Information Gaps for Risk and Ambiguity

Russell Golman, George Loewenstein, and Nikolos Gurney*

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Abstract

We apply a model of preferences about the presence and absence of information to the domain of decision making under risk and ambiguity. An uncertain prospect exposes an individual to an *information gap*. Gambling makes the missing information more important, attracting more attention to the information gap. To the extent that the uncertainty (or other circumstances) makes the information gap unpleasant to think about, an individual tends to be averse to risk and ambiguity. Yet in circumstances in which thinking about an information gap is pleasant, an individual may exhibit risk- and ambiguity-seeking. The model provides explanations for source preference regarding uncertainty, the comparative ignorance effect under conditions of ambiguity, aversion to compound risk, and a variety of other phenomena. We present two empirical tests of one of the model's novel predictions – that people will wager more about events that they enjoy (rather than dislike) thinking about.

KEYWORDS: ambiguity, gambling, information gap, risk, uncertainty

*Department of Social and Decision Sciences, Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA 15213, USA. E-mail: rgolman@andrew.cmu.edu; gl20@andrew.cmu.edu; nmgurney@andrew.cmu.edu

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1 Introduction

This paper derives both risk and ambiguity preferences from an underlying model of thoughts and feelings about information gaps, which was initially developed to explain preferences for information (Golman and Loewenstein, 2016; Golman et al., 2017). We argue that risk and ambiguity aversion arise from the discomfort of thinking about missing information regarding either outcomes or probabilities (over and above the effect of utility function curvature, which is modest at low stakes). Likewise, risk and ambiguity seeking occur in (rarer) cases in which thinking about the missing information is pleasurable. The main focus of our model is, therefore, on when and how people think about missing information, and the hedonic consequences of doing so. Here, we show that those thoughts and feelings have implications for decision making under risk and uncertainty.

We define an *information gap* (Golman and Loewenstein, 2016) as a question that one is aware of but for which one is uncertain between possible answers, and propose that the attention paid to such an information gap depends on two key factors: salience, and importance.[1] The salience of a question indicates the degree to which contextual factors in a situation highlight it. Salience might depend, for example, on whether there is an obvious counterfactual in which one does possess the missing information. The importance of a question is a measure of how much one's utility would depend on the actual answer. It is this factor – importance – which is influenced by actions like gambling.

In our model, gambling raises the importance of the gamble's associated questions (e.g., will I win, or what is my chance of winning?), which motivates one to wager on events which evoke questions that are enjoyable to think about and to not wager on events which evoke questions that are aversive to think about. A wide range of phenomena can be explained in such terms. For example, Lovallo and Kahneman (2000) find a strong positive correlation between willingness to accept a gamble and preference to delay resolution of that gamble so as to have more time to enjoy thinking about it. Lottery players often prefer to spread out drawings, perhaps in order to savor their thoughts about the possibility of winning (Kocher et al., 2014).[2] People are especially prone to insure against the loss of things

[1] Golman and Loewenstein (2016) assume that a third factor – surprise – contributes to attention when information is acquired; this assumption relates to information acquisition and avoidance (see Golman et al., 2017), but is unnecessary here. The effect of surprise on attention would, however, be relevant in situations in which the decision maker anticipates the hedonic consequences of observing the outcome of an uncertain prospect and not just the consequences of exposing oneself to this uncertainty.

[2] When faced with a risk of painful electric shocks, people generally prefer resolving this risk in one fell swoop rather than through a drawn out process, seemingly to avoid thinking about the possibility of receiving these shocks (Falk and Zimmermann, 2017).

they have an emotional attachment to (Hsee and Kunreuther, 2000), in our view because they find it unpleasant to think about losing these items. Financial professionals primed to think about the bust of a financial bubble become more risk averse (even with known probabilities in a laboratory setting) than those primed to think about a boom (Cohn et al., 2015). Self-reported feelings can be used to predict choice under risk in the laboratory (Charpentier et al., 2016). And in natural settings, it has been argued, the discomfort of thinking about risky situations is perhaps the primary motive behind risk avoidance (Loewenstein et al., 2001; see also Tymula et al., 2012).

In the case of risk, in our model, the key question about which people are uncertain – the information gap – centers around the eventual outcome when the uncertainty is resolved. For example, when deciding whether to accept a fair odds bet on a coin toss, the information gap is whether the coin flip will come out heads or tails. Thinking about the coin turning up heads (or about it turning up tails) does not seem intrinsically pleasurable or painful, but we suggest that the feeling of uncertainty about this outcome is a source of discomfort. Thus, in our model, risk aversion (even with low stakes) arises from a desire to avoid thinking about such uncertainty. Moreover, the model predicts, there will be stronger risk aversion when the outcome depends on additional uncertainties, so there will be more pronounced aversion for compound lotteries. Our predictions of risk aversion (rather than risk seeking) require information gaps to be unpleasant to think about. Despite the discomfort associated with feelings of uncertainty, however, such information gaps may be pleasurable to think about if the events under consideration have intrinsically positive valence, in which case our model would predict risk seeking. We would predict risk seeking behavior, for example, by a basketball fan for a lottery determined by his favorite basketball player making a free throw.

In the case of ambiguity, an additional key question comes into play: what the likelihoods are of obtaining different outcomes. When thinking about this question is aversive, then we expect people to be ambiguity averse; when pleasurable, they should be ambiguity seeking. Our model, therefore, provides an account of ambiguity avoidance and seeking that is different from existing accounts.

The most straightforward, novel prediction of our model is that people should be more willing to accept a risky or ambiguous gamble in situations in which they enjoy (rather than dislike) thinking about the uncertainties associated with the gamble. We test, and provide empirical support for, this novel prediction in two new experiments reported in the paper. In the first, we show that people stake larger total amounts on complementary bets that they feel good about –in this case, bets about their scores on a quiz after winning a prize for outscoring a competitor– in a situation in which there is no normative basis for doing so. In the second experiment, we show that people wager more on a $50/50$ bet between two good events than a $50/50$ bet between two bad events, specifically comparing bets on hometown baseball players getting hits versus striking out.

We proceed, in Section 2, to relate our model to the existing literature on risk and ambiguity and to distinguish our contribution. In Section 3 we introduce the formal model. Section 4 presents results applying this model to decisions under risk and Section 5 deals with ambiguity. We report experimental results supporting our theory in Section 6. Section 7 concludes. All mathematical proofs are in the appendix.

2 Relationship to Existing Literature

Our theoretical model is substantially different from prior theoretical work that has provided accounts of risk and ambiguity preferences that are inconsistent with expected utility over prizes (behavior such as low-stakes risk aversion (Rabin, 2000), the Allais (1953) common consequence and common ratio paradoxes, and the Ellsberg (1961) paradox). These other approaches typically incorporate departures from expected utility maximization, such as loss aversion (Kahneman and Tversky, 1979), non-additive probability weighting (Quiggin, 1982), and imprecise (set-valued) probabilities (Gilboa and Schmeidler, 1989). In contrast to these other approaches, our model adheres to expected utility, albeit over beliefs rather than outcomes. Our departure from traditional expected utility is that people derive utility from beliefs in addition to outcomes, and their exposure to risk or ambiguity can impact the utility they derive from their beliefs.

Our model is not intended to be a mutually exclusive alternative to these other theories of risk and ambiguity preference, which capture important psychological mechanisms and can explain many well-established behavioral patterns (see, e.g., Tversky and Kahneman, 1992; Köszegi and Rabin, 2006; Schmidt et al., 2008; Loomes and Sugden, 1982; Bordalo et al., 2012). Incorporating features from these theories would no doubt improve the predictive power of our model. For simplicity, however, we forego other behavioral assumptions and introduce utility of beliefs in an expected utility framework. With only this departure, we are able to account for a wide range of anomalous phenomena under risk and ambiguity. No model, however, including ours, can account for the full range of anomalous patterns of risk and ambiguity preference.

Our account of ambiguity preference is related to an account proposed by Frisch and

Baron (1988) according to which ambiguity aversion arises from the awareness that one is missing information that would help one to refine one's judgment of a gamble's probabilities.[3] Our account is similar to theirs in terms of focusing on missing information as the source of ambiguity preference. However, our account is more specific about how and why thinking about the information gap leads to ambiguity preference, allows for the idea that thinking about information gaps can be pleasurable, and makes the prediction that in these situations people will be ambiguity seeking.

In their superb review of the literature on ambiguity, Camerer and Weber (1992) feature prominently the notion, from Frisch and Baron (1988), that ambiguity can be thought of as "uncertainty about probability, created by missing information that is relevant and could be known."[4] Yet despite the prominence of awareness of missing information in their definition of ambiguity, in a later section of the review that summarizes many specific theories of ambiguity preference, not a single theory (nor class of theories) is based on the conception of ambiguity preference as a response to feelings about missing information. By deriving ambiguity preference from feelings about information gaps, our account is quite different from other explanations that have been proposed. For example, Ellsberg (1961), in the paper that introduced his eponymous paradox, proposed that people are pessimistic, fearing that the unknown probabilities will end up being unfavorable. Yet evidence suggests that people are often extremely optimistic in the face of uncertainty (Weinstein, 1980; Taylor and Brown, 1988). An account of ambiguity preference based on optimism and pessimism still requires an explanation (yet to be offered, as far as we know) for why people would be optimistic in those cases in which ambiguity seeking has been observed and why they would be pessimistic in those cases in which ambiguity aversion has been observed.

Other models aim to *describe* ambiguity preference but shed no light on its underlying cause. These models are intended to represent ambiguity-averse (and sometimes

[3]Similarly, Kovarik et al. (2016) propose that apparent ambiguity aversion in the Ellsberg paradox arises from not wanting to think about complexity. While our account resembles theirs, we would predict that raising the stakes in the Ellsberg gambles would lead to more ambiguity aversion, whereas their theory of complexity aversion predicts that magnifying prizes decreases its prevalence.

[4]Some models interpret ambiguity to mean that subjective odds cannot even be specified, but such a situation would be extreme. People make subjective probability judgments all the time. (Abdellaoui et al. (2011) makes a similar argument.) In our view, the distinction between ambiguity and risk is the decision maker's awareness (and uncertainty) about sources of uncertainty. With the so-called known urn in the Ellsberg paradox, the only uncertainty is about which ball will be drawn, and there is unawareness of the mechanism that will determine it. With the ambiguous urn, the decision maker is aware of an additional uncertainty about the contents of the urn in the first place. This makes a subjective probability judgment about the color of the drawn ball uncertain, but not impossible.

ambiguity-seeking) preferences, but they are not meant to be explanations for these preferences, which are seen as fundamental. For example, ambiguity preferences have been captured by assuming non-additive subjective probability weighting (as in Schmeidler's (1989) Choquet expected utility model or Tversky and Kahneman's (1992) cumulative prospect theory),[5] or imprecise (set-valued) probabilities (as in Gilboa and Schmeidler's (1989) Maxmin expected utility model, Hurwicz's (1951) α-maxmin model (see also Ghirardato et al., 2004), or Maccheroni et al.'s (2005) variational preferences model),[6] or second-order risk aversion (toward distributions of outcomes) rather than reduction of compound lotteries (as in Segal's (1987; 1990) extension of rank dependent utility, Klibanoff et al.'s (2005) smooth model, or other recursive expected utility models (Nau, 2006; Ergin and Gul, 2009; Seo, 2009)). In contrast, we aim to derive ambiguity aversion – and, in specific situations, ambiguity seeking – by considering fundamental preferences for information as well as over outcomes.

In a study that provided neural support for our interpretation of ambiguity aversion, Hsu et al. (2005) scanned the brains of subjects as they made choices involving ambiguous and unambiguous gambles. The authors found that the level of ambiguity in choices correlated positively with activation in the amygdala, a brain region that has been connected by numerous studies to the experience of fear. The authors conclude that "under uncertainty, the brain is alerted to the fact that information is missing, that choices based on the information available therefore carry more unknown (and potentially dangerous) consequences, and that cognitive and behavioral resources must be mobilized in order to seek out additional information from the environment." Additional studies have found that decision making involving ambiguous gambles, and even the perception of ambiguity in the absence of decision making, correlates with activity in the posterior inferior frontal sulcus / posterior inferior frontal gyrus (Huettel et al., 2006; Bach et al., 2009), a region of the brain that has been independently identified as responsible for attentiveness to relevant information in a task switching paradigm (Brass and von Cramon, 2004). Consistent with our information gap account, this region of the brain responds to ambiguity when information (that could potentially be known) is hidden from the observer, but not under conditions of

[5]Non-additive subjective probability weighting captures ambiguity aversion when the weights are supermodular (given linear weights for known probabilities), or, more generally, when the weights are more convex for ambiguous probabilities than for known probabilities (Wakker, 2010). These weights should not be interpreted as subjective probability judgments but merely as inputs into the decision model.

[6]Imprecise probability captures ambiguity aversion when the decision maker is cautious or pessimistic and considers worst-case scenarios. Yet in many real world decision environments, there is so much uncertainty that worst-case scenarios would render a decision maker impossibly conservative.

complete ignorance (Bach et al., 2009).

In building on a foundation of information preference, our model can help to explain when and why ambiguity preference takes different forms in different situations, including those that produce ambiguity seeking rather than aversion. One line of research (Fox and Tversky, 1995) shows that people value ambiguous and unambiguous gambles with similar subjective probabilities almost identically; it is only when the two types of gambles are compared to one-another that people become averse to ambiguity. The observation that people are more ambiguity averse when making choices between ambiguous and unambiguous gambles can be explained by the information gap account, assuming that such comparisons tend to raise the baseline attention weight on the information gap(s) relating to the probabilities associated with the ambiguous option.

Another line of research (Heath and Tversky, 1991) shows that people actually like to bet on ambiguous outcomes – e.g., a horse race – when they feel they are expert in the domain. People tend to be averse to ambiguity when they feel they are lacking information or expertise in a domain. The information gap account of ambiguity preference can easily account for these findings with a natural assumption that it is more pleasurable to think about issues one is more expert on. Betting in domains of expertise increases the attention weight on many questions about which one is confident, whereas betting on unfamiliar situations increases the attention weight on questions one is more uncertain about. We thus should expect people to have preferences over the source of uncertainty, generally preferring a familiar source to an unfamiliar source. In fact, people do prefer to bet on their vague beliefs in situations in which they feel especially competent or knowledgeable, but prefer to bet on chance when they do not (Heath and Tversky, 1991; Taylor, 1995; Keppe and Weber, 1995; Abdellaoui et al., 2011). Such 'source preference' may also help explain the common observation of relative over-investment in one's own country's (French and Poterba, 1991; Kilka and Weber, 2000), company's (Choi et al., 2005), and even locality's (Coval and Moskowitz, 1999) stock.[7]

The enjoyment of thinking about questions within one's area of expertise could also account for the prevalence of risk-seeking, especially in the domain of gambling. Gamblers often believe they have expertise on the particular events they wager on. They notoriously obey superstitions about hot or cold tables in a casino and rely on 'systems' for choosing

[7] Abdellaoui et al.'s (2011) source method can accommodate source preference in a revealed preference framework. Our model goes beyond revealed preferences and makes predictions about which sources of uncertainty people will or will not expose themselves to.

their stakes, even though many would acknowledge that the house retains a mathematical edge. von Neumann and Morgenstern (1944) explicitly disregarded the utility of gambling in capturing risk preferences with expected utility (see also Luce and Raiffa, 1957), but others have tried to incorporate intrinsic preferences for or against gambling into an expected utility framework (e.g., Fishburn, 1980; Diecidue et al., 2004). They associate a cost or benefit with a specific profile of material outcomes and probabilities (i.e., a "lottery"). A realistic behavioral model of intrinsic preferences about gambling must acknowledge that such preferences depend on the situation that gives rise to the gamble (Budescu and Fischer, 2001). In our model, the utility or disutility of gambling is not attached to the risk inherent in a gamble, but instead to the source of that risk. Particular sources of uncertainty arouse specific beliefs about those uncertainties and specific feelings – positive or negative – about those beliefs.

Like other accounts of ambiguity aversion that draw a connection between risk and ambiguity preference by assuming that ambiguity preference reflects second-order risk aversion (Segal, 1987; Klibanoff et al., 2005; Nau, 2006; Ergin and Gul, 2009; Seo, 2009), our account also proposes that both phenomena stem from the same underlying cause, but, as we have already described, introduces a novel mechanism involving thoughts and feelings about missing information. We do not doubt that other mechanisms, such as utility function curvature or a precautionary principle, also play a role in risk and ambiguity preferences. Nevertheless, we, like Caplin and Leahy (2001), Epstein (2008) and Navarro-Martinez and Quoidbach (2016), believe that affective feelings about uncertainty (i.e., information gaps) critically affect risk and ambiguity preferences. We suggest that these preferences are driven, or at least influenced, by the desire to not draw attention to questions one does not like thinking about.

3 Theoretical Framework

We use Golman and Loewenstein's (2016) question-and-answer framework and belief-based utility model. In this framework, we may define an information gap as a question that one is aware of but for which one is uncertain between possible answers. To begin, we represent a person's state of awareness with a (finite) set of activated questions $\mathcal{Q} = \{Q_1, \ldots, Q_m\}$, where each question Q_i has a set of possible (mutually exclusive) answers $\mathcal{A}_i = \{A_i^1, A_i^2, \ldots\}$. We let X designate a set of prizes. Denote the space of answer sets together with prizes as $\alpha = \mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_m \times X$. A cognitive state can then be defined by a probability measure π defined over α (i.e., over possible answers to activated questions as well as eventual prizes) and a vector of attention weights $\mathbf{w} = (w_1, \ldots, w_m) \in \mathbb{R}^m_+$. We define a question as "activated" when its associated attention weight is greater than zero. A utility function is defined over cognitive states, written as $u(\pi, \mathbf{w})$.

The probability measure reflects a subjective probability judgment about the answers to the activated questions and the prizes that may be received. The subjective probability over these prizes is in general mutually dependent with the subjective probability over answers to other activated questions. That is, material outcomes may correlate with answers about activated questions (and the answer to one question may provide information about the likelihood of different answers to another). We can consider a marginal distribution π_i that specifies the subjective probability of possible answers to question Q_i or π_X that specifies the subjective probability over prizes.

The attention weights specify how much a person is thinking about each question and, in turn, how much the beliefs about those questions directly impact utility.[8] The attention w_i on question Q_i is assumed to be strictly increasing in, and to have strictly increasing differences in, the question's importance γ_i and salience σ_i. To characterize the importance of question Q_i, we consider the probabilities of discovering any possible answer $A_i \in \mathcal{A}_i$ (or, omitting answers thought to be impossible, in the support of the individual's belief about the question, $\mathrm{supp}(\pi_i)$) and the utilities of the cognitive states $(\pi(\cdot|A_i), \mathbf{w})$ that would result from discovering each possible answer A_i.[9] We assume that the importance γ_i of question Q_i is a function of the subjective distribution of utilities that would result from different answers to the question,

$$\gamma_i = \phi\left(\Big\langle \pi_i(A_i),\, u\big(\pi(\cdot|A_i), \mathbf{w}\big)\Big\rangle_{A_i \in \mathrm{supp}(\pi_i)}\right), \tag{1}$$

that increases with mean-preserving spreads of the distribution of utilities and that is invariant with respect to constant shifts of utility.[10]

We assume that utility takes the form $u(\pi, \mathbf{w}) = u_X(\pi_X) + \sum_{i=1}^m w_i v_i(\pi_i)$.[11] The first

[8] Attention is, in principle, observable (perhaps imperfectly) through eye tracking, brain scans, and/or self-reports, but as it is, in practice, difficult to observe, we specify some determinants of attention weights.

[9] We assume here that belief updating after discovering answer A_i accords with Bayes' rule and that the attention weights do not change due to this discovery. Golman and Loewenstein (2016) assume that these attention weights are affected by surprise; all of our results are consistent with this assumption as well, but we neglect surprise for simplicity of presentation.

[10] The circularity –importance depends on utilities, utility depends on attention, and attention depends on importance– is by design. Technically, importance is a fixed point of this equation.

[11] Golman and Loewenstein's (2016) *separability*, *monotonicity*, and *linearity* properties would imply this

term describes the utility of a subjective distribution over prizes and the remaining terms describe the utilities of beliefs about each activated question, amplified by the attention weights on each of these questions.[12] We can identify as positive (neutral / negative) beliefs those for which increasing attention on the belief increases (does not affect / decreases) utility.

We further assume that the value of a belief (e.g., $v_i(\pi_i)$) depends only on the valences of the answers that are considered possible (e.g., $v_i(A_i)$ for all $A_i \in \mathrm{supp}(\pi_i)$) and the amount of uncertainty in the belief.[13] Naturally, $v_i(\pi_i)$ is increasing in $v_i(A_i)$ for each $A_i \in \mathrm{supp}(\pi_i)$.[14] Golman and Loewenstein (2016) posit a fundamental preference for clarity, which means that more uncertainty in a belief decreases its value. While entropy serves as a natural measure of the uncertainty in a belief, we need not make any assumptions quantifying uncertainty for our purposes here, and we instead simply assume a "one-sided sure-thing principle," which holds that people always prefer a certain answer to uncertainty amongst answers that all have valences no better than the certain answer (holding attention weight constant). If for all $A_i \in \mathrm{supp}(\pi_i)$ we have $v_i(\pi_i') \geq v_i(A_i)$, then $v_i(\pi_i') \geq v_i(\pi_i)$, with this inequality strict whenever π_i is not degenerate. This one-sided sure-thing principle operationalizes the assumption that uncertainty is aversive.[15] However, while we believe that uncertainty in and of itself is aversive, we still allow uncertain beliefs to have positive value if the answers considered possible have sufficiently high valence, i.e., we assume that if $v_i(A_i)$ grows large for each $A_i \in \mathrm{supp}(\pi_i)$, then $v_i(\pi_i) > 0$.

Finally, we assume that $u_X(\pi_X) = \sum_{x \in X} \pi_X(x) v_X(x)$. That is, apart from the utility derived from beliefs (and the attention paid to them), we would have expected utility over prizes. This assumption may well be unrealistically strong (it may preclude patterns of risk seeking for moderately likely losses or longshot gains, for example, when the value function over prizes is concave), but it simplifies the model so we can focus on the impact of beliefs on utility. Thus, to the extent that our account can reconcile phenomena that a traditional expected utility model cannot, the explanation will feature the utility of beliefs.

form for the utility function.

[12] If we assumed that the utility of objective outcomes was fully captured by their impact on beliefs, the first term could be left out of the model.

[13] We abuse notation by referring to the valence of answer A_i as $v_i(A_i)$, a convenient shorthand for the value v_i of belief with certainty in A_i.

[14] More precisely, we assume that if there exists $\tau : \mathcal{A}_i \to \mathcal{A}_j$ such that $\pi_i(A_i) = \pi_j(\tau(A_i))$ and $v_i(A_i) \leq v_j(\tau(A_i))$, then $v_i(\pi_i) \leq v_j(\pi_j)$ with the latter inequality strict if the former inequality is. This is Golman and Loewenstein's (2016) *label independence* property.

[15] In contrast, Savage's (1954) sure-thing principle is based on the view that uncertainty is not intrinsically attractive or aversive.

Wagering on an uncertain event is a kind of instrumental action that changes the chances of receiving various prizes, typically making them contingent on the answers to particular activated questions.[16] The obvious effect on the cognitive state is to transform the probability measure by providing new beliefs about the distribution over prizes conditional on beliefs about activated questions. A second effect is to impact attention weights because the change in prizes affects the importance of any question on which the prize is contingent. Such an action a, acting on a given cognitive state $(\pi, \mathbf{w})$, determines a new cognitive state $(\pi[a], \mathbf{w}[a])$. It specifies a map from every answer set $\mathbf{A} \in \mathcal{A}_1 \times \cdots \times \mathcal{A}_m$ to a conditional distribution over prizes in $\Delta(X)$. Along with the prior subjective judgment about the probability of each answer set, which is preserved by the action, this defines the new subjective probability measure $\pi[a] \in \Delta(\alpha)$. The new attention weights $\mathbf{w}[a]$ are determined by new values of importance as described by Equation (1). Preference between actions is determined by their impacts on the cognitive state, in accordance with the utility function $\mathrm{u}(a \,|\, \pi, \mathbf{w}) = u\left(\pi[a], \mathbf{w}[a]\right) - u(\pi, \mathbf{w})$. We can now apply the utility function to predict preferences between risky or ambiguous bets.

4 Risk

4.1 Low-Stakes Risk Aversion

People tend to be risk averse, even over low-stakes lotteries (Holt and Laury, 2002). The utility curvature needed to explain low-stakes risk aversion in a traditional expected utility model implies an absurd amount of risk aversion in high-stakes lotteries, such that, for example, an individual who at any wealth level rejects a 50-50 lottery to either gain \$110 or lose \$100 would have to reject a 50-50 lottery with a potential loss of \$1000, regardless of the potential gain (Rabin, 2000). Utility function curvature (i.e., diminishing marginal utility of money) almost certainly does play a role in risk aversion, but clearly something more is in play here, too. We suggest that betting on a lottery exacerbates the pain of thinking about an information gap by making it more important.[17]

To illustrate the information gap account for low-stakes risk aversion, consider a simplifying assumption that the value function for prizes v_X is linear over monetary prizes. (Of course, diminishing sensitivity to larger monetary prizes would be realistic, but any

[16] Actually observing the outcome of a wager is a separate action, not part of our present analysis. Decisions involving a sequence of actions, such as wagering and then observing the outcome of the wager, can be analyzed with the framework presented in our foundational paper (Golman and Loewenstein, 2016).

[17] In a rare case in which an uncertain lottery is pleasant to think about, we would suggest that risk seeking arises from the same mechanism.

differentiable value function can be well approximated by a linear function over a small neighborhood.) Consider a possible bet on a fair coin that could either pay x^* (win) or $-x^*$ (lose). Assume the decision maker has no intrinsic preference for heads or for tails (apart from the preference to win the lottery, if the bet is accepted), assigning both outcomes neutral valence. Then the decision maker will strictly prefer rejecting the bet.

Proposition 1 *Assume v_X is linear over $\mathbb{R}$. Suppose question Q_1 is about the outcome of the coin toss, so that it is independent of other questions, it is believed to be a fair coin with $\pi_1(\mathrm{H}) = \pi_1(\mathrm{T}) = \frac{1}{2}$, and both heads and tails have neutral valence, i.e., $v_1(\mathrm{H}) = v_1(\mathrm{T}) = 0$. Suppose bet b attaches prize x^* to heads and $-x^*$ to tails, so that $\pi_X^{\mathrm{H}}[b](x^*) = \pi_X^{\mathrm{T}}[b](-x^*) = 1$. Suppose not betting ($\neg b$) attaches prize 0 to both heads and tails, so that $\pi_X[\neg b](0) = 1$. There is a strict preference not to bet, $\neg b \succ b$.*

The intuition is that having to think about the outcome of the coin toss lowers utility because, according to our model, the uncertainty is aversive. Betting on the coin toss makes it more important, and the information gap would then attract more attention. The same logic also implies that if the decision maker were forced to bet on the coin toss, he would strictly prefer smaller stakes.

4.2 A Preference for Certainty

The observed patterns of non-standard risk preferences mostly seem to relate to a preference to avoid exposure to uncertainty relative to having certainty. The preference for certainty is well documented (e.g., Callen et al., 2014) and follows naturally from the information gap account. The pain of thinking about an information gap leads to what might be called *direct risk aversion*, above and beyond the risk aversion that can result from utility function curvature. There is a direct cost in the utility function simply from awareness of exposure to risk (i.e., from the existence of an information gap).[18] Direct risk aversion could underlie Gneezy et al.'s (2006) *uncertainty effect*, in which individuals value a risky prospect (say, a lottery between gift certificates worth \$50 or \$100) less than its worst possible realization (i.e., a \$50 gift certificate for sure). (See also Simonsohn's (2009) replication of the uncertainty effect.) In our model, such extreme direct risk aversion would require the uncertainty to relate to highly negative beliefs. Of course this state of affairs is rare. Given

[18] Ambiguity involves even more awareness of uncertainty than simple risk, so the information gap account also implies that there is an even larger direct utility cost from exposure to ambiguity, assuming this additional uncertainty is unpleasant to think about. Analogous to the uncertainty effect for risk, Andreoni et al. (2014) find that many subjects evaluating compound lotteries with a component that may be ambiguous actually violate (first-order stochastic) dominance as if there is a direct cost just to considering ambiguity.

the empirical facts, we might speculate that people associate the particular task of paying for a lottery over gift certificates with the danger of being suckered into a bad deal (Yang et al., 2013), which might well be a highly negative belief (see Prelec and Loewenstein, 1998; Weaver and Frederick, 2012).

4.3 Compound Risk Aversion

Seeing that people generally try to avoid exposure to an information gap, we might expect that compound lotteries – which expose an individual to multiple information gaps – are even more aversive. Indeed, the empirical evidence is clear that people do not reduce compound lotteries, or at least do not value them equivalently to their reduced form versions (Bernasconi and Loomes, 1992; Halevy, 2007; Spears, 2013; Abdellaoui et al., 2015; Armantier and Treich, 2015). This phenomenon poses a particular challenge to theories that do not allow for framing effects and that require the utility of a lottery to depend only on the possible outcomes and their probabilities. It is also a necessary consequence of the information gap account.

In this model, as long as the lotteries do not involve events with positive intrinsic valence, a compound lottery will be less preferred than an equivalent simple lottery. A lottery is traditionally defined as a known probability distribution over prizes. In our framework, we need to specify how the outcomes of the lottery depend on the answers to activated questions. We define a simple lottery as depending only on a single question, which is believed to be independent of all other questions (i.e., the beliefs about these questions are independent). Resolving uncertainty about this one question completely determines the lottery.

Definition Let Q_j be a question that is believed to be independent of all other questions. Given a sequence of prizes $x_h \in X$ with distinct valences, $v_X(x_{h_1}) \neq v_X(x_{h_2})$ for $h_1 \neq h_2$, suppose that an action a_j attaches prize x_h to answer A_j^h of question Q_j so that $\pi_X[a_j]\left(x_h|A_j^h\right) = 1$ for all h. Then we say that action a_j exposes the decision maker to a simple lottery determined by the answer to question Q_j.

We define a compound lottery as depending on multiple questions. Resolving uncertainty about a question corresponding to an early (i.e., non-terminal) stage of the compound lottery just exposes the decision maker to a new lottery based on updated beliefs.

Definition Let Q_i be a question, belief about which is pairwise dependent with belief about some other question $Q_{\tilde{i}}$. Given a sequence of prizes $x_h \in X$ with distinct valences,

$v_X(x_{h_1}) \neq v_X(x_{h_2})$ for $h_1 \neq h_2$, suppose that an action a_i attaches prize x_h to answer A_i^h of question Q_i so that $\pi_X[a_i]\left(x_h|A_i^h\right) = 1$ for all h. Then we say that action a_i exposes the decision maker to a compound lottery determined by the answer to question Q_i (and contingent on question $Q_{\tilde{i}}$).

We seek to compare a compound lottery to a simple lottery when they are materially equivalent.

Definition An action a_i that exposes the decision maker to a compound lottery is materially equivalent to an action a_j that exposes the decision maker to a simple lottery if they induce the same marginal probability distribution over prizes, $\pi_X[a_i] = \pi_X[a_j]$.

Proposition 2 *Let action a_j expose the decision maker to a simple lottery determined by the answer to question Q_j, and let a materially equivalent action a_i expose the decision maker to a compound lottery determined by the answer to question Q_i. Suppose that the answers to any question jointly dependent with Q_i (including Q_i itself) as well as the answers to Q_j all have neutral valence. Suppose questions Q_i and Q_j both have the same salience, $\sigma_i = \sigma_j$. Then the simple lottery is preferred, $a_j \succ a_i$.*

The intuition here is that the compound lottery (in contrast to the simple lottery) exposes the decision maker to additional information gaps. By assumption, these information gaps are unpleasant to think about. Putting a prize on the line to depend on the outcome of the uncertain events makes these information gaps more important. That makes the compound lottery worse than the simple lottery.

5 Ambiguity

Information gaps underlie ambiguity as well as risk. In our view, an ambiguous prospect is no different from a compound lottery. People are simply aware of and uncertain about the question, "what are the probabilities of the various outcomes?" Despite having no information from which to form an objective probability over answers to this question, we propose that the decision maker can form a subjective probability (as in Segal (1987) or Seo (2009)), but thoughts and feelings about this information gap affect choice under ambiguity.

Definition An ambiguous gamble determined by the answer to question Q_i is a compound lottery determined by the answer to question Q_i that is contingent on the question "what is the probability distribution over answers to question Q_i?"

5.1 Ambiguity Aversion

Consider the preference for the known urn in Ellsberg's problem (Ellsberg, 1961; Becker and Brownson, 1964; MacCrimmon and Larsson, 1979).[19] Even if you bet on the urn with the known proportions of balls, the proportion of balls in the other urn you could have selected is still a piece of missing information. To explain the phenomenon in terms of our model, therefore, we propose that there is a relevant question for both urns: "What is the proportion of each colored ball?" and that the attention weight is *relatively* greater for the question relating to the urn you choose. This follows from the assumption that attention weight increases in a question's importance. One knows the answer to the question for the precisely specified urn, but not for the ambiguous one.

According to our model, the desire for clarity, along with the desire to pay less attention to negative beliefs, would cause an individual to bet on the known urn rather than the ambiguous urn in the Ellsberg paradox. Much like in Ergin and Gul (2009), uncertainty aversion leads to second-order risk aversion. Corollary 1 follows from Proposition 2 from Section 4.3 and applies directly to the Ellsberg paradox. Consider Q_i to be the question of which ball is drawn from the ambiguously specified urn. Belief about this question depends on the belief about the composition of this urn. On the other hand, belief about Q_j – which ball is drawn from the known urn – is independent of all other beliefs.[20]

Corollary 1 *Let action a_j expose the decision maker to a simple lottery determined by the answer to question Q_j, and let a materially equivalent action a_i expose the decision maker to an ambiguous gamble determined by the answer to question Q_i. Suppose that the answers to any question jointly dependent with Q_i (including Q_i itself) as well as the answers to Q_j all have neutral valence. Suppose questions Q_i and Q_j both have the same salience, $\sigma_i = \sigma_j$. Then the simple lottery is preferred, $a_j \succ a_i$.*

Recognizing feelings about information gaps allows us to explain the preference for betting on the known urn rather than on the unknown urn, even when the subjective probability judgment about the odds of winning a prize is the same for both urns. Crucially, our account relies on aversion to missing information rather than a distinction between objective and subjective probabilities. Thus, consistent with Halevy's (2007) experimental

[19] Non-neutral ambiguity attitudes have been observed in many other experiments as well (e.g., Borghans et al., 2009; Ahn et al., 2014; see Trautmann and van de Kuilen, 2014).

[20] The belief that there is a one-half chance of drawing a red ball and a one-half chance of drawing a black ball from the known urn is determined by the belief about its composition, but this belief is held with certainty, and dependence on a probability zero / one event is impossible.

findings, our premise is that ambiguity preference goes hand in hand with preference over compound (objective) lotteries.[21] Additionally, consistent with Chew et al.'s (2017) experimental findings, we would predict that preference for an ambiguous gamble (over events with neutral valence) declines when there is more uncertainty about the probabilities. Similarly, we account for the fact that people prefer to bet on a single ambiguous urn than on the relationship between draws from two different ambiguous urns (Epstein and Halevy, 2017).

5.2 Comparative Ignorance Effect

Note that our explanation of ambiguity preference is inherently context dependent. In the Ellsberg paradox, ambiguity aversion arises from a desire to not pay attention to a salient information gap, combined with the opportunity to shift attention in the desired direction by placing the bet on the known urn. The description of the two urns in comparison makes salient the difference in their composition, so the questions about the composition of the urns get non-negligible attention weight. If, however, an individual is asked to price a bet on a draw from just one of the urns in isolation, the question of the composition of that urn is less salient, and so receives less attention weight. As long as the question is activated, we would expect some degree of ambiguity aversion, because taking a sure payment in lieu of the bet still does shift attention away from an uncertain prospect, but (because attention weight exhibits increasing differences in salience and importance) we would expect the degree of ambiguity aversion to be less when pricing bets on isolated urns than when pricing bets on urns that can be compared. This is precisely the comparative ignorance effect that Fox and Tversky (1995) and, following them, Chow and Sarin (2001) documented.[22]

Proposition 3 *Let action a_i expose the decision maker to an ambiguous gamble (or compound lottery) determined by the answer to question Q_i and contingent on question $Q_{\tilde{\imath}}$. Suppose that the answers to any question jointly dependent with Q_i all have neutral valence. Consider two possible baseline cognitive states $(\pi, \mathbf{w})$ and $(\pi, \hat{\mathbf{w}})$ that have the same probability judgments but with different attention weights that result from question $Q_{\tilde{\imath}}$ being more salient in the latter state than in the former, i.e., $\hat{\sigma}_{\tilde{\imath}} > \sigma_{\tilde{\imath}}$ and $\hat{\sigma}_\nu = \sigma_\nu$ for*

[21] Ambiguity preference may nevertheless be more extreme than compound lottery preference if the ambiguity makes the uncertainty more salient.

[22] Similarly, if an individual is presented with extraneous information that seems to relate to the ambiguous issue, but is not easily processed, this information activates additional questions about which the individual is uncertain. The individual can shift attention weight away from these uncertain beliefs by avoiding a bet on the ambiguous issue. Indeed, Fox and Weber (2002) find that such unhelpful information makes ambiguous bets appear less attractive.

all other Q_ν.[23] *Action a_i would be more preferable in the former cognitive state than in the latter, i.e.,* $\mathfrak{u}(a_i \,|\, \pi, \mathbf{w}) > \mathfrak{u}(a_i \,|\, \pi, \hat{\mathbf{w}})$.

Proposition 3 suggests that the comparative ignorance effect is an example of a more general phenomenon whereby a more salient information gap generates stronger ambiguity aversion. Consistent with this pattern, in a hypothetical scenario involving unknown risks of a vaccine (a scenario that subjects can intuitively grasp), salient missing information about whether the risk was high or had been eliminated made subjects more reluctant to vaccinate than when the subjects faced the same risk presented with no salient missing information (Ritov and Baron, 1990).

Other context effects have been noted as well and can be explained by our theory.[24] Studies have found that ambiguity aversion is exacerbated when others can observe the choice (Curley et al., 1986) and reduced when no others (not even the experimenter) can observe whether the bet wins or loses (Trautmann et al., 2008). The authors interpret this finding to mean that that the preference to avoid subjecting oneself to unknown risks is related to a desire to avoid social disapproval. Our model does not treat social disapproval as a fundamental, but could accommodate this phenomenon by positing, plausibly we believe, that that the possibility of social disapproval makes the unknown composition of the ambiguous urn that much more important if the bet on this urn is chosen.

5.3 Source Preference

Context dependence also helps us explain those situations in which ambiguous prospects are, in fact, preferred to risky, but clearly-defined, gambles. In our analysis of the Ellsberg paradox, the prediction of ambiguity aversion depends on shifting attention between single beliefs that all involve neutral answers but that vary in their certainty. In general, shifting attention to favorable issues or away from unfavorable issues should increase utility. That is, we predict a preference for betting on issues one likes thinking about and for not betting on issues one does not like thinking about.

Proposition 4 *Let action a_j expose the decision maker to a simple lottery determined by*

[23] Consider the latter cognitive state to result from joint evaluation of a_i alongside an action that exposes the decision maker to a materially equivalent simple lottery, and consider the former cognitive state to result from isolated valuation of a_i.

[24] For an example in the domain of risk, lotteries that are presented with narrow bracketing (and thus, we believe, made more salient) generate stronger risk aversion (Gneezy and Potters, 1997; Bellemare et al., 2005; Haigh and List, 2005; Anagol and Gamble, 2013). Proposition 3 of course implies that the salience of an information gap affects risk preferences as well as ambiguity preferences, in accord with this empirical pattern.

the answer to question Q_j, and let a materially equivalent action a_i expose the decision maker to an ambiguous gamble (or compound lottery) determined by the answer to question Q_i. Suppose questions Q_i and Q_j both have the same salience, $\sigma_i = \sigma_j$. Suppose that the answers to question Q_i and/or the answers to some question(s) that are pairwise dependent with Q_i all have the same valence υ, but now allow this valence to be positive or negative. Suppose that the answers to any other questions believed to be jointly dependent with Q_i as well as the answers to Q_j all still have neutral valence. Preference for action a_i, i.e., $\mathrm{u}(a_i \mid \pi, \mathbf{w})$, is increasing in the valence υ. Moreover, for sufficiently high υ, the ambiguous gamble is preferred to the simple lottery, $a_i \succ a_j$.

Proposition 4 implies that ambiguity-seeking behavior arises when information gaps are pleasurable to think about, i.e., in special cases in which outcomes have high valence. For example, ardent sports fans may enjoy betting on the outcome of a game they look forward to watching. They would generally prefer to bet on their home team than on other teams, and especially in comparison to a team their home team is playing against (Babad and Katz, 1991; Morewedge et al., 2016). Cases of pleasurable information gaps may often coincide with issues about which one has significant expertise. To the extent that people generally enjoy thinking about issues for which they have more expertise and dislike unfamiliar situations, Proposition 4 would account for Heath and Tversky's (1991) findings demonstrating a preference to bet on familiar rather than unfamiliar sources of uncertainty (see also Abdellaoui et al., 2011).

Also consistent with our hypothesis that gambling is correlated with the valence of an issue is the fact that people become less willing to hold risky assets after realizing a loss (Imas, 2016), as the painful experience of a loss could make thinking about another risky asset more unpleasant. This realization effect could lead to path dependent risk and ambiguity attitudes. Barberis (2011) suggests that such dynamic changes in ambiguity preference may amplify financial panics that begin with relatively modest declines in asset values.

5.4 Machina Paradoxes

Machina (2009) introduced two decision problems for which typical patterns of behavior violate the predictions of most models of choice under ambiguity, including Choquet expected utility, maxmin expected utility, α-maxmin, variational preferences, and the smooth model of ambiguity aversion (Baillon et al., 2011). As these paradoxes have been so challenging for models of ambiguity aversion to accommodate, we find it illuminating to show

how they are compatible with our model of informational preference.

Machina's "$50 : 51$ Example" presents an urn holding 50 balls colored red or yellow (in unknown proportion) and 51 colored black or green (also in unknown proportion). Table 1 displays four bets, showing the payoffs contingent upon the ball drawn. We may take 0, 101, 202, and 303 to be prizes equally spaced on the utility scale, given one's beliefs.[25] An

Table 1: Machina's $50 : 51$ Example.

	50 balls		51 balls	
Bets	Red	Yellow	Black	Green
a_1	202	202	101	101
a_2	202	101	202	101
a_3	303	202	101	0
a_4	303	101	202	0

individual chooses between a_1 or a_2, then between a_3 or a_4. Both choices involve allocating prizes between yellow and black with the remaining prizes fixed, but the contexts vary in how these remaining prizes are fixed. Bets a_2 and a_4 allocate the larger prize to black rather than yellow, which, if the individual accepts the principle of insufficient reason, means greater expected value. Bets a_1 and a_3, on the other hand, reduce how much is at stake depending on the unknown proportions in the urn. While they each reduce the stakes by the same absolute amount, bet a_1 eliminates all dependence on these uncertainties, whereas bet a_3 does not. The typical preference, $a_1 \succ a_2$ and $a_3 \prec a_4$ (at least when the magnitude of the payoffs is tuned just right), reflects a willingness to forego some material payoffs (in expectation) in order to lessen one's exposure to the unknown when the remaining exposure is minimal, but not when the remaining exposure is significant.

According to our model, choosing a bet affects utility in two ways. It determines the prize distribution corresponding to one's subjective belief about activated questions, thus directly affecting the expected value of the eventual prize. But, additionally, to the extent the distribution of prizes depends on the answers to various activated questions, a bet affects the importance of these questions, which in turn affects the utility derived from one's beliefs about these questions. As with the Ellsberg paradox, it seems reasonable to assume that all possible compositions of the urn (consistent with the known $50 : 51$ split) are subjectively judged to be equally likely and that an individual does not care about the actual proportion

[25] Actually eliciting prizes that are equally spaced on the utility scale requires, according to our model, subjects to consider random distributions of prizes that are independent of their beliefs about activated questions. There is a leap of faith in believing that subjects do not activate a question concerning which prize they will actually receive. Our analysis is not disturbed, however, if we accept this merely as an approximation.

or about which ball is drawn apart from the corresponding material payoff (i.e., all answers have neutral intrinsic valence). Drawing a black ball is thus subjectively judged to have a $\frac{.5}{101}$ greater chance than a yellow. By construction, this means that bets a_2 and a_4 each offer a gain in expected value of $.5$ over bets a_1 and a_3 respectively. On the other hand, bets a_1 and a_3 would lessen the importance of questions about the composition of the urn relative to bets a_2 and a_4 respectively. This would decrease the attention weight on the uncertain belief about the composition of the urn – a negative belief because of the uncertainty. Decreasing the attention weight on a negative belief, of course, increases utility. Our assumptions do not specify precisely how much the attention weight decreases as the stakes are reduced, but it is perfectly reasonable to think that there is diminishing sensitivity of attention weight to how much is at stake corresponding to an uncertain belief. Thus, our model can easily accommodate a greater gain in utility when rendering an uncertainty completely moot than when partially drawing down a higher-stakes exposure (and merely limiting its importance somewhat). This would allow the pattern $a_1 \succ a_2$ and $a_3 \prec a_4$.

Machina's second paradox, the "Reflection Example," involves a similar urn that is now balanced with 50 red or yellow balls and 50 black or green balls. Table 2 displays four bets, showing the payoffs in the case that each kind of ball is drawn. In this example, the prizes do not need to have equal utility increments, and it's fine to think of them as monetary payoffs. Once again, an individual first chooses between b_1 or b_2, then between b_3 or b_4.

Table 2: Machina's Reflection Example.

	50 balls		50 balls	
Bets	Red	Yellow	Black	Green
b_1	0	50	25	25
b_2	0	25	50	25
b_3	25	50	25	0
b_4	25	25	50	0

As in the $50\!:\!51$ example, both choices involve allocating prizes between yellow and black with the remaining prizes fixed, and the contexts vary in how these remaining prizes are fixed. Bets b_1 and b_3 reduce the stakes that depend on the proportion of black to green balls but increase the stakes that depend on the proportion of red to yellow balls, relative to bets b_2 and b_4 respectively. Viewed alternatively, bets b_1 and b_4 eliminate exposure to one source of uncertainty while amplifying exposure to another, relative to bets b_2 and b_3. Empirically, the most common pattern of choices (exhibited by about half of subjects) is $b_1 \succ b_2$ and $b_3 \prec b_4$, with a sizable minority (slightly above a quarter of subjects) choosing the opposite, and relatively few violating reflection symmetry (L'Haridon and Placido, 2010).

An individual who judges all possible compositions of the urn to be equally probable would determine that the expected values of the prizes associated with these four bets are all equal. Thus, according to our model, the choice between bets would hinge on which bet placed less attention weight on uncertain, negative beliefs. Once again, our model does not specify precisely how much importance, or, in turn, attention weight, decreases as the stakes associated with an uncertain belief are drawn down, and there could well be heterogeneity across the population, so the model does not rule out any pattern of behavior in this example. Still, from this perspective, the typical pattern of behavior is not surprising. If, as we hypothesized in order to explain the $50 : 51$ example, attention weight exhibits diminishing sensitivity to exposure to an uncertain belief, then eliminating a modest exposure entirely would have a greater effect than partially reducing a large exposure by the same amount. By the informational symmetry between the red/yellow composition and the green/black composition, the (negative) value of the (uncertain) belief about each should be equal. Accordingly, a greater reduction in attention weight would lead to a greater increase in utility, regardless of which uncertainty is rendered moot. That is, we would then predict $b_1 \succ b_2$ and $b_3 \prec b_4$. Thus, diminishing sensitivity of attention weight with respect to the stakes associated with an uncertain belief allows our model to accommodate both of Machina's paradoxes.

6 An Experimental Test of a Key Prediction

The 'acid test' of a new theory is to generate, and test, predictions that other theories do not predict, and which have not already been tested. Here, we report two such tests of our theory. The key prediction of the theory is that people will be more willing to bet on, and will bet more on, uncertainties that they like to think about. Study 1 confirms this prediction in a domain of ambiguity, and study 2 confirms the prediction in the domain of risk.

6.1 Study 1

We created a situation likely to produce strong feelings by having pairs of people compete on a two-part intelligence test, with one person winning and one person losing. Both individuals then had the opportunity to bet on whether they did better on the first part of the test than the second and to make the complementary bet that they did better on the second part of the test than the first. Our premise is that people who won the competition would find it more pleasurable to think about the test, so we predicted that winners would be willing to bet more, in total, on the two complementary bets than would losers.[26]

Of course, subjects could not be randomly assigned to the conditions of winning or losing the competition, so there could be a selection effect. More intelligent people tend to be more risk tolerant (Dohmen et al., 2010), so winners might bet more because of their intelligence. To rule out such a selection effect, we controlled for idiosyncratic risk preferences by also offering subjects a third bet on a random event involving rolls of dice.

We recruited subjects for time slots, deliberately scheduling two subjects for each slot. The subjects were recruited separately (so most did not know one-another) and participated for a show-up fee of $10 and the opportunity to win additional money and/or prizes through incentivized choices.

The two subjects first competed against one-another on a math quiz to win a non-monetary prize. The math quiz was derived from previous GRE tests. It consisted of 18 multiple-choice questions, divided into two clusters of 9 problems. One cluster consisted of traditional math problems (e.g., if $5x + 32 = 42x$, what is the value of x?), and the other consisted of quantitative comparison problems (e.g., which is greater: 54% of 360 or 150?). The order of the two clusters was randomly determined, and subjects were given 6 minutes to work on each cluster, with a warning one minute from the end of each 6 minute interval. The warning instructed them that they had a minute left, and encouraged them to guess as needed to give some answer to each question, since there was no penalty for incorrect answers. Upon completion, quizzes were scored immediately. Subjects were informed of their total score on all 18 problems, but, crucially, were not told their score breakdown on each cluster of the quiz. (By encouraging guessing, we aimed to make subjects more uncertain about which questions they answered correctly, even if they could accurately assess their own ability.) To increase the salience of winning and losing, the subject in each pair who received the higher score was given a tangible bonus prize – a succulent plant with a retail value of $3-$5.

Subjects were then told that they would be presented with three gambles and were told that only one of them would count, to be determined randomly. Each gamble was presented sequentially, with no preview of what subsequent gambles would consist of. The first gamble was presented as follows:

[26] We used a quiz for the competition not to induce feelings of competence, but simply because our subjects –mostly Carnegie Mellon students– would care about the results. Thus, winning the competition should increase the valence of both possible answers to the question, "which part of the test did I score higher on?" Winning the competition should not affect the attention to this question, nor should it affect feelings of competence to predict the actual answer.

Gamble 1 depends on your performance on the quiz. Gamble 1 will pay equal to your wager if your score on the quantitative comparison questions is greater than or equal to your score on the problem solving questions.

Please indicate how much you are willing to wager. You can wager up to half your money ($5). If you win the bet, then you will get back double the amount you wager. If you lose the bet, then you will lose the amount you wager.

How much do you want to wager?

nothing $1 $2 $3 $4 $5

The second gamble was the complementary bet that paid only if their score on problem solving questions was greater than or equal to that on the quantitative comparisons questions. The third gamble involved two rolls of a ten-sided die. It paid out if the second roll was greater than or equal to the first roll. The amount that subjects wanted to stake on each of these gambles was elicited in the same way as it was for the first (with the knowledge that only one of their choices would count). An exit survey collected their attitudes regarding the task and prize as well as demographic data.

Subjects were 102 individuals from Pittsburgh area universities (48 males, 54 females, $M_{\text{age}} = 24.75$) who were recruited using the Carnegie Mellon University Center for Behavioral Decision Research Participant Pool. One subject was excluded from the analysis because he achieved a perfect score on the test, from which he could infer that his score on the two parts would be equal.[27]

The top rows of Table 3 present means and t-tests comparing amounts gambled and selected other variables as a function of whether an individual did or did not win the prize. As hypothesized, individuals bet much more on gamble 1 and the total of gamble 1 and 2 when they won the prize. (They did not bet more on gamble 2, perhaps because of cognitive dissonance after having just chosen a wager on the complementary bet.) As predicted by the theory, and helping to rule out selection effects and mood effects, winning the prize for performance on the math quiz had no significant impact on the amount subjects staked on the third, unrelated, gamble.

Not surprisingly, subjects who won felt better about their performance and reported that they performed better than they would have expected to. An unexpected finding was that those who won the prize reported liking it more than those who did not, perhaps indicating

[27]We have reported all measures, conditions, and data exclusions. A sample size of 100 was initially chosen arbitrarily and then increased to include one additional pair after one subject's data had to be excluded.

Table 3: Mean amounts gambled and self-reported feelings if subjects won or lost the pairwise competition.

	gamble 1	gamble 2	total (1 & 2)	gamble 3	feeling about performance	performance relative to expectation	liking of prize
lost	\$1.84	\$.84	\$2.68	\$1.38	3.5	3.3	2.2
won	\$2.92	\$.96	\$3.88	\$1.65	5.1	4.2	2.5
significance	$p = .005$	$p = .67$	$p = .004$	$p = .38$	$p < .001$	$p < .001$	$p = .007$
correlation with total points	$r = .24$ $p = .014$	$r = -.007$ $p = .95$	$r = .23$ $p = .02$	$r = -.03$ $p = .79$	$r = .54$ $p < .001$	$r = .17$ $p = .09$	$r = .04$ $p = .73$

denigration of the prize by those who didn't receive it. The bottom row, which presents Pearson correlations between total points earned on the exam and the same variables, shows that amount staked on the first gamble was positively related to total points earned on the exam, and that feeling about performance on the exam was also strongly correlated with actual performance.

Table 4 presents regressions of total amount staked in gambles 1 and 2 on selected variables. Specification 1 simply reiterates that winning the prize increased the total amount staked, by approximately \$1.20 (out of a total maximum of \$5.00). Specification 2 shows that winning the prize still increases amount gambled, even after controlling for total score (total score is significant if entered by itself; $p < .05$). Specification 3 shows that winning the prize is no longer statistically significant (although the magnitude of the point estimate is not reduced much) once feeling about performance on the test is included in the equation. This supports our theory's account of the effect – i.e., that people like to bet on events they feel good about. Specification 4 shows that the effect of winning the prize is still significant after controlling for demographics (age and gender). Females did stake considerably less (\$1.10) on the two gambles than did men, and females did considerably worse on the test than males (10.3 correct versus 11.9 correct; $p < .05$). Finally, specification 5 uses amount staked on gamble 3 to control for generalized risk-taking. This does not affect the other coefficients substantially, but the coefficient on the new term is large and highly significant, and the adjusted R^2 of the overall equation more than doubles.

The significant effect of winning the prize even after controlling for gambling on the dice rules out the alternative explanation that people who do better on the quiz are simply more risk tolerant in general. However, another potential confound would arise if people who do better on the quiz are better able to predict the part of the quiz that they scored higher on. If they could better identify whether gamble 1 or gamble 2 was likely to pay

Table 4: Regressions of total amount staked in gambles 1 & 2.

	1	2	3	4	5
Won Prize	1.2*** (.41)	.96** (.47)	.71 (.46)	.95** (.41)	.84** (.37)
Total Score		.07 (.06)			
Feeling About Performance			.31** (.15)		
Female				-1.1*** (.41)	-.85** (.37)
Age				-.002 (.03)	-.01 (.02)
Amount Staked on Gamble 3					.60*** (.12)
Constant	2.7*** (.29)	3.3*** (.64)	1.6*** (.59)	3.5*** (.79)	2.6*** (.72)
Adjusted R^2	.071	.072	.10	.12	.30

$N = 101$
Standard errors in parentheses
$p < .10$, ** $p < .05$, *** $p < .01$

out, they could rationally bet \$5 just on the high-expected-value gamble, and their total betting would then exceed other subjects' average total betting. To rule out this confound, we check whether subjects who won the prize earned more from their bets than subjects who lost. We find no significant relationship between winning the prize and earnings from the bets.[28]

Our finding of an increased preference to make bets when one feels better about the subject of the bets is a new kind of source preference for exposure to ambiguity. Heath and Tversky (1991) first demonstrated a source preference involving expertise or familiarity. Their experiments showed that people prefer to make bets when they are more knowledgeable about the subject of the bets. In contrast, our experiment shows that people prefer to make bets when they enjoy thinking about the subject of the bets. Of course, in many situations a subject that a person is knowledgeable about is one that the person enjoys thinking about and vice versa. In our experiment, however, there is no reason to believe that people who win the prize for performance on the test are more knowledgeable about which section of the test was harder or easier for them. Performance on the test obviously indicates a cognitive ability, i.e., competence in math, but it does not indicate metacognitive ability, i.e.,

[28]We thank Emir Kamenica for suggesting that we check this.

competence in self-assessment of mathematical knowledge. Winning or losing the prize thus manipulates people's feelings about their performance without affecting their knowledge about their performance. Heath and Tversky's competence hypothesis cannot account for our finding.

6.2 Study 2

We also designed a lab-in-the-field study to again see if people do, in fact, bet more money on uncertainties that they like to think about than uncertainties they do not like to think about. We identified hometown sports teams' performance as an issue that many people have strong feelings about; in particular, we examined participants willingness to bet on baseball player performance during regular season play.

We assumed that Pittsburgh Pirates baseball fans like thinking about Pirates getting hits and dislike thinking about Pirates striking out. Our theory predicts that Pirates fans' willingness to bet on a gamble over which of two Pirates has more hits (positive-valence information gap) in a fixed period should be greater than their willingness to bet on a gamble over which of two Pirates has more strikeouts (negative-valence information gap) in the same fixed period. To test this prediction, we asked visitors of the Three Rivers Arts Festival, a free community fair held in Downtown Pittsburgh, to complete a 1-page study in which they were given an opportunity to place a bet on the performance of two players.

In our between-subjects study, we gave participants the opportunity to make a simple bet: whether Josh Bell or Josh Harrison (two of the top players on the Pirates) would get more hits (in the positive condition, or, instead, strikeouts in the negative valence condition) during the four week period leading up to the All-Star Game. (If they both were to get the same number of hits (or strikeouts respectively), the bet would neither win nor lose.) We gave participants a \$25 credit to bet with and promised that one in every five participants would receive their balance (i.e., \$25 plus or minus their bet, depending on whether they won or lost) in the form of an Amazon.com gift card emailed to them after the four weeks were up. We first elicited participants' familiarity with the two players (and informed subjects who had no idea who the players were that they were two of the top hitters for the Pirates). We then asked subjects to decide and report how much they wanted to bet *before knowing who they were betting on*. We then determined the player that participants were betting on via a coin flip. (Participants first picked one of the two players to associate with heads and were allowed to make a clean flip of the coin themselves. If heads came up, their bet was for the player they picked. If tails came up, their bet was for the other player. This

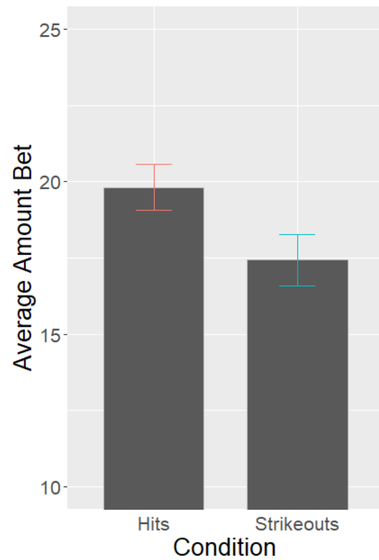

Figure 1: Average amount bet on gambles about hits and about strikeouts.

way there could be no doubt that the chances of winning the bet, before knowing who they were betting on, were $50/50$.)

After the bet was made, participants completed a brief set of demographic questions. Of the 193 people that completed the study, 50% were male (n = 96), the average age was 36.57 (min = 18, max = 86), and 61% (n = 118) had at least a bachelor's degree.[29] Figure 1 shows the average amount bet in each condition. As predicted, subjects bet significantly more in the positive-valence condition than in the negative-valence condition. Table 5 presents regression analyses of willingness to bet on the two different gambles. Specification 1 simply regress the amount bet on the study condition. Specification 2 includes demographic controls. A specification including measures of fandom is not included in the regression table because the controls did no meaningfully impact the measures. In short, these specifications show that study participants were willing to bet significantly more in the positive valence bet (hits) than the negative valence bet (strikeouts, which serves as the regression constant).

7 Conclusion

Preferences over beliefs (and the attention paid to them) create preferences for or against risky and ambiguous gambles. This information gap account of attitudes toward risk and

[29]We have reported all measures, conditions, and data exclusions. Our sample size was determined by the number of subjects we were able to recruit during the arts festival and was finalized before we examined the data.

Table 5: Regressions of amount bet on the baseball gamble.

	Dependent variable:	
	Amount bet in \$US	
	(1)	(2)
Hits	2.390^{**}	2.201^{*}
	(1.128)	(1.145)
Age		-0.043
		(0.036)
Education		-0.062
		(0.369)
Male		1.110
		(1.140)
Constant	17.429^{***}	18.883^{***}
	(0.820)	(2.134)
Observations	193	187
R^2	0.023	0.037
Adjusted R^2	0.018	0.016
Residual Std. Error	7.822 (df = 191)	7.768 (df = 182)
F Statistic	4.490^{**} (df = 1; 191)	1.754 (df = 4; 182)
Note:		*p<0.1; **p<0.05; ***p<0.01

ambiguity makes sense of low-stakes risk aversion, the difference between comparative and non-comparative responses to ambiguity vis a vis risk, and the sensitivity of ambiguity preference to the source of the uncertainty. It is consistent with empirically documented patterns of behavior that have been difficult for other theories to reconcile. We have established the following testable predictions:

- H1 Individuals prefer to avoid actuarially fair lotteries that do not involve events that they particularly enjoy thinking about.
- H2 Individuals prefer an equivalent simple lottery to a compound lottery that does not involve events that they enjoy thinking about.
- H3 Individuals prefer to wager on uncertainties they enjoy thinking about (i.e., that depend on positive beliefs) than on objectively random events, but prefer such random bets to wagers that depend on negative beliefs.
- H4 Individuals forced to choose among wagers that depend on negative beliefs prefer to wager on an uncertainty that is less salient.

Timing effects are not part of our formal model, and intuitions about the effects of time delay runs in both directions. From one point of view, it seems intuitive that the costs (or benefits) associated with thinking about negative (or positive) beliefs would scale with the amount of time that an individual spends thinking about them. To the extent the pleasures or pains of focusing on an information gap account for risk and ambiguity preferences, we should then expect that some time delay between exposure to uncertainty (risk or ambiguity) and resolution of that uncertainty would strengthen risk and ambiguity preferences. On the other hand, there is substantial evidence that the feelings associated with uncertainty are strongest right before uncertainty is going to be resolved (van Winden et al., 2011). This suggests that short- and long-term time discounting will dictate whether time delay strengthens or weakens risk and ambiguity preferences. Although we are reluctant to offer any general predictions about the effect of time delays, to the degree that time delay intensifies risk or ambiguity preferences, we would speculate that the effects would be stronger for people who discount the future less.

The primary determinant of risk and ambiguity preference in our model is how people feel when they think about the information they are missing about a gamble. These feelings are likely to be a function of a wide range of factors, including the outcomes, associated probabilities, the vividness of outcomes, the individuals feeling of expertise, any contextual factors (e.g., residual sadness or elation) which affect the individuals emotional reactions, and a variety of individual dispositional factors. Another tenet of our model is that feelings, and hence preferences, should depend on the salience of the missing information – the information gap. Salience is, in turn, likely to depend on situational factors, decision framing, and the existence of counterfactuals that highlight the information gap. We have shown that these effects can make sense of a variety of already established empirical effects, and also provided experimental evidence in support of a key, previously untested, prediction. Many other predictions will, we hope, be tested in future empirical research.

Appendix

Proof of Proposition 1

Linearity of v_X implies that $u_X(\pi_X[b]) = u_X(\pi_X[\neg b])$. However, because bet b spreads out the utilities that would result from discovering either heads or tails, it increases γ_1, which implies that $w_1[b] > w_1[\neg b]$. By the one-sided sure-thing principle, we know that $v_1(\pi_1) < 0$ (regardless of whether the bet is taken) because the belief about the coin flip is not degenerate (i.e., because it is uncertain). Accepting the bet would increase attention weight on a negative belief and would thus lower utility, so $\neg b \succ b$. ∎

Proof of Proposition 2

Denote by $\mathcal{Q}^E$ the set of all questions believed to be jointly dependent with question Q_i. Actions a_i and a_j determine subjective probability measures $\pi[a_i]$ and $\pi[a_j]$ and attention weight vectors $\mathbf{w}[a_i]$ and $\mathbf{w}[a_j]$ such that:

1. $\pi_{\mathcal{A}}[a_i](\cdot) = \pi_{\mathcal{A}}[a_j](\cdot)$;
2. $\pi_X[a_i](\cdot) = \pi_X[a_j](\cdot)$;
3. $w_i[a_i] = w_j[a_j]$ and $w_j[a_i] = w_i[a_j]$;
4. for any ν such that $Q_\nu \in \mathcal{Q}^E$, we have $w_\nu[a_i] \geq w_\nu[a_j]$ with strict inequality for $\nu = \tilde{\imath}$;
5. for any $\nu \neq j$ such that $Q_\nu \in \mathcal{Q} \backslash \mathcal{Q}^E$, we have $w_\nu[a_i] = w_\nu[a_j]$.

The first condition holds because instrumental actions determine prizes, but not beliefs. The second condition holds because the actions are materially equivalent. Condition 3 follows from the assumption that Q_i and Q_j have the same salience together with the observations that the same material importance is given to each question when the corresponding action is taken (because the questions have the same subjective probabilities and the actions attach the same prizes) and that neither question is important when the other action is taken. The crucially important fourth condition applies because only question Q_i has dependence on $\mathcal{Q}^E \backslash \{Q_i, Q_j\}$, so only action a_i can increase the importance of these other questions.

Lastly, condition 5 holds because questions outside of $\mathcal{Q}^E$ are independent of both Q_i and Q_j.

Because questions Q_i and Q_j have the same subjective probabilities (following from material equivalence) as well as the same (neutral) valences for all possible answers, it can be shown (using the assumptions of label independence and linearity with respect to attention weights) that the utility cost of an increase in attention weight on one is equal to the utility cost of the same increase in attention weight on the other.

Any uncertain belief about a question in $\mathcal{Q}^E$ must be a negative belief because certainty would be a neutral belief and the one-sided sure-thing principle applies. Thus, by the assumption of monotonicity with respect to attention weights, the increase in attention weight on question $Q_{\tilde{\imath}}$ that occurs for action a_i (according to condition 4) causes a decrease in utility. $\blacksquare$

Proof of Proposition 3

As in Proposition 2, bet a_i attached to question Q_i makes question $Q_{\tilde{\imath}}$ more important and thus increases the attention weight on a negative belief. By the assumption that attention weight exhibits increasing differences in salience and importance, the decrease in utility due to this effect is worse in cognitive state $(\pi, \hat{\mathbf{w}})$ when question $Q_{\tilde{\imath}}$ is more salient than in cognitive state $(\pi, \mathbf{w})$. $\blacksquare$

Proof of Proposition 4

By our construction, utility exhibits increasing differences in the value of a belief and the attention weight on it. For sufficiently high v, even an uncertain belief will be a positive belief. In this case, increasing the attention weight on it increases utility, so the bet a_i becomes favored relative to a_j. $\blacksquare$

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