Demand Estimation with Social Interactions and the Implications for Targeted Marketing*

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Abstract

The role that social interactions play in the purchase decisions of consumers is of growing interest to marketers. Consumers’ decisions not only depend on information they receive from others, but also on the decisions made by other members of their social group. However, the extension of choice models to incorporate a peer’s choice forces the modeler to confront a variety of empirical challenges in separating out correlated behavior within a group from actual causal social interactions within a group. Furthermore, even if causal social interactions exist, estimation of the effect of a peer’s decision on a consumer is complicated by the fact that peers often coordinate decisions. This paper defines an empirical equilibrium model with a flexible heterogeneity structure to confront these challenges of modeling demand from groups of consumers. To validate the model and explore implications for marketing mix decisions, we apply it to a data set of golfers who frequently play together. We find that more than 50 percent of the demand from the consumers is attributable to the social interaction. In addition, we find that marketing to one consumer actually increases the returns to marketing to a peer, such that the firm will not focus all of its marketing effort on a single group member unless there is a strong asymmetry between the individuals within a group.

Keywords: decision-making, interdependent preferences, consumption, discrete choice, social interactions, targeted marketing, customer relationships.

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1 Introduction

There are many contexts where individuals make decisions in groups. For example, individuals may coordinate to eat together, watch a movie together, or choose the same cellular phone company to take advantage of in-network calling discounts. In these examples, coordination requires customers to weigh both their personal preferences and their partners’ to find a choice with which all individuals are comfortable. In other words, they are seeking an equilibrium solution to a coordination game.

This paper incorporates a coordination game directly into an econometric model to estimate demand from groups of customers as well as analyze the marketing implications of these social interactions. This falls within the broad class of empirical models of discrete games defined by Bresnahan and Reiss (1991). The structural model assists the estimation of causal effects and enables the analysis of counterfactual scenarios in which a firm may wish to alter the marketing mix variables to all, or some subset, of customers. With regard to estimation, the game explicitly defines the endogenous relationship between partners’ choices and uses this relationship to form the likelihood. After estimation, the game allows us to quantify the demand complementarities between

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3 See Chintagunta et.al. (2006) and related commentaries for a general discussion of how structural models in marketing facilitate both estimation and counterfactual analyses.
individuals and analyze policies that may target either specific groups or specific individuals within groups.

It is important to contrast our estimation approach with others in the marketing and economics literature. Marketing academics have used a statistical approach to estimate correlations in parameters and unobservables across participants with interrelated decisions. For instance, Yang and Allenby (2003) uses a spatial autoregressive mixture model and data augmentation to allow for a flexible set of correlations in a Bayesian MCMC approach. While this approach is very effective in fitting data and characterizing the joint distribution of agents’ choices, it is not well suited to extracting the causal effect of one agent’s decision on another’s. The primary reason is that estimated correlations may be driven by either causal effects or confounding factors that may also lead to correlations in individuals’ choices.

The economics literature has focused specifically on the identification of causal effects in the presence of other confounding factors. Manski (1993) and Moffitt (2001) describe three primary factors that hamper identification of causal peer effects. One confound is endogenous group formation. This arises when individuals that form a group share common characteristics that create a correlation in choices. Correlated unobservables confound identification of causal effects between agents by generating related choices even when two agents do not affect one another. Simultaneity bias creates an endogeneity between partners’ decisions that exaggerates estimated effects. It arises

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4 Most studies of joint decision-making in marketing focus on spouses providing input into a single decision, rather than coordinating their respective decisions. These are typically conjoint studies that first survey individuals separately, then resurvey them jointly (e.g. Krishnamurthi (1988), Arora and Allenby (1999) and Aribarg et.al. (2002))
because a given individual, A, may choose an option because his partner, B, is also choosing that option. However, B’s choice is also influenced by A’s, so econometrically, an unobservable in A’s choice will not be independent of B’s affect on A. Statistical approaches in marketing have been designed to directly address the simultaneity problem, but they have been unable to separate the sources of correlations into endogenously determined group characteristics, correlated unobservables and causal effects. This literature in economics argues for the use of exclusion restrictions between individuals’ structural outcome equations to non-parametrically identify the causal peer effect. Within marketing, Nair, Manchanda and Bhatia (2006) provide a nice description of these issues and show how reduced-form estimation of a linear model with fixed effects and exclusion restrictions can be used to uncover causal effects in these contexts.5

Our estimation approach addresses these issues as follows. First, the simultaneity issue is addressed through the inclusion of the simultaneous move coordination game in the econometric model. This allows us to remove the bias arising when the endogeneity of a partner’s decision is ignored. Second, we account for endogenous group formation biases by estimating individual-level heterogeneity in preference parameters that are allowed to be correlated across individuals. Finally, we argue that scanner panel data can be used to explicitly separate a joint decision from one that may appear to be joint because of correlated time varying unobservables.

5 Yang et.al. (2006) also use an exclusion restriction together with the statistical approach of Yang and Allenby (2003), but they do not describe how it overcomes these confounds. In addition, the excluded variable from a wife’s demand for a television show is the fact that she is not male. There are two potential problems with this. First, sex is a binary variable, so including her sex in her demand equation is actually including the husband’s variable. Second, demographics describe heterogeneity and are not exogenous policy variables that enter the structural equation.
After estimation, we illustrate how the estimated parameters of the coordination game can be used. The estimated coefficients and their marginal effects can be used to determine the magnitude of the effect of one individual’s choice on another’s, but the model allows this complementarity to be quantified as a percent of overall demand. This can assist a marketer in assessing the importance of demand complementarities. If these complementarities are in fact important, a different set of marketing policies may be necessary. For example, an advertisement can be expected to have a direct effect on an individual that sees it, but if the individual cares about his partner’s decision, it also has an indirect effect if the partner sees it. Furthermore, even if the partner does not see the ad, there is also a multiplier effect arising because the ad increases his utility, which in turn increases his partner’s utility, which in turn increases his utility even more. The equilibrium model allows the total effect to be quantified.

The equilibrium model also allows us to assess the effectiveness of targeting policies. Rossi, McCulloch and Allenby (1996) illustrate that targeting policies can be very effective, but in the context of groups, it is unclear whether a firm should target specific groups, or specific individuals within groups. If demand complementarities exist and individuals are homogenous within groups, we illustrate that a firm would be best off to target all individuals within a selected set of groups. However, if there are asymmetries within groups, it may only be optimal to target a subset of the members of the group.

We follow the hierarchical Bayesian MCMC approach of Rossi, McCulloch and Allenby (1996) to enable targeting, but the group choice context requires us to devise a unique approach for estimating the covariance matrix of the heterogeneity distribution. Off-diagonal elements of the heterogeneity matrix are essential for properly accounting for
endogenous group formation characteristics, but restrictions between off-diagonal elements are also necessary. We follow Barnard, McCulloch and Meng (2000) by decomposing the covariance matrix into a correlation matrix and matrix of standard deviations, but are unable to directly apply their approach. The reason is that restrictions in the covariance matrix imply that its determinant is not a quadratic function of the unknown correlation in the “one at a time” Gibbs sampler. We therefore show how their Griddy Gibbs approach can be adapted to accommodate restrictions causing the determinant to be a higher order polynomial.

To validate our model and illustrate how our model can be used to explore unique features of marketing to groups of customers, we apply it to a data set of pairs of golfers. We focus on pairs, because this is the most common group size in which individuals coordinate decisions to golf, see a movie or eat a meal. Coordination in larger groups occurs but is less common, more difficult to manage and can require different modeling assumptions. In this application, we find that the equilibrium model does in fact resolve a significant simultaneity bias.

Next, we use the individual-level parameter estimates to illustrate the targeting implications. For some pairs, targeting one partner greatly increases the incentive to target the other, while for other pairs the effect is small. In an exercise where we allow the firm to allocate a scarce demand shifter to half the customers in the data, we show that it is optimal to allocate it to both partners in 41 percent of the pairs, and that for

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6 We deal with pairs in which it is reasonable to assume that they know each others preferences or can easily communicate them to one another. When groups grow larger, decisions are more likely to be made based on expected choices of partners, rather than coordinated decisions of partners. In these cases, the models defined by Brock and Durlauf (2001 and 2003) are better suited.
another 19 percent of pairs, it is optimal to only allocate the scare demand shifter to one partner. This approach of allocating a scarce number of potential demand increases across customers is common in service industries such as restaurants, hotels or golf courses. Employees often face tradeoffs in which increasing service to one customer comes at the cost of another customer. Airlines also frequently face the problem of allocating a scarce number of upgrades. Should priorities rest with one particular partner from each group or should firms make only some top percentile of groups a priority and treat both members of these groups equally? Our analysis suggests this is an empirical question.

To summarize, our paper makes the following contributions to the empirical analysis of marketing to groups of customers. We specify an equilibrium model that allows us to overcome an important simultaneity bias that arises when trying to estimate the effect of partners’ decisions. We also show that our model allows measurement of demand complementarities within groups and enables the evaluation of counterfactual marketing policies that may or may not be targeted. We describe how scanner panel data can be used to identify literally joint decisions as opposed to decisions which appear joint because of correlated unobservables. Finally, we devise a method for imposing correlations in the covariance of the heterogeneity distribution in Bayesian MCMC models. This is important for how we control for endogenously formed group characteristics, but can be applied generally to any research where the heterogeneity distribution in a Bayesian MCMC model is restricted.

The paper proceeds as follows. In the next section we specify an equilibrium model for consumers that coordinate decisions. In section 3, we define the econometric
specification of the model and discuss identification of our model and compare it to other approaches in the literature. Section 4 describes our empirical application and details of the data. Section 5 presents model estimates and their implications. Section 6 conducts counterfactual analyses evaluating the prospects of targeting when individuals make joint decisions. Section 7 concludes.

2 An Equilibrium Model of Joint Decision-Making

This section specifies a random utility model for an individual’s choice when the choice is affected by whether or not a partner also makes the same choice. For illustration purposes, we restrict our analysis to the case of two individuals in the group; however the model can be extended to larger groups.

The utility that individual $A$ in group $g$ receives at time $t$, conditional on preferences $\gamma_{Ag}$, is:

$$u(y_{Ag}, y_{Bg}, \epsilon_{Ag}; \gamma_{Ag}) = \begin{cases} v_0 + \epsilon_{0,Ag} & \text{if } y_{Ag} = 0 \\ v_{1,Ag} (y_{Bg}, \gamma_{Ag}) + \epsilon_{1,Ag} & \text{if } y_{Ag} = 1 \end{cases}$$

(1)

where $v_{1,Ag}$ represents the non-stochastic portion of the indirect utility from choosing a firm’s good or service. $v_0$ is the utility of the outside good, which is normalized to 0 . $\epsilon_{Ag}$ is an extreme value distributed individual- and time-specific shock to preferences. We assume this is independent across individuals and time. $B$ is $A$ ’s partner, and $y_{Ag}$ is an indicator equal to one, when individual $i \in \{A, B\}$ chooses the firm’s good or service.
\( y_{Agt} \) is chosen to maximize \( u(y_{Agt}, y_{Bgt}, \varepsilon_{Agt}; \gamma_{Agt}) \). From this point forward, we will suppress the subscripts \( g \) and \( t \).

The endogeneity of the partner’s choice can be recognized by noting that \( y_B \) is also a function of \( y_A \). This implies that \( \varepsilon_A \) is correlated with \( y_B \).

One assumption we will impose in the model is that partners do in fact prefer to consume together. If this were not the case, we would not expect them to be partners unless the group formation was truly exogenous to consumption. We therefore assume:

\[
\begin{align*}
\{ v_{iA} (y_B = 1; \gamma_A) & \geq v_{iA} (y_B = 0; \gamma_A) \} \\
\text{and} \quad (2)
\{ v_{iB} (y_A = 1; \gamma_B) & \geq v_{iB} (y_A = 0; \gamma_B) \}
\end{align*}
\]

### 2.1 Joint Decision-Making

Given the interdependencies in partners’ choices, we model an equilibrium joint decision. We assume actions are simultaneous and that the game involves complete information.\(^7\)

Specifically, partners know each others preferences, \( \gamma \), and the realizations of the preferences shocks, \( \varepsilon \). Depending on the information, there are four possible outcomes in this simple two-by-two game: \( (y_A, y_B) \in \{(1,1), (1,0), (0,1), (0,0)\} \).

We begin by defining, then graphically depicting, the equilibrium conditions for each of these outcomes:

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\(^7\) We also assume consumers are myopic. Identifying social interactions models of forward-looking agents with complete information is complicated by the fact that there is not an analytical expression for the integral (expectation) over future realizations of unobservables.
where $v_{1,y,t}$ abbreviates $v_{1,y} (y_B; y_A)$. Figure 1 graphically depicts the inequalities that form these equilibria over the space defined by $\zeta_A = \varepsilon_{1,A} - \varepsilon_{0,A}$ on the horizontal axis and $\zeta_B = \varepsilon_{1,B} - \varepsilon_{0,B}$ on the vertical axis. Equilibrium (1,1) corresponds to regions II, III, V, and VI. Equilibria (1,0) and (0,1) correspond to regions IX and I respectively. Equilibrium (0,0) corresponds to regions IV, V, VII, and VIII.

Notice that region V arises in both (1,1) and (0,0). This implies that under the specific set of $\varepsilon$s defined by region V, there are multiple equilibria. This creates two problems in estimation. First, each equilibrium involves a distinct likelihood function. Without a way to select between these equilibria, we cannot estimate the model structurally. Furthermore, there is not a unique reduced-form of the model, so purely statistical estimation will not allow ex-post recovery of structural parameters. Second, even if we were able recover the model parameters, we would be unable to make counterfactual predictions if we did not know which equilibrium would be selected.

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8 This graphical approach follows that introduced by Bresnahan and Reiss (1991) and has been used in the entry literature to illustrate equilibria and the potential for multiple equilibria. In this case, the complementarities between the agents imply that the region of multiple equilibria involves neither or both consuming, whereas the competition between firms implies that the region of multiple equilibria involves one entrant.
The horizontal and vertical axes in the figure define the net shock to preferences for consuming, $\zeta_i = \varepsilon_i - \varepsilon_{-i}$, for an individual $A$ and $B$ respectively. The dashed lines define indifference between consuming and not when an individual’s partner is not consuming. The solid lines define indifference between consuming and not when an individual’s partner is consuming. Regions II, V, and VI represent coordinated consumption in that at least one partner is consuming only because the other is consuming. In region V, it is also possible that neither consume, but this is Pareto dominated by both consuming. Region III defines cases in which partners coincidentally consume together. Regions IV, VII, and VIII correspond to neither consuming. Regions I and IX correspond to only one partner consuming.

Equilibrium selection is therefore required. In this paper, we assume the players will always select the Pareto dominant equilibria. The inequalities in Equation (2), and the inequalities defining region $V$ in Figure 1 imply that $v_{11A} + \varepsilon_{1A} > \varepsilon_{0A}$ and $v_{11B} + \varepsilon_{1B} > \varepsilon_{0B}$. This implies that when the $\varepsilon$s fall in region $V$, both players will always be better off if equilibrium $(1,1)$ is selected. For the remainder of this paper, we will therefore assume that when this problem of multiple equilibria arises, the Pareto dominant equilibria is always selected. Such equilibrium selection rules are common. For example, Berry
(1992) uses an assumption that the equilibrium maximizing total surplus is chosen in an entry game. The condition of Pareto dominance is even more plausible because it requires that all agents are better off.

3 Empirical Model and Identification

3.1 Data Likelihood

By restricting ourselves to the case of Pareto dominant pure strategy equilibria, we can structurally estimate the equilibrium model defined above. Figure 1 depicts the two-dimensional space of the preference shocks, \( \{(e_{1A} - e_{0A}), (e_{1B} - e_{0B})\} \), which must be integrated over to form the likelihood function. The likelihoods for the four possible outcomes, \((y_A, y_B) \in \{(0,0), (1,0), (0,1), (1,1)\}\), can be expressed as function of inequalities as follows:

\[
\begin{align*}
\Pr(1,1) &= \Pr(v_{11A} + e_{1A} > e_{01A}) \Pr(v_{11B} + e_{1B} > e_{01B}) \\
\Pr(1,0) &= \Pr(v_{10A} + e_{1A} > e_{01A}) \Pr(v_{11B} + e_{1B} < e_{01B}) \\
\Pr(0,1) &= \Pr(v_{11A} + e_{1A} < e_{01A}) \Pr(v_{10B} + e_{1B} > e_{01B}) \\
\Pr(0,0) &= 1 - \Pr(1,1) - \Pr(1,0) - \Pr(0,1)
\end{align*}
\]

Notice that the primary difference between these probabilities and the equilibrium conditions defined above in Equation (3) is that the probability of outcome \((0,0)\) involves integration over a smaller space because this outcome is Pareto dominated in region \(V\). The likelihood function for a given group, \(g\), over the \(T_g\) periods that we observe it is:
\[
L_r \left( y_{Ag}, y_{Bg}; \gamma_g \right) = \prod_{i=1}^{T_r} \left[ \sum_{A=0}^{1} \sum_{B=0}^{1} \Pr \left( A, B; \gamma_g \right) \left\{ y_{Ag} = A \left\{ y_{Bg} = B \right\} \right. \right] \tag{5}
\]

If agents are homogenous, there will only be two parameters to estimate in the simple model without covariates: \( \{ v_{11}, v_{10} \} \). As depicted in Equation (4) above, there are three separate means to identify these two variables. This implies that exogenous variables are not necessary to identify the model parameters. However, exogenous variables can easily be included in Equation (1), and, if there are exclusion restrictions between partners’ indirect utilities of consuming, these variables can provide non-parametric identification.

### 3.2 Heterogeneity Structure

In practice, consumers will differ in their utilities both across and within groups.

Understanding the distribution of this heterogeneity and whether or not individuals within a pair are similar or different can assist in specifying marketing policies and targeting consumers. We therefore define the indirect utilities of consuming alone and jointly to be drawn from a population distribution defined by:

\[
\begin{bmatrix}
  v_{10,A} \\
  v_{11,A} \\
  v_{10,B} \\
  v_{11,B}
\end{bmatrix}
\sim
h\left(\begin{bmatrix}
  \nu_{10} \\
  \nu_{11}
\end{bmatrix}, \Psi\right) \tag{6}
\]

s.t.

\[
\begin{align*}
  v_{11,A} &> v_{10,A} \\
  v_{11,B} &> v_{10,B}
\end{align*}
\]

where the \( \nu \)'s represent the means of these indirect utilities and \( \Psi \) is a covariance matrix.

The latter expression in (6) restates the restriction that individuals prefer to consume
jointly relative to consuming alone. The easiest way to impose this is to specify the
difference between these utilities, $\gamma_1 = v_{11} - v_{10}$, to be distributed with a strictly positive
support, (e.g. log-normal). The researcher can then estimate the distribution of $\gamma_1$ and the
distribution of a parameter describing either $v_{11}$ or $v_{10}$. Typically, one would estimate the
latter, but in this case it is better to estimate $v_{11}$ and infer $v_{10}$ from $v_{11}$ and $\gamma_1 = v_{11} - v_{10}$.

We therefore define the following heterogeneity distribution for individual-level
parameters:

$$
\begin{bmatrix}
v_{11.A} \\
\ln(v_{11.A} - v_{10.A}) \\
v_{11.B} \\
\ln(v_{11.B} - v_{10.B})
\end{bmatrix}
= 
\begin{bmatrix}
\gamma_{0.A} \\
\ln\gamma_{1,A} \\
\gamma_{0.B} \\
\ln\gamma_{1,B}
\end{bmatrix}
\sim N
\begin{bmatrix}
\gamma_0 \\
\tilde{\gamma}_1 \\
\gamma_0 \\
\tilde{\gamma}_1
\end{bmatrix},
\Sigma = 
\begin{bmatrix}
\sigma_0^2 & \sigma_{01} & \tilde{\sigma}_{00} & \tilde{\sigma}_{01} \\
\sigma_{01} & \sigma_{11} & \tilde{\sigma}_{01} & \tilde{\sigma}_{01} \\
\tilde{\sigma}_{00} & \tilde{\sigma}_{01} & \sigma_{01}^2 & \sigma_{11} \\
\tilde{\sigma}_{01} & \tilde{\sigma}_{01} & \sigma_{01} & \sigma_{11}^2
\end{bmatrix}
$$

(7)

With this distribution, we still only have two mean parameters to estimate, but we also
have a covariance matrix to estimate. However, notice that there are many restrictions in
the covariance matrix. Instead of the typical ten parameters in a four-by-four covariance
matrix, there are only six. This occurs because we have to assume both partners are
drawn from the same distribution (i.e. assignment as partner $A$ or $B$ is arbitrary). To put
these restrictions in the covariance matrix, we decompose it into a matrix of standard
deviations and a correlation matrix, such that $\Sigma = SRS$, where:

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9 Contact the corresponding author for estimates under the alternative assumption.
The standard deviations reflect the different types of agents observed in the data. They are primarily identified across pairs, but taking into account both partners’ parameters. One question that arises is whether we have enough restrictions to identify a four-by-four correlation matrix despite the fact that we still only have three moments at the group level. To see that the answer is yes, first consider the case where we restrict

\[ \gamma_{14} = \gamma_{18} = \gamma_1 \]  

(i.e. symmetric “partner effects”). Under this restriction, we are back to a three-by-three and can therefore clearly estimate the upper left correlation matrix consisting of parameters \( \rho_{01} \) and \( \tilde{\rho}_{00} \). \( \tilde{\rho}_{01} \) is not estimated because under this symmetric specification, \( \tilde{\rho}_{01} = \rho_{01} \). Extending to the full four-by-four correlation matrix \( R \) adds the parameter \( \tilde{\rho}_{11} \) and \( \tilde{\rho}_{01} \). \( \tilde{\rho}_{11} \) measures the correlation between the partners’ “partner effects.” In the symmetric “partner effect” case just mentioned, the implicit assumption is that \( \tilde{\rho}_{11} = 1 \). Allowing \( \tilde{\rho}_{11} \neq 1 \) therefore allows asymmetric “partner effects” in the data. The existence of only three moments does not allow us to reject the assumption of \( \tilde{\rho}_{11} = 1 \) at the group level, however, we can reject this assumption and estimate \( \tilde{\rho}_{11} \) and \( \tilde{\rho}_{01} \) at the population (i.e. cross-group) level. In a hierarchical Bayesian model, the identification of this parameter will rely heavily on the estimated priors of the individual-

\[
S = \begin{bmatrix}
\sigma_0 & 0 \\
\sigma_1 & \sigma_0 \\
0 & \sigma_1 \\
\end{bmatrix},
R = \begin{bmatrix}
1 & \tilde{\rho}_{01} & \tilde{\rho}_{01} \\
\rho_{01} & 1 & \tilde{\rho}_{01} \\
\tilde{\rho}_{00} & \tilde{\rho}_{01} & \rho_{01} \\
\tilde{\rho}_{01} & \tilde{\rho}_{11} & \rho_{01} \\
\end{bmatrix}
\]

(8)
level parameters. We illustrate this in our empirical application below. This can however be relaxed if there are explanatory variables that only affect one partner.

The heterogeneity structure defined in this section can be estimated using simulated maximum likelihood, or it could be used to specify priors on individual-level parameters in a hierarchical Bayesian model. We define the Bayesian model here, but actually use both in our empirical application below in section 5.

### 3.3 Hierarchical Bayesian MCMC Specification

To use Bayesian methods to estimate our model, we begin by specifying priors and describing the sampling approach for each parameter.

The heterogeneity distribution defined in Equation (7), will serve as the prior for the individual-level parameters. As is common for logit models, we use a random-walk Metropolis-Hastings (MH) algorithm to draw the individual-level parameters.

The priors over the mean population parameters are assumed to be independent and normal/log-normal:  

\[
\begin{bmatrix}
\gamma_0 \\
\ln \gamma_i
\end{bmatrix} \propto N\left( \begin{bmatrix} 0 \\
0 
\end{bmatrix}, \begin{bmatrix} 100 & 0 \\
0 & 100 
\end{bmatrix} \right).
\]

Because the size of this vector differs from that of the individual level parameters, \( \{\gamma_{0A}, \ln \gamma_{1A}, \gamma_{0B}, \ln \gamma_{1B}\} \), we also draw these parameters using MH.

The restrictions in the covariance matrix prevent us from using the conjugacy of the typical inverse-Wishart prior for \( \Sigma \). We therefore estimate the standard deviations, \( S \), and the correlation matrix, \( R \), following the separation strategy defined in Barnard, McCulloch and Meng (2000). We specify the prior over the standard deviations to be
independent and log-normal: 
\[
\begin{bmatrix}
\ln s_0 \\
\ln s_1
\end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \right).
\]

We draw these parameters using MH as well. We specify the prior over \( R \) to be joint uniform, \( p(R) \propto 1 \), then draw each correlation using a “one-at-a-time” Griddy Gibbs approach.

The restrictions in the correlation matrix prevent us from directly applying Barnard, McCulloch and Meng (2000). Specifically, we cannot derive the support of correlations \( \rho_{01} \) and \( \tilde{\rho}_{01} \) by their method because these parameters enter \( R \) twice in the lower-diagonal. This implies that \( |R(r)| \) will not be a quadratic function in \( r \) for \( r \in \{ \rho_{01}, \tilde{\rho}_{01} \} \).

Furthermore, checking \( |R_{4,4}(r)| > 0 \) is not sufficient to assure positive definiteness of \( R \).

Each sub-matrix \( R_{m,m}(r) \) for \( m \in \{1, \ldots, 4\} \) must satisfy the condition \( |R_{m,m}(r)| > 0 \).

However, the application of Griddy Gibbs provides a convenient way to determine the support. We begin by defining a support \( G \) with equally spaced lengths of \( GNS \) over the interval \([-1,1]\) for a given correlation \( r \). We then calculate \( |R(G)| \) element by element. Next, we recover a vector \( G_r \subseteq G \) such that \( |R(g)| > 0, \ \forall \ g \in G_r \). Given this support, we can directly apply Griddy Gibbs to obtain a draw from the conditional

\[
f(r \mid y, S, \rho_{j\neq r}, \gamma_0, \gamma_1),
\]

where \( \rho_{j\neq r} \) is a vector of all other correlations.

3.4 Comparison with Other Approaches to Identifying Social Interactions

Identifying the influence of one individual on another can be quite challenging. The estimation approach we defined above is designed to extract interdependent preferences from revealed preference data. We focus our discussion of identification on these
instances. However, it is important to note that if the researcher has access to potential customers to conduct a survey, interdependent preferences can also be extracted by surveying them independently, then jointly as in the case of Arora and Allenby (1999).

3.4.1 Linear Models

Perhaps the earliest work to recognize the econometric issues specific to social interactions is Manski (1993). The primary concern was the simultaneity of decisions. In linear models this is problematic because exclusion restrictions are required to identify the effects. Given appropriate restrictions, structural parameters can be recovered from estimates of the reduced-form of the model.

Perhaps the most closely related paper to the present uses this approach. Yang et.al. (2006) define a model of spousal television viewing to recover structural parameters from an estimated reduced form. Their approach differs from the typical linear model in that they use a spatial autoregressive mixture model and data augmentation, following Yang and Allenby (2003), to obtain posteriors on model parameters.

The primary difference between their approach and the model specified in this paper is that their dependent variable aggregates the consumption of each of the agents, rather than considering the discrete decision to consume separately, jointly, or not at all. This prevents them from separating joint decision-making from interdependent preferences. This is an important distinction because if individuals influence each other’s preferences but do not care about each other’s choices, there will not be indirect effects of marketing variables. For instance, if an advertisement changes a customer A’s preferences, A may then influence B’s preferences. However, if B also sees the ad, it will not have an
indirect effect on A and will not be influenced by the fact that A saw the ad. Furthermore, the multiplier that additionally increases A’s demand because A’s increased desire to purchase increased B’s is not existent in purely interdependent preferences without a causal effect of partners’ decisions.

There are two other econometric issues that can arise when identifying social interactions, whether the model is linear or non-linear: endogenous group formation and correlated unobservables. Endogenous group formation arises when individuals share common characteristics that lead them to be grouped together in the first place. In the model above, this would be a common preference parameter in the partners’ $\gamma$s, or a correlation between $\gamma_A$ and $\gamma_B$. By specifying a model that applies to panel data, the random coefficients specification allows these correlations to be identified, and hence separated from the causal effects. This follows the approach taken in Bhatia, Manchanda, and Nair (2006) where fixed effects account for the endogenous group formation.

Observation of the time of consumption in the panel data also assists identification by distinguishing truly joint decisions from apparently joint decisions that arise due to correlations in time varying unobservables. In some cases, the data may report observations jointly. In other cases, the precise timing of purchases allows correlated unobservables to be ruled out because there would have to be an unreasonably high degree of correlation for individuals to repeatedly make purchases within minutes or seconds of one another. In the golf application we study in section 5 below, unobservables would have to be correlated within the short span of time between which two golfers swipe membership cards for their round of golf. Essentially, we could allow
unobservables to be correlated at the month, day and even hour levels and still be able to assure the decisions really are joint consumption occasions.

The typical identification approach in linear models has also been applied to a number of empirical models of network effects in which the dependent variable is nonlinear, and the social interaction variable is linear. For instance, Tucker (2006) considers individual’s adoption of a video conferencing technology as a function of the installed base of other workers and managers with the technology. Similarly, Nam, Manchanda and Chintagunta (2006) evaluate the adoption of an on-demand satellite movie technology as a function of the number of neighbors that have adopted. Both these papers use exclusion restrictions to instrument for the linear variable summarizing other agents’ decisions. In other cases, researchers have used the decisions of agents in the same geographic region to summarize potential network effects (e.g. Manchanda, Xie and Youn (2004) and Bell and Song (2004))

3.4.2 Non-linear Models

When estimating a non-linear model, exclusion restrictions are not required for non-parametric identification. However, estimation is complicated by the multiple equilibria problems we describe above.

Yang and Allenby (2003) developed the Bayesian spatial autoregressive approach that enabled the flexible estimation of the reduced-form in Yang et.al. (2006). Unlike Yang et.al. (2006), the model in Yang and Allenby (2003) is designed for discrete choice applications and is applied to the decision to buy Japanese or non-Japanese cars. In their case, they do not recover the structural parameters, but use the spatial autoregressive
process to pick up correlations arising from each of four possible sources: causal effects, simultaneity, endogenous group formation, and correlated unobservables. They consequently identify groups of agents (e.g. geographically and demographically defined networks) making similar decisions. Knowledge of this joint distribution of agents’ decisions can be useful in many marketing applications and the methods have already been extended and implemented elsewhere. However, as described in the introduction, the respective contributions of each of these factors to the correlations cannot be identified.

Brock and Durlauf (2003 and 2001) consider structural estimation of discrete choice models, but do so under an assumption that an individual’s choice is affected by his or her belief about others. They acknowledge that this assumption is “clearly problematic in describing interactions between a pair of best friends,” where it is likely that they know each others preferences, unobservables and decisions. In marketing, we are likely to be concerned both with interactions in larger communities (as has been the focus of Brock and Durlauf’s work) and in small decision-making units such as households or groups of friends or associates. For these latter cases, an assumption of complete information (as made in the model in this paper), is more appropriate.

Bajari, Hong and Ryan (2004) propose an estimator that applies to peer effects models when agents have complete information. Their estimator requires that multiple equilibria are observed and that there exists a variable which determines the equilibria, but does not affect the payoffs of the agents. The equilibrium selection equation in their model adjoins the estimation of agent payoffs in much the same way that Heckman (1979)’s
selection equation adjoins the estimation of an outcome equation. The authors operationalize the approach for a simple website entry game.

While Bajari, Hong and Ryan (2004) is a significant advance in the estimation of these types of games, it is not well suited to the model in this paper. Specifically, if agents are willing to always choose the Pareto dominant equilibria when multiple equilibria may arise, there will never be variation in the equilibria chosen to identify the equilibrium selection equation.

4 Empirical Application

To validate our model, explore how model parameters are derived from data, and evaluate potential targeting strategies, we apply our model to a sample of pairs of golfers. Golf is certainly an activity where many individuals prefer to play with a friend or colleague, as opposed to alone or with an unknown partner. Furthermore, it is a useful example because many pairs will not be spouses, so it is less likely that partners will be maximizing an identical or related utility function under the same constraints.\(^\text{10}\)

4.1 Group Definition

Our model and estimation approach is designed for groups that are known ex-ante. We define the groups ex-ante based on the precise time of golf revealed by them swiping a scanner card. We define golfers to be paired with another golfer if they swiped at least four times within 2 minutes of one another between January 1, 2000 and December 31, 2000. The empirical example of golf has also been analyzed in a dynamic decision context by Hartmann (2006). As noted previously, it is intractable to include forward-looking behavior in this model. We therefore ignore the dynamic aspects of the data to explore identification of the model presented in this paper.
2001. Over 80 percent of the groups found using this procedure were groups of two, so we decided to focus only on these groups. This left us with 86 mutually exclusive pairs, which we observe over time periods ranging from 59 to 730 days. Note that this group is defined as a pair of customers that like to purchase together and therefore coordinate their decisions. When at the course, they may in fact become part of a foursome, but the matching of pairs to create foursomes is a decision by the course, not the golfers themselves. Our focus is on the customers’ coordination efforts, so we therefore ignore the highly random process by which pairs are grouped together by the course to become foursomes.

For proof of concept, we only use the outcome, or dependent variables, in estimation. Once we have estimated pair-specific parameters of the model, we display the data to provide intuition for how the parameters are determined. Additional explanatory variables complicate this intuition and are not crucial to illustration of how the model works.

Table 1 presents summary statistics for the pairs. We observe the pairs an average of 474 days. Of the 40,791 pair and day combinations we observe, 5.8 percent of these observations involved at least one partner golfing. Within these 2,353 times a partner golfed, approximately 40 percent were instances of joint consumption, while 60 percent of the time a golfer was playing without his partner. This willingness to consume alone, as well as with their partner, is what will allow identification of the preference for joint consumption.

---

11 We also tried using 3, 4 and 5 minute intervals for defining the groups and found little difference.
5 Model Estimates and Implications

To validate our model and illustrate its advantages, we apply it to the golf data and use both Bayesian MCMC estimation and maximum likelihood. We estimate the full model using Bayesian MCMC to obtain individual-level parameters that illustrate the importance of our model for targeting decisions. We also use maximum likelihood to validate that priors are not driving our estimates and to analyze the data without our model.

5.1 Bayesian Estimation

Table 2 reports the estimates of the MCMC implementation of the model. Table 2 only describes the posteriors on population parameters, as there are too many individual-level parameters to report. The table is divided into two sections: symmetric and asymmetric partner effects. As described in section 3.2, the symmetric version of the model is cleanly identified at the individual-level, however the version that allows asymmetric

---

### Table 1
Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>At Least One Consumes</td>
<td>40,791</td>
<td>0.058</td>
<td>0.233</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Both Consume</td>
<td>2,353</td>
<td>0.404</td>
<td>0.491</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Only One Consumes</td>
<td>2,353</td>
<td>0.596</td>
<td>0.491</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Number of Time Periods</td>
<td>86</td>
<td>474</td>
<td>196</td>
<td>59</td>
<td>730</td>
</tr>
</tbody>
</table>
partner effects relies more heavily on the prior (or population distribution of parameters) for identification.

From the means of the population parameters, we can see that individuals get a lower utility from golfing with their partner (i.e. Joint Consumption Utility) than from not golfing at all. This is expected because individuals are frequently observed to not golf. The parameter describing the “partner effect” suggests that individuals do get a substantial disutility from their partner not joining them when golfing. Examination of the 5th and 95th percentiles of these posteriors illustrate that these parameters are in fact significantly different from zero. The standard deviations of the population parameters illustrate that there is significant heterogeneity in both the Joint Consumption Utility and Partner Effect.

The mean parameters are similar across both the symmetric and asymmetric cases, but the interesting factors lie in the correlation matrices. Figure 2 illustrates box plots extending between the 1st, 5th, 95th and 99th percentiles of the posterior marginal distributions of each of the correlation coefficients. In the symmetric model, there is not a significant correlation between a given partner’s Joint Consumption Utility and Partner Effect, but there is a significantly negative correlation between the Joint Consumption Utilities of the two partners. This implies that more avid golfers tend to pair up with less avid golfers.

In the version of the model that allows for asymmetric partner effects, none of the correlation coefficients is significantly different from zero. Furthermore, the estimated distributions are much wider than those for the model with symmetry imposed for the
partner effects. As described above, this is because the present data is not sufficient to non-parametrically identify partner effects at the individual level. What does this mean? Whether symmetric or asymmetric partner effects are allowed, the average partner effects and joint consumption utilities can be identified. This is evident in the nearly identical mean parameters. However, if a researcher wishes to examine asymmetric partner effects, the estimates will be highly dependent on the priors. To relate this back to the exclusion restrictions argued for by Manski (1993) and Moffit (2001), while the equilibrium model can resolve simultaneity to estimate average peer effects, the exclusion restrictions allow for asymmetric peer effects to be identified in the heterogeneity distribution.
# Table 2
## Model Estimates Using MCMC

### Posteriors of Population Parameters

<table>
<thead>
<tr>
<th>Symmetric Partner Effects</th>
<th>Mean</th>
<th>5th pctl</th>
<th>95th pctl</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Means</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint Consumption Utility</td>
<td>$\gamma_0$</td>
<td>-1.75</td>
<td>-1.82</td>
</tr>
<tr>
<td>ln(Partner Effect)</td>
<td>$\ln \gamma_1$</td>
<td>1.00</td>
<td>0.93</td>
</tr>
<tr>
<td><strong>Standard Deviations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint Consumption Utility</td>
<td>$\gamma_0$</td>
<td>0.81</td>
<td>0.72</td>
</tr>
<tr>
<td>ln(Partner Effect)</td>
<td>$\ln \gamma_1$</td>
<td>0.38</td>
<td>0.33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation Matrix</th>
<th>Joint Consumption Utility</th>
<th>ln(Partner Effect)</th>
<th>Joint Consumption Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{0,\ell}$</td>
<td>1.00</td>
<td>-0.06</td>
<td>-0.47</td>
</tr>
<tr>
<td>$\ln \gamma_{1,\ell}$</td>
<td>-0.06</td>
<td>1.00</td>
<td>-0.06</td>
</tr>
<tr>
<td>$\gamma_{0,\ell B}$</td>
<td>-0.47</td>
<td>-0.06</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Asymmetric Partner Effects</th>
<th>Mean</th>
<th>5th pctl</th>
<th>95th pctl</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Means</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint Consumption Utility</td>
<td>$\gamma_0$</td>
<td>-1.76</td>
<td>-1.84</td>
</tr>
<tr>
<td>ln(Partner Effect)</td>
<td>$\ln \gamma_1$</td>
<td>0.96</td>
<td>0.88</td>
</tr>
<tr>
<td><strong>Standard Deviations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint Consumption Utility</td>
<td>$\gamma_0$</td>
<td>0.64</td>
<td>0.50</td>
</tr>
<tr>
<td>ln(Partner Effect)</td>
<td>$\ln \gamma_1$</td>
<td>0.52</td>
<td>0.44</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation Matrix</th>
<th>Joint Consumption Utility</th>
<th>ln(Partner Effect)</th>
<th>Joint Consumption Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{0,\ell}$</td>
<td>1.00</td>
<td>0.02</td>
<td>-0.18</td>
</tr>
<tr>
<td>$\ln \gamma_{1,\ell}$</td>
<td>0.02</td>
<td>1.00</td>
<td>-0.16</td>
</tr>
<tr>
<td>$\gamma_{0,\ell B}$</td>
<td>-0.18</td>
<td>-0.16</td>
<td>1.00</td>
</tr>
<tr>
<td>$\ln \gamma_{1,\ell B}$</td>
<td>-0.16</td>
<td>0.18</td>
<td>0.02</td>
</tr>
</tbody>
</table>
5.2 Maximum Likelihood Estimation

We use maximum likelihood estimation to compare our model to other specifications to demonstrate the model’s ability to account for biases arising from simultaneity and endogenous group formation. We also use maximum likelihood to test for asymmetries in the partner effect and verify that the priors in the MCMC specification are not driving the estimates.

The effects of accounting for simultaneity bias can be observed by comparing Table 3 estimates in the column labeled “Homogenous Equilibrium” with the estimates in the column labeled “Homogenous Simple Logit.” The “Homogenous Equilibrium” estimates the model and likelihood defined in Equation (5), without the heterogeneity specification defined in Equation (7). The “Simple Logit Model” assumes that a partner’s choice is
exogenous, so the likelihood is just the standard logit probabilities that are independent across partners and groups. We see from the comparison that accounting for the simultaneity with the equilibrium model greatly reduces the utility of consuming together (“Joint Consumption Utility”) from 0.303 to -1.714. We expect this type of correction because the “Simple Logit Model” does not account for the fact that when Partner A decides to golf because B is also golfing, that B is golfing because Partner A had a strong intention to golf anyway. This spurious correlation overstates their desire to golf with one another. It is important to point out that the likelihoods of the first two specifications in Table 3 cannot be compared with the others because they estimate a model with a single binary dependent variable and an explanatory variable as opposed to our model which has four possible outcomes and no explanatory variables.

Table 3

<table>
<thead>
<tr>
<th>Model Estimates</th>
<th>Homogenous Simple Logit</th>
<th>Heterogeneous Simple Logit</th>
<th>Homogenous Equilibrium</th>
<th>Heterogeneous Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Means</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint Consumption Utility $\gamma_s$</td>
<td>0.303</td>
<td>0.763</td>
<td>-1.714</td>
<td>-1.664</td>
</tr>
<tr>
<td>ln(Partner Effect) $\ln\gamma_l$</td>
<td>1.460</td>
<td>1.678</td>
<td>0.771</td>
<td>0.968</td>
</tr>
<tr>
<td>Standard Deviations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint Consumption Utility $\gamma_s$</td>
<td>0.763</td>
<td>0.829</td>
<td>0.817</td>
<td>0.817</td>
</tr>
<tr>
<td>ln(Partner Effect) $\ln\gamma_l$</td>
<td>1.678</td>
<td>0.385</td>
<td>0.536</td>
<td>0.536</td>
</tr>
<tr>
<td>Correlation Matrices</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint Consumption Utility $\gamma_{ss}$</td>
<td>$\rho_{01}$</td>
<td>$\rho_{01}$</td>
<td>$\rho_{01}$</td>
<td>$\rho_{01}$</td>
</tr>
<tr>
<td>ln(Partner Effect) $\ln\gamma_{ls}$</td>
<td>1.00</td>
<td>-0.09</td>
<td>-0.29</td>
<td>1.00</td>
</tr>
<tr>
<td>Joint Consumption Utility $\gamma_{as}$</td>
<td>$\rho_{01}$</td>
<td>$\rho_{01}$</td>
<td>$\rho_{01}$</td>
<td>$\rho_{01}$</td>
</tr>
<tr>
<td>ln(Partner Effect) $\ln\gamma_{la}$</td>
<td>-0.09</td>
<td>1.00</td>
<td>-0.09</td>
<td>1.00</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>-9285</td>
<td>-8,186</td>
<td>-11,556</td>
<td>-10,539.33</td>
</tr>
</tbody>
</table>

Joint Consumption is the utility of consuming with a partner. ln(Partner Effect) is the log-normal distribution of the additional utility of consuming with a partner relative to consuming alone. The heading labeled mean reports the mean of the heterogeneity distribution. The standard errors for the means of the distribution are reported in parentheses. The Standard Deviations and Correlation Matrices are the decomposed Variance-Covariance matrix for the heterogeneity distribution over these parameters.
The other confound we need to overcome arises from the fact that partnerships are likely to be formed by individuals that have similar preferences. We account for this by incorporating the heterogeneity structure defined in Equation (7). The last two columns replicate the Bayesian analysis above. In the symmetric partner effect model, the estimated correlation coefficients are very similar to the Bayesian estimates, and are significant. Once again the negative correlation between Joint Consumption Utilities implies that more avid golfer tend to group with less avid golfers. Because the golfers pair up with less similar golfers, the endogenous group characteristics unaccounted for in the Homogenous Equilibrium Model bias the Joint Consumption Utilities downward.

The model allowing for asymmetric partner effects is similar to the Bayesian version above as well. The correlations here are also not significantly different from zero (they are not reported in the table). However, it is useful to point out that this model nests the symmetric partner effect model and has a better likelihood. In fact a likelihood ratio test rejects the assumption of symmetric partner effects at a confidence of 97 percent, even though the correlation matrix is not as cleanly identified.

### 5.3 Assessing Consumption Complementarities

To quantify the implications of these estimates, it is useful to evaluate the predicted probabilities of various outcomes and measure the percent of demand that is attributable to preferences for joint consumption. Table 4 illustrates that just about 93 percent of the time, neither partner consumes. 3.9 percent of the time only one consumes. As expected, coincidental joint consumption occurs rarely at 0.1 percent of the time. Coordinated consumption can occur by one partner choosing to consume because he knows the other
will consume (regions II and IV in Figure 1) or by both partners consuming when they would not have otherwise (region V in Figure 1). The former occurs 0.7 percent of the time and the latter occurs 2.5 percent of the time. While these numbers may appear small, demand from a given pair that is attributable to joint consumption is more than 56% of expected demand on average. There is also substantial heterogeneity in this multiplier effect across pairs as the standard deviation of 0.23 suggests. Within-group demand complementarities are therefore an important source of demand in this industry.

Table 4
Demand From Joint Consumption

<table>
<thead>
<tr>
<th>Demand Attributable to Joint Consumption</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob Neither Consumes</td>
<td>0.928</td>
<td>0.059</td>
</tr>
<tr>
<td>Prob Only One Consumes</td>
<td>0.039</td>
<td>0.040</td>
</tr>
<tr>
<td>Prob. Both Coincidentally Consume Togther</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>Prob One Consumes when would Not have Otherwise</td>
<td>0.007</td>
<td>0.010</td>
</tr>
<tr>
<td>Prob Both Consume when would Not have Otherwise</td>
<td>0.025</td>
<td>0.024</td>
</tr>
<tr>
<td>Demand Attributable to Joint Consumption</td>
<td>0.057</td>
<td>0.053</td>
</tr>
</tbody>
</table>

The table reports the mean probabilities of various outcomes. The last two rows in bold refer to expected demand measures, which differ from probabilities as specified in Equation (10).

6 Application to Targeting Customer Groups

Our final analysis is to evaluate targeted marketing strategies. There is a growing literature considering how to target individual customers, but no counterfactuals
evaluating how to target marketing policies when individuals make joint decisions. We first use estimates from the asymmetric model above to demonstrate the role of asymmetries on the marginal effects of shifting golfers’ utilities. Then we demonstrate the role of asymmetries in targeting by evaluating how the firm will optimally allocate a scarce number of demand shifters.

### 6.1 Asymmetric Responses to Marketing Variables

In Table 5, we list the partners with the greatest within-pair differences in their reactions to changes in marketing variables. We define the reactions by partner A using the following derivative:

\[
\frac{dE[q_g]}{dx} = \frac{d\gamma_{0A}}{dx} \left\{ 2 \times Pr_{A=0, B=1} \ Pr_{A=1, B=1} + Pr_{A=0, B=0} \ Pr_{A=1, B=0} - Pr_{A=1, B=1} - Pr_{A=0, B=1} \right\}^{12}
\]

where

\[
E[q_g] = 2 \times Pr_{A=1, B=1} + 1 \times \left\{ Pr_{A=1, B=0} + Pr_{A=0, B=1} \right\}
\]

In all cases, the partner with the greatest derivative has a higher utility of golfing with his partner (i.e. the joint consumption utility reported in the estimates above), a lower incremental utility for golfing with his partner as reflected in the column labeled “Partner Effect” (i.e. the partner effect reported in the estimates above), and a higher utility of golfing alone (i.e. the difference between the parameters estimated above).

---

12 We do not have an x variable, so we cannot measure \( \frac{d\gamma_{0A}}{dx} \). This is actually better for our purposes so we can illustrate the general trends without confusing the implications based on differences between customers \( \frac{d\gamma_{0A}}{dx} \)’s. When we report derivative, we therefore report the expression in brackets.
### Table 5

**Marketing Effects for Asymmetric Partners**

<table>
<thead>
<tr>
<th>Pair Number</th>
<th>Partner A</th>
<th>Partner B</th>
<th>Derivatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partner A</td>
<td>Alone</td>
<td>With Partner</td>
<td>Partner Effect</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>-1.73</td>
<td>-0.58</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>-2.35</td>
<td>-1.07</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>-2.58</td>
<td>-1.66</td>
</tr>
<tr>
<td>4</td>
<td>53</td>
<td>-2.56</td>
<td>-1.16</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>-4.73</td>
<td>-2.10</td>
</tr>
<tr>
<td>6</td>
<td>29</td>
<td>-4.29</td>
<td>-1.78</td>
</tr>
<tr>
<td>7</td>
<td>23</td>
<td>-4.76</td>
<td>-2.77</td>
</tr>
<tr>
<td>8</td>
<td>73</td>
<td>-4.80</td>
<td>-2.26</td>
</tr>
</tbody>
</table>

The first column in the table reports the row number of the table. The second column reports a pair identifier. Columns titled "Alone" report the expected value of the individual's indirect utility of consuming alone. Columns titled "With Partner" report the expected value of the individual's indirect utility of consuming with a partner. Column titled "Partner Effect" report the difference in these parameters, which is the incremental indirect utility of consuming with a partner. Columns under the heading Derivative report either the derivatives derived using Equation (8) or the difference between these derivatives for the partners.

One way to summarize the above information is to note that individuals with a greater utility for golfing have a greater responsiveness to marketing activities generally. We explore this more by asking whether increasing a partner’s likelihood of golfing, which should increase the individual’s likelihood of golfing, also increases their responsiveness to marketing variables. We explore this by providing a small increase in the intercept of each player’s partner (a 0.1 increase) and evaluating how that impacts their responsiveness to marketing. We find that if a partner receives an intercept increase of 0.1, it increases their responsiveness to a 0.1 intercept increase by 6 percent on average. The minimum increase is 1.2 percent and the maximum increase is 9%. Targeting one partner therefore implies that the returns to targeting the other are increased.

We now consider scenarios where it may be optimal to target both partners in a pair, scenarios where it is optimal to target one partner, and others where it is not advisable to
target either partner in the pair. The fact that targeting one partner increases the returns to targeting another will be instrumental in this analysis.

To do this, we suppose that the firm can allocate the intercept increase of 0.1 to each of 86 of the 172 customers observed in the data. We allocate it to those customers with the greatest resulting increase in expected demand. If customers within groups were identical, the fact that targeting one increases the returns to targeting the other will imply that the firm should target both partners in the 43 most profitable groups. However, if asymmetries exist (in either their intercepts or their partner effects as estimated above), it may be optimal to only target one customer in some groups.

To analyze targeting when asymmetries are allowed, we proceed customer by customer, and allocate the intercept to the most dominant partner of a given pair first. That customer’s partner may also receive the increase if the incremental value of giving it to him is greater than giving it to the more dominant partner of a pair that has not yet received an intercept increase. We will proceed until all 86 have been allocated.

The result of the above allocation of intercept increases is that both partners were given an increase in 35 of the 86 pairs. In another 16 pairs, only one partner was given an increase, leaving 35 pairs in which neither partner received a demand increase.

As stated in the introduction, allocating scarce demand shifters is fairly common in service industries. As an example, take the well-known case of a casino with a customer database providing comps. Should the casino allocate their scarce comps to both a husband and wife, or should it target a particular spouse and provide comps to a greater number of couples? Our preceding analysis suggests there are not obvious rules of thumb
and that it really is an empirical question of whether both, one, or neither partner in a pair should be targeted.

7 Conclusions

This paper develops an estimable equilibrium model of demand for groups of customers. We add to the growing literature on interdependent preferences by resolving a significant simultaneity bias when agents have complete information. We find a multiple equilibria problem in this model but show that a unique Pareto dominant equilibrium exists. Our model defines complementarities between partners’ decisions and our application reveals that these customer-to-customer relationships are an important source of demand. In addition, the model illustrates how targeting incentives change when customers’ purchase decisions depend on a partner’s decision. The model reveals that targeting marketing activities to a single member of a group can greatly increase the returns to targeting other group members in some cases, but not others. The model therefore also serves as an input to a firm’s decision of whether to target all members in a group or focus on a single dominant member.
References


