome of the bargaining game. The advantage of this approach is that the model can allow for various bargaining processes, which may involve lengthy negotiations or reach agreements right away. It has been shown that the Nash solution is a good approximation to the equilibrium outcome in non-cooperative strategic alternating bargaining games with either time-preference or uncertain termination of the bargaining (e.g., Binmore et al. 1986).

The key assumption behind the Nash solution concept is that, for auto loan  $i$ , the monthly payment  $p_i$  observed from the data will maximize a joint-value function as follows:

$$v(p_i) = (u^c(p_i) - u^c(r_i^c))^{\alpha_i} \cdot (u^f(p_i) - u^f(r_i^f))^{1-\alpha_i} \quad (1)$$

In this value function,  $u^c(p_i)$  represents the payoff function for the consumer, and  $u^f(p_i)$  the payoff function for the finance manager. The reservation price for the consumer is  $r_i^c$ , and for the finance manager is  $r_i^f$ . Thus,  $u^c(p_i) - u^c(r_i^c)$  represents the surplus for the consumer and  $u^f(p_i) - u^f(r_i^f)$  the surplus for the finance manager. Finally,  $\alpha_i$  is the consumer's relative bargaining power, which ranges from 0 to 1, and the finance manager's bargaining power is  $1 - \alpha_i$ . To maximize the joint-value function,  $p_i$  has to be within the range where both the consumer and the finance manager enjoy positive surpluses; otherwise, the

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<sup>15</sup> Consumers could instead face a menu of loan schedules, each with a unique combination of loan amount, loan length and monthly payment. Even in this case, there can still be room for negotiation on the actual monthly payments, after consumers have selected the loan amount and length. The identification issue will be further discussed in section 4 in the paper.

negotiation will break down. The larger the bargaining power of one party, the larger the surplus it will gain from the bargaining.

To model the reservation prices, I use the institutional details that a bank or a credit union will offer the finance manager a bank buy rate which is based on some loan and consumer characteristics (e.g., credit score)  $X_i$ . This buy rate determines a monthly payment  $\underline{p}(X_i)$ . I assume that the finance manager will not accept a monthly payment lower than  $\underline{p}(X_i)$ ,<sup>16</sup> and therefore

$$r_i^f = \underline{p}(X_i) \quad (2)$$

Next, I assume there is a maximum interest rate, which uniquely determines a monthly payment  $\bar{p}(X_i)$ , which the consumer can obtain from outside sources (e.g., from her own bank). The consumer will not accept a monthly payment higher than  $\bar{p}(X_i)$ . Therefore, the reservation for consumers is to pay  $\bar{p}(X_i)$ ,

$$r_i^c = -\bar{p}(X_i) \quad (3)$$

### 3.2. Payoff Functions

The consumer's payoff decreases with the monthly payment. Suppose she evaluates the cost of the payment without biases, her payoff function would be  $u^c(p_i) = -p_i$ , and her surplus from the negotiation is  $\bar{p}(X_i) - p_i$ . Likewise, if the finance manager can evaluate the monetary return of the payment without biases, his payoff function would be  $u^f(p_i) = p_i$ , and his surplus is  $p_i - \underline{p}(X_i)$ .

The model allows for two types of biases with numbers, the extent of which will be estimated from the data. The first type captures the discontinuity in the perceived value when payments cross \$100 marks. This helps explain the high level of bunching at \$99- and \$00-ending loans. Let  $d(p_i)$  be the digit in the hundredth place for payment  $p_i$ , e.g.,  $d(\$400) = 4$  and  $d(\$399) = 3$ . I use a specification  $\delta^c \cdot [d(\bar{p}(X_i)) - d(p_i)]$  to model the discontinuity in the consumer's perceived value. Model parameter  $\delta^c$  captures the discontinuous change in consumer's payoff every time payment  $p_i$  crosses \$100 marks. This bias captures the effect that \$399 may be perceived to be much lower than \$400. Similarly, the discontinuity

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<sup>16</sup> This assumption can be violated if the auto dealer is willing to take a loss from financing so that it can gain from selling the car and add-on services. In the data sample, I exclude loans with APRs lower than 1.9% (see the discussion in the data section) to avoid mis-specifying the reservation prices for those loans.

for the finance manager is specified as  $\delta^f \cdot [d(p_i) - d(\underline{p}(X_i))]$ . Model parameter  $\delta^f$  captures the discontinuous change in finance manager's payoff when payment  $p_i$  crosses \$100 marks.<sup>17</sup>

The second type of bias is that the sensitivity to \$1 change in payment may differ as the payment increases to the next \$10 level. This bias helps explain the observed payments bunching at \$9- and \$0-endings, as well as the increasing number of loans from \$1- to \$8-ending. Let  $[p_i]_{10}$  be the number that rounds up payment  $p_i$  to the next \$10 level. For example,  $[\$389]_{10} = [\$390]_{10} = \$390$ . I use a specification,  $10^{1-\rho^c} \cdot ([p_i]_{10} - p_i)^{\rho^c} - ([p_i]_{10} - p_i)$ , to allow for curvature in the consumer's payoff function. The functional form ensures that this bias goes away when  $p_i$  ends in \$0, and that the payoff is monotonic in  $p$ .<sup>18</sup> Depending on the parameter  $\rho^c$ , the consumer's payoff function can be concave ( $0 < \rho^c < 1$ ), convex ( $\rho^c > 1$ ) or linear ( $\rho^c = 1$ ) within the \$10 range<sup>19</sup>. The payoff curvature captures the effect that the perceived difference between payments with \$6- and \$7-endings can be different from that between \$5- and \$6-endings. Likewise, the effect on the finance manager is represented by  $([p_i]_{10} - p_i) - 10^{1-\rho^f} \cdot ([p_i]_{10} - p_i)^{\rho^f}$ , where model parameter  $\rho^f$  determines the curvature of the finance manager's payoff function.

Combining the two biases, the consumer's payoff function is

$$u^c(p_i) = -p_i + \delta^c \cdot [d(\bar{p}(X_i)) - d(p_i)] + \left(10^{1-\rho^c} \cdot ([p_i]_{10} - p_i)^{\rho^c} - ([p_i]_{10} - p_i)\right) \quad (4)$$

and the finance manager's payoff function is

$$u^f(p_i) = p_i + \delta^f \cdot [d(p_i) - d(\underline{p}(X_i))] + \left(([p_i]_{10} - p_i) - 10^{1-\rho^f} \cdot ([p_i]_{10} - p_i)^{\rho^f}\right) \quad (5)$$

To better illustrate the effect of the biases on the payoff functions, I plot several examples under different parameter values in Figure 3. The length of arrows represents the payoff difference for \$1 change

<sup>17</sup> For the consumer,  $d(\bar{p}(X_i))$  is included as a scaling constant, so that when  $p_i = \bar{p}(X_i)$ , the consumer payoff discontinuity will become zero. Similarly, for the finance manager,  $d(\underline{p}(X_i))$  is also a scaling constant to restrict the manager's payoff discontinuity to zero when  $p_i = \underline{p}(X_i)$ .

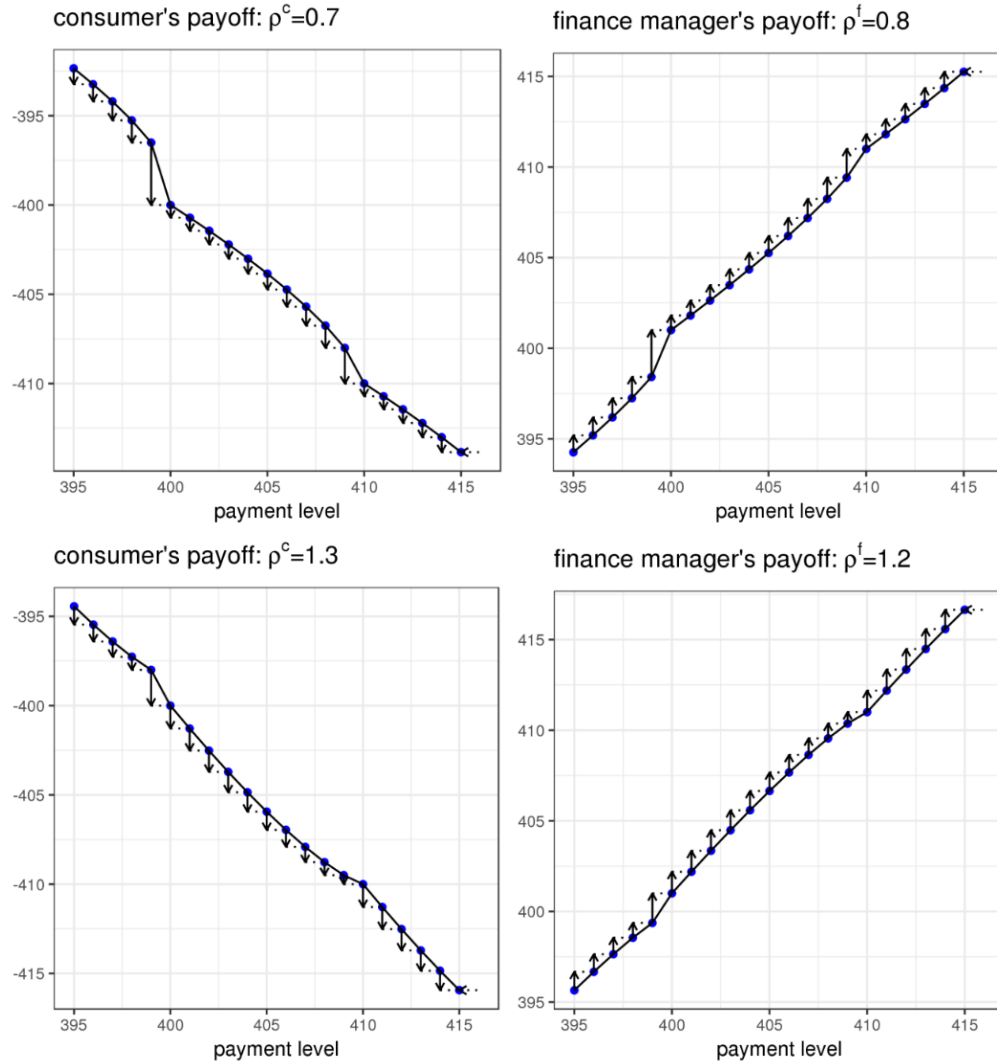
<sup>18</sup> A more general specification is  $-([p_i]_{10} - \lambda \cdot ([p_i]_{10} - p_i)^{\rho^c})$ . I choose  $\lambda = 10^{1-\rho^c}$  to satisfy the following conditions: 1) When  $\rho^c = 1$ ,  $\lambda = 1$  as such the payoff function is linear. 2) To ensure payoff monotonicity,  $\lambda$  needs to satisfy  $0 < \lambda < 10 \cdot 9^{-\rho^c}$ .

<sup>19</sup> This can be seen by examining the first and second order derivative of the consumer's payoff function within the \$10 range. The first order derivative is  $\frac{\partial u^c(p_i)}{\partial p_i} = -10^{1-\rho^c} \cdot \rho^c \cdot ([p_i]_{10} - p_i)^{\rho^c-1} < 0$ . The sign of the second order derivative depends on  $\rho^c$ ,  $\frac{\partial^2 u^c(p_i)}{\partial p_i^2} = 10^{1-\rho^c} \cdot \rho^c \cdot (\rho^c - 1) \cdot ([p_i]_{10} - p_i)^{\rho^c-2}$ . It is negative when  $0 < \rho^c < 1$ , which corresponds to a concave consumer's payoff function within the \$10 range, and positive when  $\rho^c > 1$ , which corresponds to a convex consumer's payoff function,



in payment. I assume that the payoff discontinuities over \$100 marks are  $\delta^c = 1.5$  and  $\delta^f = 1$ . The curvature of the payoff function is determined by  $\rho^c$  and  $\rho^f$ . When  $\rho^c = 0.7$ , consumers are more sensitive to the change in payment as it increases to the next \$10 level (Figure 3, top left), with the largest perceived difference between \$9- and \$0-ending payments. When the payment crosses \$400, there is an additional drop of  $\delta^c$ . When  $\rho^c > 1$ , consumers are less sensitive to payment changes with larger ending digits (Figure 3, bottom left). Similarly, the top right diagram of Figure 3 shows that with  $\rho^f = 0.8$ , the finance manager is more sensitive to payment changes with larger ending digits but the curvature is less than that of the consumer's payoff as  $\rho^f > \rho^c$ . The bottom right diagram of Figure 3 shows that when  $\rho^f = 1.2$ , finance managers are less sensitive to payment changes with larger ending digits.

**Figure 3. Examples of Consumer's and Finance Manager's Payoff Functions**



### 3.3. Bargaining Power

The relative bargaining power  $\alpha_i$  in equation (1) can be heterogeneous among consumers. For example, minority or lower income consumers may be more likely to have a lower bargaining power. The consumer bargaining power is specified as follows

$$\alpha_i = \frac{1}{1 + \exp(\mu_a + X_i\beta + \epsilon_i)} \quad (6)$$

where  $X_i$  includes a vector of loan characteristics including the loan amount and loan length, and consumer characteristics including credit score, age, zip-code level household income, house value, and the proportion of African Americans and Hispanics in the population. The stochastic component  $\epsilon_i$  captures the heterogeneity in bargaining power beyond what is explained by  $X_i$ . I assume that it follows a normal distribution, i.e.,  $\epsilon_i \sim N(0, \sigma_\epsilon^2)$ . The parameters  $\mu_a, \beta$  and  $\sigma_\epsilon$  govern the distribution of bargaining power in the consumer population.<sup>20</sup>

Note that the Nash solution concept predicts that the final monthly payment depends on the bargaining power of the consumer relative to the finance manager, without the need for details about how the two parties bargain back-and-forth. The model does not require multiple rounds of negotiations. If the finance manager has full information on the joint value function, he could offer the payment predicted by the model at the beginning and, if the consumer also has full information, she would immediately accept. In this sense, the proposed bargaining model can generalize to environments where back-and-forth price negotiations are not frequently observed.

### 3.4. Payments Bunching and Differential Interest Rates

In this section, I discuss how the bargaining model with number biases can rationalize the data patterns of bunching payments and differential interest rates using a simple stylized model. Note that in the model a consumer with a large bargaining power  $\alpha_i$  can push the monthly payment closer to the lower bound  $\underline{p}(X_i)$  and away from the upper bound  $\overline{p}(X_i)$ . This is the case even though the consumer and the finance manager have number biases. The number biases, however, create discontinuities in the payoff functions and make the final monthly payment different from that when the biases do not exist.

#### 3.4.1. Bunching Payments

To illustrate how bunching payments come from the model, I will use a simplified version of payoff functions, where there are only discontinuities in the payoff function between \$9- and \$0-ending payments.

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<sup>20</sup> Dealer attributes, such as the dealership for different car manufacturers or the size of the dealer, could also affect the relative bargaining power. These attributes are not observed from the data.

With only consumer biases, payments bunch at \$9-ending and few payments end at \$0. Let  $z$  represent a \$9-ending payment, and  $z + 1$  is \$0-ending with \$1 more each month. Suppose the consumer's payoff function has a discontinuity of  $\delta^c$  at  $z$ , that is,  $u^c(p) = -p$  if  $p = z$  and  $-p - \delta^c$  if  $p = z + 1$ . Assume that the finance manager's payoff is linear with no bias, that is,  $u^f(p) = p$ . Nash bargaining solution concept predicts that the payment  $p$  maximizes the joint value function. I focus on comparing the likelihood of the payment to be at  $z$  or  $z + 1$  by evaluating  $\log\left(\frac{v(z+1)}{v(z)}\right) = \alpha \cdot \log\left(1 - \frac{1+\delta^c}{\bar{p}-z}\right) + (1 - \alpha) \cdot \log\left(1 + \frac{1}{z-\underline{p}}\right)$ . Since  $\log\left(1 - \frac{1+\delta^c}{\bar{p}-z}\right)$  decreases with the bias  $\delta^c > 0$ ,  $\log\left(\frac{v(z+1)}{v(z)}\right)$  is more likely to be negative, compared with the case when  $\delta^c = 0$ . This implies that loan payments are more likely to bunch at  $z$  (\$9-ending). In contrast, there will be few payments at  $z + 1$  (\$0-ending).

With the same logic, the bias of the finance manager will lead to bunching at \$0-ending loans, represented by  $z + 1$ . Only when both consumers and finance managers are subject to biases (i.e.  $\delta^c > 0$  and  $\delta^f > 0$ ), can payments bunch at \$9 and \$0. Furthermore, the number of loans with payments ended at \$9 will increase as the consumer's number bias  $\delta^c$  becomes larger. The increase is drawn from loans with payments ending at \$0, \$1, and so on, when the bias does not exist. Similarly, the number of loans with payments ending at \$0 is higher with larger bias  $\delta^f$  from the finance manager, and the increase is drawn from loans with payments ending at \$9, \$8, and so on.

The intuition of the bunching pattern is that, with a large drop in the payoff function for consumers from \$9- to \$0-ending payments, it is hard for the finance manager to increase payments from \$9- to \$0-ending or beyond. Therefore, there are more loans with payments bunched at \$9-ending digits. Likewise, a large drop in the payoff function for the finance manager from \$0 to \$9 makes it hard for consumers to bargain down from \$0-ending payments, leading more payments to bunch at \$0-ending digits.

### 3.4.2. Difference in Interest Rates

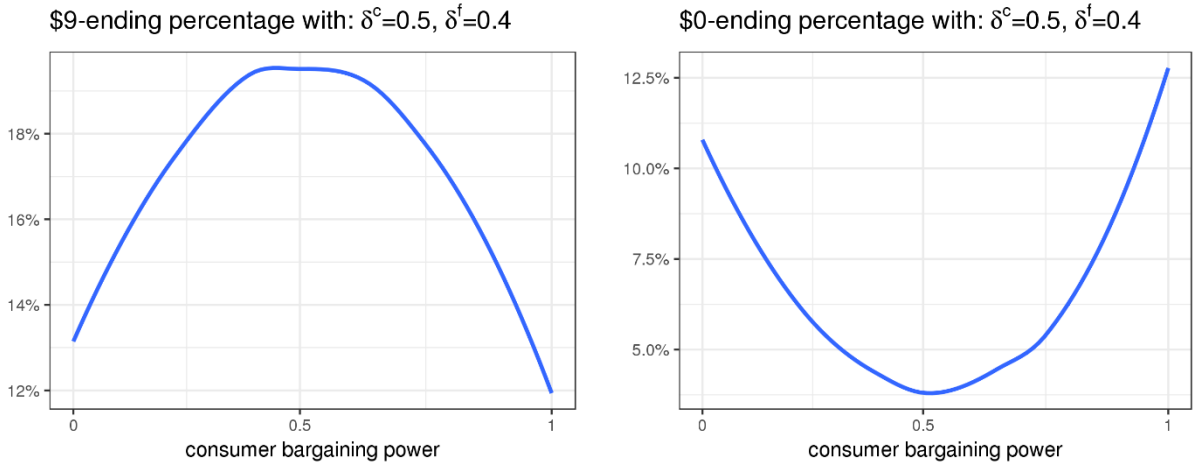
The systematic interest rate difference between \$9- and \$0-ending loans reflects the bargaining power difference for consumers with these loans. Using the same simplified example above, I explore how the bargaining power  $\alpha$  influences the likelihood of a loan payment to settle at  $z$  or  $z + 1$ . This is done through examining how  $\log\left(\frac{v(z+1)}{v(z)}\right) = \alpha \cdot \log\left(1 - \frac{1+\delta^c}{\bar{p}-z}\right) + (1 - \alpha) \cdot \log\left(1 + \frac{1+\delta^f}{z-\underline{p}-\delta^f}\right)$  changes with the bargaining power. As the bargaining power varies, the implied interest rates will also be different for loans with  $z$  and  $z + 1$  payments.

Bargaining power has two opposite effects on  $\log\left(\frac{v(z+1)}{v(z)}\right)$ . The *first* effect is that, when  $\alpha$  is low, the payment level is closer to the consumer's reservation price  $\bar{p}$  and farther away from the finance

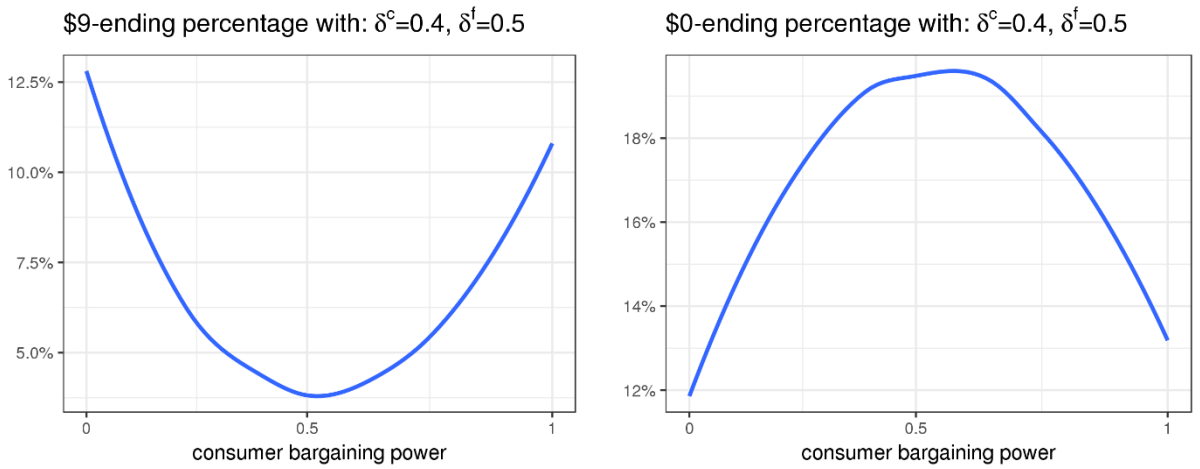
manager's reservation price  $\underline{p}$ . Therefore,  $\log\left(1 - \frac{1+\delta^c}{\bar{p}-z}\right)$  and  $\log\left(1 + \frac{1+\delta^f}{z-\underline{p}-\delta^f}\right)$  are both smaller, and thus  $\log\left(\frac{v(z+1)}{v(z)}\right)$  is more likely to be negative. As such the payment is more likely to be set at  $z$ . The *second* effect is that, when  $\alpha$  is low, the relative weight of  $\log\left(1 - \frac{1+\delta^c}{\bar{p}-z}\right)$  (which is negative) is smaller and the weight of  $\log\left(1 + \frac{1+\delta^f}{z-\underline{p}-\delta^f}\right)$  (which is positive) becomes larger. Thus,  $\log\left(\frac{v(z+1)}{v(z)}\right)$  is more likely to be positive, and there will be more loan payments bunching at  $z + 1$ .

**Figure 4. Bunching at \$9- and \$0-ending Payments and Bargaining Power**

Panel (A). Bunching patterns when the consumer's bias is larger ( $\delta^c = 0.5$ ,  $\delta^f = 0.4$ ).



Panel (B). Bunching patterns when the finance manager's bias is larger ( $\delta^c = 0.4$ ,  $\delta^f = 0.5$ ).



Which effect dominates depends on the bargaining power and the extent of the discontinuities in the consumer's and the finance manager's payoff functions. I use a simulation exercise to illustrate the relationship. First, I assume the payoff discontinuities are  $\delta^c = 0.5$  and  $\delta^f = 0.4$  (i.e. the consumer's bias is larger). The finance manager's reservation price  $\underline{p}$  is drawn uniformly from 400 to 500, and the consumer's reservation price is  $\bar{p} = \underline{p} + 50$ . Panel (A) of Figure 4 plots the proportions of simulated payments ended at \$9- and \$0-ending digits at different levels of  $\alpha$ . When the overall bargaining power is high among consumers (i.e., their  $\alpha$ 's are in the region of 0.5-1), the first effect prevails. That is, the proportion of \$9-ending payments decreases among consumers with higher  $\alpha$  within the range (see the left diagram). In contrast, the proportion of \$0-ending loans increases among consumers with higher  $\alpha$  (see the right diagram). Given that the interest rates are negatively related to the bargaining power, these results suggest that, when consumers' bargaining power is high in general, those who pay \$9-ending loans are more likely to pay a higher interest rate, and those who pay \$0-ending loans are more likely to pay a lower interest rate, when compared with the others.

In the region where consumers' bargaining power is low overall (i.e., their  $\alpha$ 's are in the region of 0-0.5), the second effect prevails, and therefore the proportion of \$9-ending loans increases and the proportion of \$0-ending loans decreases, among consumers with higher  $\alpha$ . Consequently, we should observe those who pay \$9-ending loans are more likely to pay a lower interest rate, and those who pay \$0-ending loans are more likely to pay a higher interest rate, when consumers' bargaining power is low in general.

Next, I assume the payoff discontinuities are  $\delta^c = 0.4$  and  $\delta^f = 0.5$  (i.e. the finance manager's bias is larger), and repeat the simulation. Panel (B) of Figure 4 graphically illustrates the results. The data pattern is opposite to that in Panel (A). That is, when consumers' bargaining power is high in general (i.e., their  $\alpha$ 's are in the region of 0.5-1), the model predicts those who pay \$9-ending loans are more likely to pay a lower interest rate, and those who pay \$0-ending loans are more likely to pay a higher interest rate. In contrast, when the overall bargaining power is high among consumers (i.e., their  $\alpha$ 's are in the region of 0-0.5), \$9-ending loans are more likely to pay a high interest rate and \$0-ending loans are more likely to pay a low interest rate.

To conclude, the relationship between the \$9- and \$0-ending loans and their interest rates depends on the consumer bargaining power and the extent of the biases of both sides in my model. Note that the model is flexible enough to predict not only the relationship I observe in the data, but also when the relationship is the opposite. Consequently, it can be applied to different general contexts when prices are determined by the two-sided bargaining.

## 4. Model Estimation

The data that I use for estimating the proposed model includes the monthly payment  $p_i$ , and the loan and consumer characteristics  $X_i$ . The set of model parameters is  $\Theta = \{\delta^c, \delta^f, \rho^c, \rho^f; \mu_a, \beta, \sigma_\epsilon\}$ . The first four parameters govern the number biases in the payoff function, and the latter three determine the bargaining power distribution. In this section, I discuss the estimation strategy, the details of the estimation procedure, and the model identification.

### 4.1 Moment Conditions with Equality Constraints

I use the simulated method of moments (SMM) for model estimation because deriving a likelihood function is challenging with the stochastic term  $\epsilon_i$  entering the joint value function non-linearly (equation (6)). Another advantage of using the SMM is that consistent estimates can be obtained with a finite number of simulations to construct the moment conditions. I utilize the first and second moment conditions to identify the mean and dispersion of bargaining power. In the estimation, I draw  $\epsilon_i^{sim}$  for every loan from the distribution  $\epsilon_i \sim N(0, \sigma_\epsilon^2)$ , where  $sim = 1, \dots, NS$ . Given  $\epsilon_i^{sim}$ , I simulate the monthly payment,  $p_i^{sim}(X_i, \Theta)$  based on observed covariates  $X_i$  and assumed model parameters  $\Theta$ . Let  $p_i^s(X_i, \Theta) = \frac{1}{NS} \sum_{sim=1}^{NS} p_i^{sim}(X_i, \Theta)$ , and let  $\Theta^0$  be the true parameters. The first and second moment conditions are as follows:

$$E[p_i - p_i^s(X_i, \Theta^0) | X_i] = 0 \quad (7.a)$$

$$E \left[ (p_i - E(p_i | X_i))^2 - (p_i^s(X_i, \Theta^0) - E(p_i^s(X_i, \Theta^0)))^2 \middle| X_i \right] = 0 \quad (7.b)$$

where  $p_i$  is the observed payment.  $E(p_i | X_i)$  is the average observed monthly payments, and  $E(p_i^s(X_i, \Theta^0))$  is the average simulated monthly payments. At true model parameters  $\Theta^0$ , the differences between the true and the simulated payment as well as between the variance of true and simulated payments, are uncorrelated with instruments  $X_i$ . The estimated  $\hat{\Theta}$  set the sample analog of moments as close as possible to zero.

With the moment conditions alone, however, it is still difficult to pin down the number biases parameters. This is because these parameters are uniquely mapped to the distribution of loans with different ending digits. To estimate the number biases parameters, I impose a set of linear equality constraints while minimizing the criterion function constructed from the moment conditions. Let  $e(p_i)$  be the ending digit of payment  $p_i$ , i.e.,  $e(p_i) = p_i - \lfloor \frac{p_i}{10} \rfloor * 10$ , where  $\lfloor x \rfloor$  is an operator that removes decimal places from  $x$  (e.g.  $\lfloor 29.9 \rfloor = 29$ ). Also, let

$$E[d] = \frac{1}{N} \sum_{i=1}^N \{e(p_i) = d\}, \text{ and}$$

$$E[\widehat{d}](\Theta) = \frac{1}{N} \sum_{i=1}^N \frac{1}{NS} \sum_{sim=1}^{NS} \{e(p_i^{sim}(X_i, \Theta)) = d\}$$

for all ending digits  $d = 0, 1, \dots, 9$ , where  $\{\cdot\}$  is an indicator function that takes the value of 1 if the logical expression inside the bracket is true, and 0 otherwise. The equality constraint I impose in the estimation is

$$E[d] = E[\widehat{d}](\Theta) \quad (8)$$

That is, the proportion of payments ending at each digit  $d$  is the same among observed and simulated payments. These equality constraints help identify the number biases parameters.

## 4.2 Details of the Estimation Procedure

Before estimating the model parameters, I estimate the consumer reservation price,  $-\bar{p}(X_i)$ , and the finance manager reservation price,  $\underline{p}(X_i)$ , as the first step (see equations 2 and 3). I assume that the finance manager reservation price is determined by the bank buy rate, which is the cost of the loan for the dealer. The bank buy rate is approximated by the lower bound of APRs for a given loan type that has a similar loan amount, length, credit score, and time period, in the data<sup>21</sup>. For loan types with few observations in data, this method will give an imprecise approximation. To solve this problem, I estimate the relationship of the bank buy rate and relevant covariates<sup>22</sup> in a regression, using data from loans types with at least 50 observations. The regression coefficients are then used to predict the bank buy rate for all loan types, including ones with few observations in data. To estimate the consumer reservation price, I assume that the interest rate gap between the consumer and finance manager reservation varies only across time periods but not among consumers.<sup>23</sup> I estimate the gap in each period of the data<sup>24</sup>. The consumer reservation interest rates are equal to the estimated gap plus the bank buy rate. With the estimated bank buy rate and consumer reservation interest rate, I calculate the consumer and finance manager reservation prices, which are expressed as monthly payments, using the observed loan amount and loan length.

<sup>21</sup> Empirically, I only use loans above the 5<sup>th</sup> percentile of the APRs, among loans of the same type, to avoid outlier issues. Loans are similar if borrowers have the same credit score, within a range of loan amount (within \$5000) and loan length (within 1 year) and originated in the same month.

<sup>22</sup> I use third-order polynomials of credit score, loan amount and loan length, plus year-month fixed effects, as covariates.

<sup>23</sup> This assumption is reasonable if the interest rate from the outside source that a consumer can obtain the auto loan also uses the same rule that determines  $\underline{p}(X_i)$ , plus a fixed markup. To the consumer, because she will have to search for the outside source and apply separately, there is also an additional cost to seek a loan from this source. The fixed markup plus the additional cost is represented by the difference between  $\bar{p}(X_i)$  and  $\underline{p}(X_i)$ , which does not vary by consumer types. If this assumption is violated, the error of measuring  $\bar{p}(X_i)$  will attribute to the bargaining power in the estimation. For example, consumers with a low reservation price, such as those who obtain a pre-approval loan from their own bank, will be treated as those who have a high bargaining power in the model.

<sup>24</sup> Similar to using the 5<sup>th</sup> percentile as the lower bound, I only use loans below the 95<sup>th</sup> percentile of APRs to avoid outlier issues. This way, the gap between lower and upper bounds covers 90% of all observed interest rates.

With  $\bar{p}(X_i)$  and  $\underline{p}(X_i)$ , I can simulate monthly payment  $p_i^{sim}(X_i, \Theta)$  given simulated  $\epsilon_i^{sim}$ , which maximizes the joint value function in equation (1). As there are discontinuities in the payoff functions,  $p_i^{sim}(X_i, \Theta)$  cannot be solved analytically using the first-order condition. Since all the monthly payments in the data are integers (e.g., \$399), in the model estimation I calculate the joint value for each integer value between  $\bar{p}(X_i)$  and  $\underline{p}(X_i)$ , and choose the one with the highest value as the simulated payment.

Finally, I use a two-step feasible GMM estimation method. In step 1, I set the weighting matrix  $W$  to be the identity matrix and compute estimate  $\hat{\Theta}_{(1)}$ . In step 2, I calculate the optimal weighting matrix

$$\hat{\Sigma} = \left( \frac{1}{N} \sum_1^N g(p_i, X_i, \hat{\Theta}_{(1)})^T g(p_i, X_i, \hat{\Theta}_{(1)}) \right)^{-1},$$

where  $g(p_i, X_i, \hat{\Theta}_{(1)})$  is an  $N \times K$  matrix that represents the sample moments ( $N$  is the number of loans and  $K = 18^{25}$  is the number of moments I use). This way it takes account of the variances and covariance between the moment conditions. Model estimates  $\hat{\Theta}$  are re-computed with the updated weighting matrix.

### 4.3. Identification

#### 4.3.1. Identification of Bargaining Power Parameters

With  $\bar{p}(X_i)$  and  $\underline{p}(X_i)$  that are computed in the first step, parameters associated with the relative bargaining power,  $\{\mu_a, \beta\}$ , are identified from how close the realized monthly payment  $p_i$  is to  $\bar{p}(X_i)$  relative to  $\underline{p}(X_i)$ . If the average payment across all consumers is close to  $\underline{p}(X_i)$ , it implies that the overall consumer bargaining power is large, which identifies  $\mu_a$ . If the average payment of consumers with specific  $X_i$  is closer to  $\underline{p}(X_i)$  than other consumers to their lower bound payments, this implies that the consumer bargaining power associated with  $X_i$  is larger, which identifies  $\beta$ . Furthermore, the identification of the variance  $\sigma_\epsilon$  comes from the variation of monthly payments from consumers with the same  $X_i$ .

#### 4.3.2. Identification of Number Biases Parameters

The identification for the number biases parameters  $\{\delta^c, \delta^f, \rho^c, \rho^f\}$  comes from the distribution of the number of loans ending at different digits. The simplified example in Figure 4 is a good illustration. Given that  $\mu_a$ ,  $\beta$ , and  $\sigma_\epsilon$  are identified, the distribution of  $\alpha$ 's across consumers is identified. Suppose  $\alpha$ 's are populated in the low bargaining power region (i.e. between 0 and 0.5). If the loan payments of the majority of consumers whose expected bargaining power, i.e.,  $E(\alpha_i | X_i, \mu_a, \beta, \sigma_\epsilon)$ , is low end at \$9, while that of

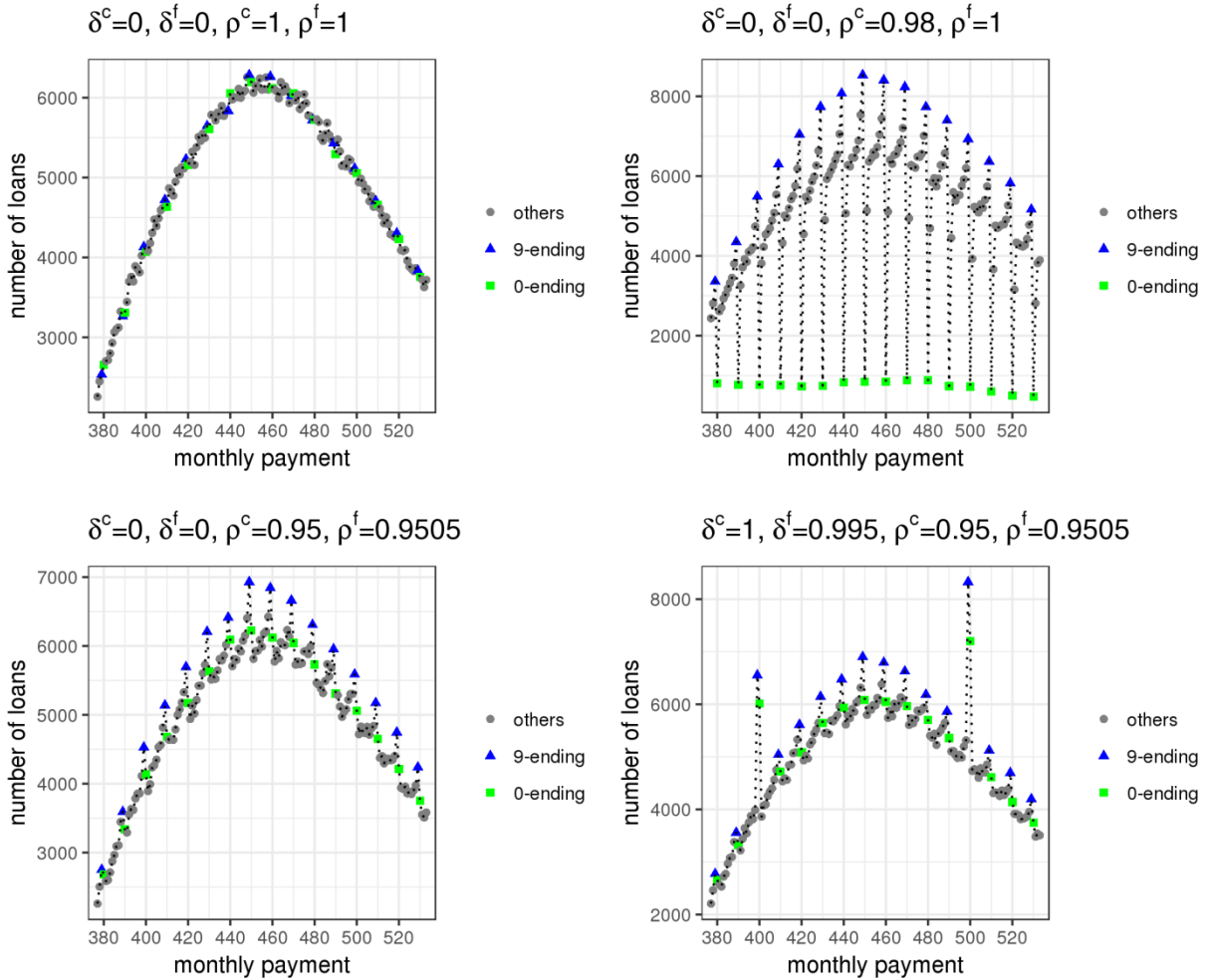
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<sup>25</sup> I use 9 instruments for model estimation, including constant, loan amount, loan length, credit score, age, African American percentage, Hispanic percentage, income, and average house value. With first and second order moment conditions (Equation 7a and 7b), there are a total of  $K = 9 * 2 = 18$  number of moments.



consumers whose expected bargaining power is high end at \$0, this implies that the extent of consumers' biases is smaller than that of finance managers' (i.e., Panel (B) of Figure 4). Suppose  $\alpha$ 's are in the high bargaining power region (i.e. between 0.5 and 1). In this case the above bunching pattern will imply the opposite for the number biases.

**Figure 5. Bunching Patterns with Different Number Biases Parameters**



To illustrate the identification argument beyond the simplified example which only focuses on \$9- and \$0-ending loans, Figure 5 plots the distribution of the simulated monthly payments under different sets of biases parameters, with bargaining power  $\alpha$  drawn from a uniform distribution between 0 and 1. I start off with a benchmark case where there are no number biases for consumers or finance managers, i.e.,  $\delta^c = 0, \delta^f = 0, \rho^c = 1, \rho^f = 1$ . As shown in the top left diagram, the distribution of payments is smooth without loan payments bunching at any ending digits. When  $\delta^c = 0, \delta^f = 0, \rho^c = 0.98$ , and  $\rho^f = 1$ , i.e., the only bias is that consumers become more sensitive to payment change at larger ending digits, payments will

bunch at \$9-ending and there are very few \$0-ending loans, as shown in the top right diagram. This is because payments that would have ended at \$0 (with \$1 more) in the benchmark case will end up at \$9 now. Also, the number of loans with larger ending digits is increasing in the \$10 range. Bunching at both \$9- and \$0-endings happens when consumers and finance managers are both more sensitive to payment change with larger ending digits, as shown in the bottom left diagram using parameters  $\delta^c = 0, \delta^f = 0, \rho^c = 0.95$ , and  $\rho^f = 0.9505$ . As payment goes from \$9- to \$0-ending, consumers have a large payoff drop while finance managers have a large payoff gain, leading to bunching at both \$9- and \$0-endings. In all of the above cases, bunching at \$99- and \$00-ending digits are not more prominent, which is inconsistent with the data observation (see Figure 1). The bottom right diagram of Figure 5 demonstrates the case when  $\delta^c = 1, \delta^f = 0.995, \rho^c = 0.95$ , and  $\rho^f = 0.9505$ . That is, both consumers' and finance managers' payoff functions have a discontinuity at \$100 marks. In this case, we observe a higher level of bunching over \$100 marks.

#### 4.3.3. Monte Carlo Study

I use a Monte Carlo study to show that the proposed estimation strategy can successfully recover the true parameters. I simulate 100,000 loans by randomly drawing loan amount and loan length from the data, and simulate the monthly payment for each loan from the model using the “true” parameter values, as shown in Column (1) of Table 5.

I estimate the model using the simulated data set. I use bootstrapping and perform the estimation 100 times, each with 100,000 loans from resampling the data set. The average parameter estimates from the 100 estimations and their bootstrapped standard errors are reported in Column (2) of Table 5. The parameter estimates are very close to the true values, with small standard errors, showing that the true model parameters can be recovered with the proposed estimation strategy.

Without using the equality constraints in equation (8), however, the number bias parameters are not well identified. Column (3) of Table 5 shows that  $\delta^c$  and  $\delta^f$  are under-estimated. Furthermore, all of the number biases parameters have large standard errors. This shows that the payments bunching data pattern, captured by the equality constraints of the number of loans at each ending digit, is crucial to pin down the number biases parameters.

Finally, even if researchers are only interested in estimating the distribution of consumers' bargaining power, accounting for number biases in the payoff function is still important. To illustrate this point, I estimate a bargaining model that imposes no biases for consumers and finance managers (i.e.  $\delta^c = 0, \delta^f = 0, \rho^c = 1, \rho^f = 1$ ). Results are shown in Column (4) of Table 5. The bargaining power estimates

are significantly different from the true values, leading to incorrect inference of bargaining power distribution among different consumer groups.

**Table 5. Monte Carlo Simulation**

	True Parameters	Proposed Estimation Strategy: Estimate (s.e.)	No Equality Constraints: Estimate (s.e.)	No Number Biases: Estimate (s.e.)
	(1)	(2)	(3)	(4)
$\rho^c$ : Consumer's payoff curvature	0.8900	0.8911 (0.0079)	0.8779 (0.0812)	
$\rho^f$ : Finance manager's payoff curvature	0.8915	0.8927 (0.0094)	0.8799 (0.1048)	
$\delta^c$ : Consumer's payoff discontinuity at \$100	0.7400	0.7340 (0.0861)	0.5552 (0.3108)	
$\delta^f$ : Finance manager's payoff discontinuity at \$100	0.7200	0.7141 (0.0857)	0.5576 (0.3255)	
$u_a$ : Bargaining power constant	-0.7500	-0.7278 (0.0790)	-0.7294 (0.0752)	-0.6376 (0.0469)
$\sigma_e$ : Standard deviation of bargaining power	0.8190	0.8230 (0.0643)	0.7626 (0.0799)	0.7394 (0.0408)
$\beta_1$ : Loan amount in bargaining power function	-0.0500	-0.0500 (0.0010)	-0.0492 (0.0015)	-0.0458 (0.0008)
$\beta_2$ : Loan length in bargaining power function	0.2000	0.1969 (0.0131)	0.1946 (0.0120)	0.1764 (0.0075)

## 5. Results

In this section, I will first discuss model estimation results for number biases and bargaining power parameters. For the ease of computation, the model is estimated from a randomly selected sample of 1 million loans. I will also discuss several alternative explanations for the observed data patterns. Next, I will use the estimation results to conduct counterfactuals.

### 5.1. Model Estimation Results

Model estimation results are reported in Table 6. The first four parameters represent the number biases of consumers and finance managers. The curvatures of the payoff functions for both parties,  $\rho^c$  and  $\rho^f$ , are significantly smaller than 1, indicating that the sensitivity to a \$1 change in payment increases with a larger ending digit (i.e., when payments are closer to the next \$10 level), and it is the highest when the payment moves from \$9- to \$0-ending. For consumers, the payoff drop for a \$1 increase from a \$9-ending payment is \$1.27, significantly larger than \$1. It represents the perceived payoff difference between \$9- and the next

\$0-ending payments,  $10 - 10^{1-\rho^c} \cdot (10 - 9)^{\rho^c}$ . The gap from a \$1 change in payment monotonically decreases at smaller ending digits, and it is the smallest from \$0- to \$1-ending payments at \$0.90, significantly smaller than \$1. The payoff function for finance managers is similar to that of consumers.

**Table 6. Estimation Results**

	Estimate	Standard Error
$\rho^c$ : Consumer's payoff curvature	0.8963	0.0029
$\rho^f$ : Finance manager's payoff curvature	0.8981	0.0028
$\delta^c$ : Consumer's payoff discontinuity at \$100	0.7841	0.0311
$\delta^f$ : Finance manager's payoff discontinuity at \$100	0.7555	0.0307
$u_a$ : Bargaining power constant	5.1985	0.1401
$\sigma_\epsilon$ : Standard deviation of bargaining power	1.1665	0.0098
$\beta$ : Bargaining power covariates:		
Loan amount (in \$1000)	-0.0264	0.0041
Loan length (in years)	0.0563	0.0054
Credit score	-0.0090	0.0002
Age	0.0022	0.0013
Income (in \$1 million)	-1.2464	0.0634
African American percentage	0.5032	0.0433
Hispanic percentage	0.7147	0.0401
Average house value (in \$1 million)	-0.0614	0.0029

There is a further discontinuity in the perceived payoff functions for both consumers and finance managers when payments cross \$100 marks, captured by  $\delta^c$  and  $\delta^f$ . For consumers, the perceived difference for a \$1 increase from \$99- to \$00-ending loans is \$2.05, which equals the payoff drop of \$1.27 from \$9- to \$0-ending plus the additional discontinuity of  $\delta^c = 0.78$ . The level of discontinuity is similar for finance managers, whose perceived difference between \$00- and \$99-ending loans is \$2.02. The additional discontinuities in payoffs contribute to the higher levels of bunching around \$100 marks. The existence of the discontinuities is consistent with findings in the prior literature in consumers' payoff functions (e.g., Stiving and Winer 1997, Lacetera et al. 2012). I show that, even for finance managers who have rich experience in negotiations, they are still prone to human biases the same as consumers. This paper therefore adds to the existing literatures that document psychological biases among professionals or experts in high-stake decision making, such as lawyers, professional traders, and managers in a multinational corporation (Coval and Shumway 2000, Birke and Fox 1999, Workman 2012; see Goldfarb et al. 2012 for a review of the behavioral models on managerial decision-making).

The rest of the parameters in Table 6 govern the distribution of bargaining power among consumers. The range of  $\alpha_i$  in equation (6) is between 0 and 1. After transformation, the average bargaining

power for consumers is 0.77. That is, the overall bargaining power of consumers is larger than that of finance managers. One of the possible reasons is that, if the negotiation breaks down, the dealer will lose not only the interest markup but also the profit from selling the vehicle and other follow-up services. Consumers therefore may have more power when they negotiate financing terms. Regarding other parameters, since  $X_i$  in equation (6) appears in the denominator, consumers' bargaining power is negatively correlated with the covariates that have positive parameter estimates. Table 6 shows that the bargaining power is higher for consumers with a larger loan amount, shorter loan length, and higher credit score. These results are quite intuitive. A consumer who needs a larger loan amount is likely to purchase more expensive vehicles and thus can have more power when negotiating financing terms. A loan request from a consumer with a higher credit score and a shorter loan length is more likely to be accepted by more banks or credit unions, leading to a higher consumer bargaining power. Note that these characteristics have been controlled for when I estimate  $\bar{p}(X_i)$  and  $\underline{p}(X_i)$ , thus the results imply that the monthly payments of those consumers are distributed more densely toward the lower bound of the range.

Consumers who live in areas with lower average income, lower house value, and higher minority representation, and consumers who are older have a lower bargaining power. The results for minority consumers have a strong policy implication. To quantify the parameters, I compare the predicted payments for an African American and a Hispanic consumer with that for a Caucasian consumer, while holding the other variables at the sample average.<sup>26</sup> Results show that the African American consumer pays 1.70%, or \$443, higher total interest payment than the Caucasian consumer. This number is close to that documented in Cohen (2012), who used class action litigation data from five captive lenders to show that African Americans on average paid between \$347 and \$508 more than Caucasians in markup. The Hispanic consumer's payment is 2.50%, or \$653, higher than that of the Caucasian consumer, all else equal.

There can be multiple potential reasons why minority consumers are charged a higher markup. The first is that there are measurement errors for the estimates of  $\bar{p}(X_i)$  and  $\underline{p}(X_i)$ . Suppose banks and credit unions charge a higher buy rate for minority consumers, the estimated  $\underline{p}(X_i)$  for these consumers is downward-biased and, as a result, the model will wrongly attribute their higher monthly payments to the lower bargaining power. This, however, is inconsistent with the institutional reality, as banks and credit unions cannot discriminate minorities when setting the buy rate.<sup>27</sup> Furthermore, they do not have information on the race of consumers when they evaluate loan requests. The second possible reason is that

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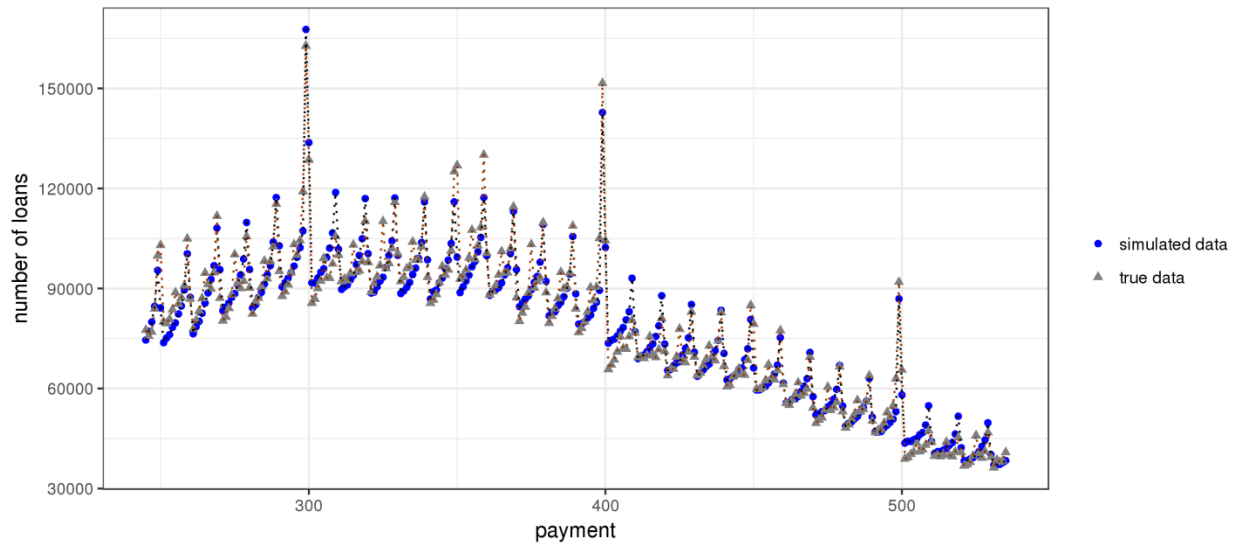
<sup>26</sup> The payment for the African American (Hispanic) consumer is calculated by fixing the African American (Hispanic) variable to 1. The payment for the Caucasian consumer is calculated by fixing both African American and Hispanic variables to 0.

<sup>27</sup> Equal Credit Opportunity Act (ECOA), enacted in 1974, makes it unlawful for any creditor to discriminate against any applicant on the basis of race, color, region, national origin, sex, marital status or age.

minority consumers are less resourceful or less informed about alternative financing sources. Finally, the higher markup can be due to the propensity to discriminate against African Americans and Hispanics among finance managers. If so, finance managers can be more aggressive when negotiating with minority consumers, who will end up paying more than Caucasian consumers on average. I cannot disentangle which of these two latter explanations is the real reason. Nevertheless, the results confirm the existence of a significant payment gap for consumers with different races (and other characteristics such as income levels) due to the discretionary dealer markup practice.

I simulate the monthly payment for each loan using the estimation results. Figure 6 plots the number of loans at each payment level in the true and the simulated data. The two distributions match quite well. In particular, simulated payments also bunch at \$9- and \$0-endings. In addition, the number of loans increases from \$1- to \$8-ending payments within the \$10 range.<sup>28</sup> Finally, the level of bunching at \$99- and \$00-ending payments is more significant around \$100 marks.

**Figure 6. Distribution of Monthly Payments for Actual and Simulated Data**



In addition to matching the overall bunching patterns, the model also provides an explanation for the heterogeneous levels of bunching among different consumer groups. As shown in Figure 4(A), when the consumer's relative bargaining power is high, which is the case in my empirical application, low bargaining power consumers are more likely to get \$9-ending loans. Since African American and Hispanic consumers are estimated to have a lower bargaining power, they are more likely to get \$9-ending loans,

<sup>28</sup> I have imposed the equality constraints in the model estimation. In the simulation, however, I do not impose any constraints. It still matches well with the data at each payment level. Also, the simulation replicates the overall trend and not just the number of loans at different digits in data.

which is consistent with the statistics shown in Table 2. Higher income and higher credit score consumers, on the other hand, have a higher bargaining power and are less likely to get \$9-ending loans.

The same mechanism explains the systematic interest rate difference for \$9- and \$0-ending loans. Given that low bargaining power consumers are more likely to get \$9-ending payments, the average interest rate for the loans will be higher than that for other loans. Similarly, consumers with \$0-ending loans have a higher bargaining power and therefore they have a lower interest rate on average. From the simulated data, the difference in interest rate between \$9- and \$0-ending loans is 0.116%.

To further evaluate the model performance, I estimate the bargaining model with two alternative payoff specifications. They differ in how to incorporate the number bias within the \$10 range. In the first alternative specification, payoffs change linearly within the \$10 range, with a discrete drop in payoffs occurring over \$10 marks. This specification is similar to the ones used in Lacetera et al. (2012) and Stiving and Winer (1997). Although this model can successfully replicate the bunching patterns in simulations, it fails to replicate the increase from \$1- to \$9-ending loans (Figure 1). The second specification uses an alternative functional form to capture payoff curvature within the \$10 range. Although this model can generate the increase from \$1- to \$9-ending loans as well as the bunching patterns at \$9- and \$0-endings, the fit is clearly not as good as the main model. Details are discussed in Appendix C.

## **5.2. Alternative Explanations**

The proposed model is built upon the assumptions that consumers and finance managers negotiate monthly payments and that both parties have number biases. I have shown how the model can explain the unique patterns in data. However, there may be other explanations that can also rationalize the patterns. In this subsection, I will discuss several alternative explanations.

*Promotional effects.* Auto dealers may run promotions with advertised payments ending at \$99 or \$00. This may explain why the bunching phenomenon exists. If this is the reason, however, there should be no systematic difference in the interest rates for these two types of loans. In particular, the interest rate for \$99-ending loans should not be higher than other loans. Furthermore, it cannot explain why there is an increasing number of loans with larger ending digits. In addition, I find from data that the bunching phenomenon is quite stable over time. This is in contrast with auto dealer promotion activities that are periodic in nature.

*Consumer biases only.* One may attempt to come up with a more flexible consumer payoff function in lieu of finance manager biases to explain the data patterns. Suppose consumer payoffs have a large drop from \$9 to \$0 as well as from \$0 to \$1, payments can bunch at both \$9- and \$0-endings. However, such specification would imply that the average interest rate for \$0-ending loans is higher than \$1-ending loans.

This is inconsistent with the empirical evidence, as \$0-ending loans actually have the lowest interest rate. Without allowing finance managers to also have the number biases, it is difficult to rationalize the large difference between the interest rates for \$9- and \$0-ending loans.

*Focal point effect.* Alternatively, one may attribute the \$0-ending bunching to a focal point effect. Based on this explanation, the roundedness of the payments may facilitate negotiations, which will lead to a higher number of loans at \$0-ending. However, this explanation cannot explain bunching at \$9-ending digits. It also cannot explain why \$0-ending loans have the lowest average interest rate.

Finally, one may question the validity of the key model assumption that bargaining leads to the observed bunching phenomenon. I have argued that the discretionary dealer markup policy in the auto finance market means that there is room for negotiating monthly payments. I run a “placebo” test using data from the mortgage loan market also provided by Equifax Inc. The dataset consists of 7.3 million mortgages originated in 2014 across the United States. Similar to auto loans, mortgage loans are an important consumer installment loan with monthly payments. The discretionary markup policy, however, does not exist in the mortgage loan market. Loans are directly provided by banks or credit unions and are subject to much tighter regulations. Therefore, the monthly payments do not come from a bargaining setting where number biases could play a role. I find no evidence for the bunching phenomenon in data. The proportions of monthly payments at \$0- and \$9-ending digits are both exactly 10%. There is also no difference in the APR for loans ending at \$0 or \$9. These results support for the bargaining assumption in the proposed model that applies to the auto loan market.

### **5.3. Counterfactuals**

I use the model estimates to investigate two issues using counterfactual analysis. The first is to explore the effect of number biases on loan payments. The second is to quantify the payment changes under alternative non-discretionary markup policies for minority consumers.

#### **5.3.1. Effect of Number Biases**

Even though biases with numbers may be a human tendency, there are potential ways to mitigate their influence on the decision making process. In the auto finance setting, for example, dealers may direct the attention of finance managers away from the monthly payment and highlight the total markup profit or interest rate instead, so that managers will no longer be influenced by the biases. Likewise, financial education for consumers may help de-bias their perception of numbers.

To explore the effects of de-biasing, I construct a counterfactual payoff function without number biases and use it as the benchmark case. The payoff functions are linear and continuous everywhere, with payoff changing by \$1 for each \$1 increase in payment. I set the parameters  $\delta$ 's to 0 and the parameters  $\rho$ 's



to 1. With these adjustments, however, the overall payoff levels are also changed. For example, without the discontinuity over \$100 marks, the consumer payoffs will become higher than the estimated payoff function. To remove the effect from this level change, I adjust the constant term in the counterfactual payoff function, so that the payoff at any loan payment level is the same under the initial and the counterfactual payoff functions.<sup>29</sup> By doing so, the difference in payments reflects the impact of non-standard payoff functions with curvature and discontinuity instead of changes in the level of payoff.

I simulate the monthly payments under the counterfactual scenario. The payments are compared to those in alternative scenarios where either consumers or finance managers, or both, have the number biases. The difference in payments represents the impact of the biases. Results are shown in Table 7. Panel A reports the change in payments when consumers are biased (but not finance managers), relative to the benchmark case without biases. Biased consumers will pay 0.025% less, with total payments reduced by \$203 million. Therefore, consumers' number biases are beneficial for consumers by lowering their payments.

**Table 7. Effect of Number Biases on Payments**

		Bargaining Power Percentile (from High to Low)				
	Overall	0 – 20%	20 – 40%	40 – 60%	60 – 80%	80 – 100%
Average Bargaining Power	0.77	0.96	0.90	0.83	0.70	0.44
Panel A: Effect of Number Biases for Consumers (Finance Managers not Biased)						
Change in Total Payment (\$million)	-202.9	-12.2	-22.4	-41.5	-61.9	-65.0
% Change	-0.025%	-0.007%	-0.014%	-0.026%	-0.039%	-0.040%
Panel B: Effect of Number Biases for Finance Managers (Consumers not Biased)						
Change in Total Payment (\$million)	101.9	10.4	16.7	22.5	28.0	24.3
% Change	0.013%	0.006%	0.010%	0.014%	0.018%	0.015%
Panel C: Effect of Number Biases for Both Consumers and Finance Managers						
Change in Total Payment (\$million)	-33.0	0.5	-4.0	-7.5	-10.6	-11.35
% Change	-0.004%	0.0003%	-0.002%	-0.005%	-0.007%	-0.007%

The result that consumers pay less when having the number biases is counter-intuitive. One may view biases as a negative factor in the bargaining process by intuition. For example, since consumers'

<sup>29</sup> The procedure is done in several steps. First, I simulate the monthly payment for each loan using the estimates from the proposed model with number biases. I calculate the perceived payoff value, and then adjust the constant term in the counterfactual payoff function so that the payoff of the simulated payment is the same under the biased and the counterfactual de-biased payoff functions. Lastly, I simulate the counterfactual payments for each loan using the adjusted de-biased payoff functions.

number biases lead to bunching at \$9 with a higher interest rate, one may conclude that removing such bias should benefit consumers. This intuition is in general supported in the psychological literature, where biases are generally considered to make people worse off. Researchers often propose ways to de-bias consumers for a better decision-making strategy (Larrick 2004). Furthermore, studies of the 9-ending retail prices in the marketing literature implicitly suggest that firms can take advantage of consumers' number bias with numbers to charge a price higher than when the bias does not exist. My results show the opposite. The key difference from the previous literature is that in this study prices (monthly payments) are set through two-sided negotiations and not decided by firms as in other retail settings. With biases, the perceived drop for \$1 increase in payment from \$9-ending is larger than \$1, especially over \$100 marks. The large drop in payoff makes it harder for finance managers to push the payments higher from \$9-ending. In other words, the biases create a psychological hurdle for consumers so that they are more resistant to payments crossing the hurdle. Although the bias in the payoff function at small ending digits has the opposite effect (since the perceived drop for \$1 increase is smaller than \$1 at those digits), overall the first effect prevails and consumers benefit from having the number biases.

Having number biases is also beneficial for finance managers. As shown in Panel B of Table 7, when compared with the benchmark case, dealers will receive 0.013%, or \$102 million, higher loan payments when their finance managers are biased (and consumers are not). The reason is similar – the large drop in payoff for \$1 change from \$0-ending digits, especially at \$100 marks, makes it hard for consumers to push down payments from \$0-ending. Although the biases also make it easier for consumers to push down payments at smaller ending digits, the total effect is still positive for finance managers. When both parties are biased, consumers will pay 0.004%, or \$33 million in total, less compared to the benchmark case.

The effect of number biases is systematically different for consumers of different bargaining power. In the empirical application where the consumer bargaining power is overall high, the decrease in payments for biased consumers is more significant among low bargaining power consumers. For example, Panel A of Table 7 shows that 32% of the total decrease comes from consumers whose bargaining power is at the bottom 20<sup>th</sup> percentile, while only 6% is from consumers whose bargaining power is at the top 20<sup>th</sup> percentile. The effect of biases is stronger for low bargaining power consumers because they are the ones mostly likely to get \$9-ending payments. When both parties have number biases, consumers with the bottom 20<sup>th</sup> percentile bargaining power pay \$11.4 million less loan payments. In contrast, consumers with the top 20<sup>th</sup> percentile bargaining power will pay \$0.5 million more.

Because of the discretionary dealer markup policy, low bargaining power consumers pay higher markups than high bargaining power consumers do. Consumer's number biases, however, help reduce the

gap in markups between the two types of consumers. For example, Table 7 shows that, consumers' number biases reduce the difference in the markup between consumers with the top 20<sup>th</sup> and bottom 20<sup>th</sup> percentile bargaining power by 0.29%, or \$52.9 million in total. Similarly, the gap is reduced by 0.06%, or \$11.8 million in total, when the biases exist for both consumers and finance managers.

### 5.3.2. Non-Discretionary Dealer Markups

The discretionary markup policy in the auto loan market is controversial and has been under intense regulatory scrutiny. A series of class-action lawsuits were filed challenging this practice against most of the captive auto lenders in the U.S. as well as some large auto lending financial institutions (Munro et al. 2005). Since created in 2011, the Consumer Finance Protection Bureau (CFPB) has taken action against several large auto lenders with large settlements.<sup>30</sup> These lawsuits claimed that the practice authorizes dealers to charge subjective markups that result in disparate impact among minority consumers. My estimation results provide evidence in support of these claims. In this section, I investigate the effects of two alternative policies that compensate dealers with non-discretionary markups, and quantify the change of payments for minority consumers. I also show that, without considering the influence of biases in the bargaining model, one would underestimate the impact of the policy changes for minority consumers.

Under non-discretionary markup compensation policies, markups do not vary among consumers because of the difference in their relative bargaining power. Minority consumers would be charged the same markup as their Caucasian counterparts, all else equal. Under the first counterfactual policy, auto dealers are compensated by a fixed percentage of the loan amount,<sup>31</sup> so that consumers with the same loan amount get the same level of markup. The markup percentage is calculated to be at the level that the total amount that auto dealers make from arranging auto financing is the same as under the current discretionary markup compensation. Under the second policy, the level of markup is a fixed percentage of the bank buy rate that is based on credit score, loan amount, and loan length but not on consumer demographics such as ethnicity. Similarly, the percentage is calculated to achieve the same level of total dealer markup profit as under the current compensation policy. Both of these counterfactual policies are easy to implement in practice. Dealers are not worse off under the counterfactual policies because of the same level of markup profit. Since the bank buy rate does not change, banks and credit unions are also not worse off.

Under the non-discretionary markup compensation, there is a shift in the consumer payments among different groups of consumers. Not surprisingly, low bargaining power consumers benefit from the non-discretionary policies. Table 8 reports the change in payments for consumers in different racial groups

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<sup>30</sup> For example, Ally Financial Inc. paid \$98 million for the settlement in 2013, and Honda paid \$24 million in 2015.

<sup>31</sup> One bank had adopted this compensation policy, citing the CFPB's guideline as the reason, but later reverted back to the discretionary markup practice.

under the new policies. When dealers are compensated by a fixed percentage of the loan amount, consumers from predominantly African American areas (i.e. with more than 40% of the population) pay 1.37% lower total payments, and consumers from high Hispanic population neighborhoods (i.e. with more than 40% of the population) pay 1.35% lower total payments. The aggregate payment decrease among minority consumers is quite substantial, with \$452 million in total for African American consumers and \$275 million in total for Hispanic consumers. When dealers are compensated by a fixed percentage of the bank buy rate, the reduction in payments is slightly higher, with \$473 million in total for African American consumers and \$300 million in total for Hispanic consumers. In contrast, consumers from predominantly Caucasian neighborhoods (i.e. with more than 97% of the population) under the counterfactual policies will have to pay \$445 million and \$484 million more. To conclude, the new non-discretionary policies will have a significant economic benefit for minority consumers, while Caucasian consumers will pay a higher monthly payment to compensate for the policy change.

**Table 8. Counterfactual Results from Non-Discretionary Dealer Markups**

Change in Payment under Non-discretionary Markup Compensation	Fixed Markup by Loan Amount			Fixed Markup by Bank Quote		
	Total Payment (\$million)	Total Payment (%)	Payment per Loan (\$)	Total Payment (\$million)	Total Payment (%)	Payment per Loan (\$)
Predominantly African American Areas (>40%)	-451.8	-1.37%	-373.8	-472.6	-1.44%	-391.0
Predominantly Hispanic Areas (>40%)	-274.5	-1.35%	-373.4	-300.3	-1.47%	-408.5
Predominantly Caucasian Areas (>97%)	444.9	0.48%	120.2	483.9	0.52%	130.8

I have shown that, if the number biases were not incorporated in the bargaining model, the bargaining power estimates would be biased. Consequently, the counterfactual estimates about payment changes will be biased. For the first non-discretionary markup compensation, I find that the payment changes from African American consumers are under-estimated by \$36.8 million, or 8.1%. Similarly, the payment changes from Hispanic consumers are under-estimated by \$12.1 million, or 4.4%. The results are similar for the second non-discretionary markup compensation, with the payment changes under-estimated by \$36.7 million for African American consumers and \$11.8 million for Hispanic consumers if the number biases were not considered. This comparison suggests that the proposed model is important to correctly quantify how the current discretionary dealer markup policy has caused over-payments from minority consumers.

## 6. Conclusion and Discussion

This paper investigates how number biases with numbers affect bargaining outcomes in the auto finance market. The proposed bargaining model that incorporates the biases from both consumers and finance managers can explain the puzzling data patterns of bunching payments and differential interest rates. I use a large data set of 35 million auto loans in this study. Two types of biases in the payoff function are identified: a larger perceived difference from \$1 change at larger ending digits and an additional payoff discontinuity between \$99- and \$00-ending payments. Counter-intuitively, having such biases are actually beneficial in a bargaining setting, as consumers will pay less than when their biases are removed. Similarly, auto dealers will receive a higher markup profit when their finance managers are subject to the number biases.

From the policy perspective, this paper sheds light on the debate about the discretionary markup practice in the auto finance market. I evaluate alternative non-discretionary policies, where dealers are compensated by a fixed percentage of loan amount or bank buy rate, and quantify the economic impact of the policy changes for minority consumers. Counterfactual suggests that African American consumers pay \$452-473 million more in total payments, and Hispanic consumers pay \$275-300 million more in total payments than they would under a non-discretionary policy. Incorporating biases is important in recovering unbiased bargaining power estimates. Failure to do leads to the change in payments to be under-estimated by \$37 million for African American consumers and \$12 million for Hispanic consumers.

The insights from this study have broad implications beyond the auto finance market. Knowing that number biases exist not only among consumers but also among employees can be useful for firms to better understand what factors drive negotiated prices in many other settings, including estate sales, auto sales, online retail platforms (e.g., Taobao.com in China), and B-to-B environments where price negotiations are common. The result that consumers' perceived value has a large drop when crossing a threshold suggests that \$9-ending prices are stickier than other digits in most retail environments. Beyond \$9-ending prices, the result that consumers' sensitivity toward price change is lower at small digits also suggests that the demand elasticity may vary across different ending digits in the price.

Although I use a representative-agent framework to model biases, the model can be generalized to incorporate richer heterogeneities. In the proposed model, biases and bargaining power jointly determine the level of bunching. Suppose there is a large variation of bunching across different consumer groups beyond what could be explained by the difference in their bargaining power, the remaining variation can be attributed to the heterogeneity in biases. This is not the case in my empirical application. Thus, the biases parameters are assumed to be the same for simplicity.

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## Appendix A: Calculate Auto Loan APR

For each auto loan, I observe loan amount, loan length and scheduled monthly payment. The implied APR (annual percentage rate) can be backed out from these three attributes. The discounted value of each monthly payment sums to the loan amount. For a given loan amount and loan length, the higher the APR is, the larger the scheduled monthly payment is.

$$\frac{pmt}{1+r} + \dots + \frac{pmt}{(1+r)^n} = amount \quad (A1)$$

$pmt$  is the scheduled monthly payment,  $r$  is APR/12,  $n$  is the length of loan in months, and  $amount$  is the loan amount.

Rearranging terms of equation (A1), I get:

$$pmt[(1+r)^n - 1] - amount * r * (1+r)^n = 0 \quad (A2)$$

I solve for APR  $r$  by newton's method. Let  $f(r) = amount * r * (1+r)^n - pmt[(1+r)^n - 1]$ . The first order derivative of  $f(r)$  is  $f'(r) = amount * n * (1+r)^{n-1} * r + amount * (1+r)^n - pmt * n * (1+r)^{n-1}$ .

Starting from an initial guess of  $r_0 = 6\%$ , I calculate the next approximation  $r_1$ .

$$r_1 = r_0 - \frac{f(r_0)}{f'(r_0)}$$

The process is repeated as

$$r_{k+1} = r_k - \frac{f(r_k)}{f'(r_k)}$$

until  $r_{k+1}$  and  $r_k$  are sufficiently close,  $|r_{k+1} - r_k| < e^{-10}$ .

## **Appendix B: APR Analysis with Machine Learning Method**

The regression model suggests that the loan APR is systematically different depending on the monthly payment ending digit. For robustness, I have also implemented a machine learning method, Extreme Gradient Boosted Trees, to predict APR based on the following features, credit score, loan amount and length. Boosted tree models bootstrap a multitude of decisions trees, and the final prediction is based on an aggregate across multiple trees. Decision tree types of algorithms consider complex interactions among features, which may be hard to accommodate with a traditional regression approach. It does so in a way that balances in-sample accuracy and out-of-sample prediction. The sample is randomly partitioned into training, validation and testing sample. Various models are trained on the training data set, and the one that performs the best on the validation data set is selected, which prevents over-fitting. Finally, in the testing sample, the selected model is used to predict APR with observed features, including credit score, loan amount and loan length. The difference of the actual APR and the predicted APR in the testing data set is used to assess whether APR is systematically different for loans with different ending digits. The results are close to those from the regression model. In the testing data set, the APR for \$9-ending loans is 0.039% higher than predicted, and it is 0.023% lower for \$0-ending loans than predicted. The difference between the two is 0.062%. More generally, the APR is higher for loans with larger ending digits from \$1 to \$9.

## Appendix C: Alternative Payoff Specifications

In this appendix, I present and discuss results from two alternative payoff specifications.

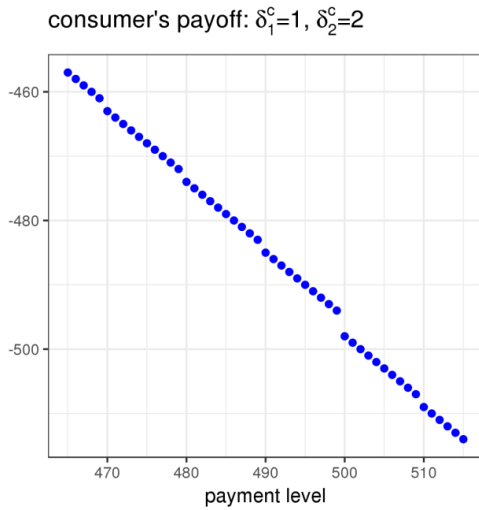
### 1. Direct Discontinuity

In this alternative specification, the payoff function is linear within the \$10 range, with a discrete drop in payoff when payments cross \$10 marks. The discrete drop captures the bias that the perceived difference between \$9- and \$0-endings can be larger than 1. Let  $d_1(p)$  be the hundreds digit, and  $d_2(p)$  be the tens digit of payment  $p$ , e.g.,  $d_1(234) = 2$ ,  $d_2(234) = 3$ . Let  $\delta_1^c$  and  $\delta_2^c$  denote the level of payoff discontinuity at \$10 marks and \$100 marks respectively. Consumer's payoff function is

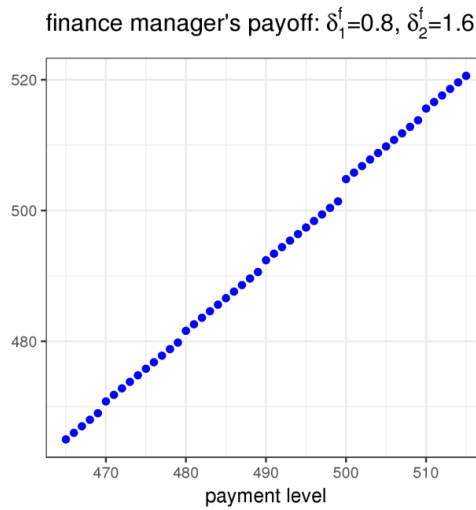
$$u^c(p_i) = -p_i + \delta_1^c \cdot \left[ (10 \cdot d_1(\bar{p}(X_i)) + d_2(\bar{p}(X_i))) - (10 \cdot d_1(p_i) + d_2(p_i)) \right] + \delta_2^c \cdot [d_1(\bar{p}(X_i)) - d_1(p_i)].$$

Figure D1 shows an example of the consumer's payoff function.

**Figure D1**



**Figure D2**



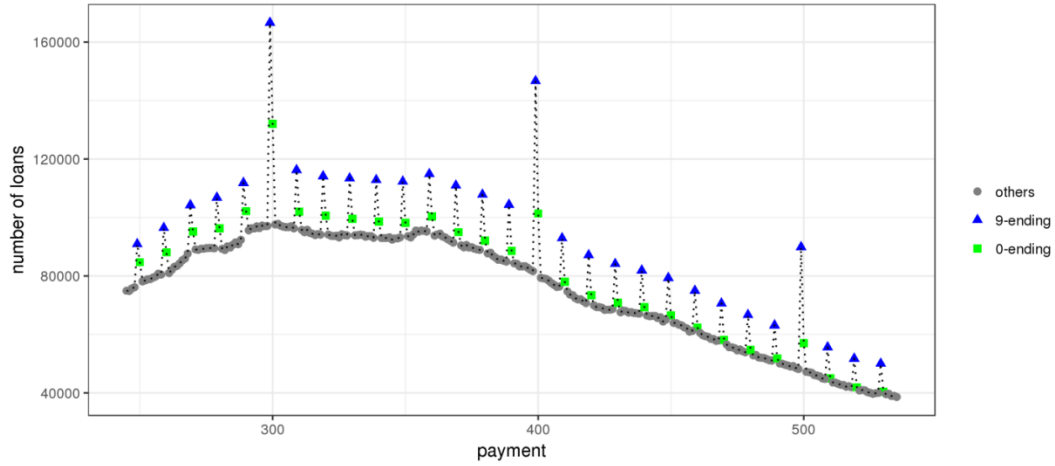
Finance manager's payoff increases as the monthly payment increases, with payoff discontinuity of  $\delta_1^f$  and  $\delta_1^f + \delta_2^f$  at \$10 marks and \$100 marks respectively. Finance manager's payoff function is

$$u^f(p_i) = p_i + \delta_1^f \cdot \left[ (10 \cdot d_1(p_i) + d_2(p_i)) - (10 \cdot d_1(\bar{p}(X_i)) + d_2(\bar{p}(X_i))) \right] + \delta_2^f \cdot [d_1(p_i) - d_1(\bar{p}(X_i))].$$

Figure D2 shows an example of the finance manager's payoff function.

I estimate the model and simulate payments for each loan with the parameter estimates. Figure D3 presents the numbers of loans at each level of simulated payments. Although this model can successfully reproduce the bunching at \$9- and \$0-endings, with the payoff function being linear within each \$10 range, it fails to capture the number bias within the \$10 range and cannot reproduce the increasing pattern from \$1 to \$9 (Figure 1).

**Figure D3**



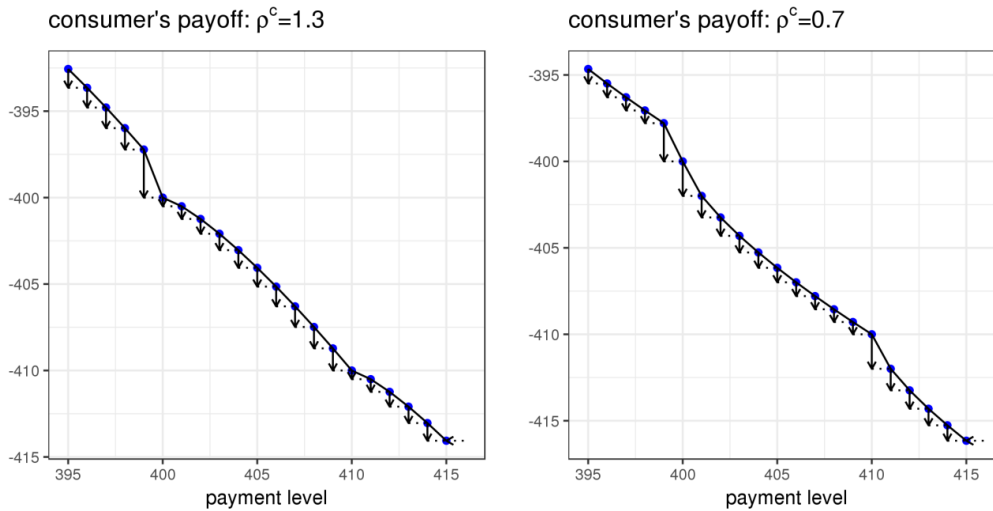
## 2. Alternative Payoff Curvature Specification within the \$10 Range

The empirical observation of an increasing number of loans from \$1- to \$9-ending motivates a payoff specification with curvature. Consider an alternative way to specify the payoff curvature:

$$u^c(p_i) = -p_i + \delta^c \cdot [d(\bar{p}(X_i)) - d(p_i)] + \left( (p_i - [p_i]_{10}) - 10^{1-\rho^c} \cdot (p_i - [p_i]_{10})^{\rho^c} \right).$$

Same as the main model,  $\delta^c$  captures the level of perceived value drop when payment  $p$  crosses \$100 marks.  $[p]_{10}$  rounds down payment  $p$  to the nearest \$10 level,  $[p]_{10} = \left\lfloor \frac{p}{10} \right\rfloor * 10$ , e.g.,  $[\$234]_{10} = \$230$ .  $\rho^c$  governs the amount of payoff curvature within the \$10 range. Figure D4 shows examples of the consumer's payoff function. When  $\rho^c > 1$ , the perceived difference for a \$1 change in payment is larger with a larger ending digit, and the opposite is true when  $0 < \rho^c < 1$ .

**Figure D4**



Finance manager's payoff function is analogous to the consumer's payoff function, with  $\rho^f$  governing the payoff curvature within the \$10 range.

$$u^f(p_i) = p_i + \delta^f \cdot \left[ d(p_i) - d(\underline{p}(X_i)) \right] + \left( 10^{1-\rho^f} \cdot (p_i - \lfloor p_i \rfloor_{10})^{\rho^f} - (p_i - \lfloor p_i \rfloor_{10}) \right).$$

Model estimates under this alternative payoff specification suggest that the consumer's payoff function is concave with  $\rho^c = 1.1039$ , which is associated with greater sensitivity to payment change as the payment gets closer to the next \$10 level, same as the main model. Figure D5 presents the numbers of loans with each level of simulated payments. Although this model can reproduce the increasing pattern from \$1 to \$9, the fit is not as good as the main model.

**Figure D5**

