Direct Marketing on a Social Network

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Abstract

Direct Marketing (DM) - sending promotional messages to individual customers - is increasingly used by marketers as a result of the explosive growth of customer databases. Most current methods to calculate optimal budgets for such DM campaigns consider customers in isolation and ignore word-of-mouth communication (WOM). When the customer base forms a network (as is the case in telecom or social network databases) ignoring WOM clearly leads to suboptimal DM budgets. This paper develops a model to help address this challenge. We assume that firms know the network structure formed by customers but do not know (or are not allowed to use) data on individuals' connections. Under this scenario, we compare the optimal campaign of a monopolist to that of firms competing in simultaneous-move or sequential-move games. The analysis provides two key insights: (i) we show that ignoring the existence of WOM leads to significant profit loss for firms and this is more so under competition. In particular, knowing the "density" of consumer connections is crucial for the design of optimal campaigns. (ii) Competition in DM exhibits strong first-mover advantages and, even in a simultaneous-move game between identical firms, highly asymmetric outcomes are possible. Specifically, at low levels of competition, the game only has symmetric equilibria, while at high levels of competition, one of the firms tends to saturate most of the network. The paper also explores three extensions. First, we generalize our model of social influence and show that our main qualitative insights remain valid. This remains to be the case for a model where DM activity endogenously grows the customer base. Finally, we also examine a somewhat simplified model in which firms know individuals' network positions. In this case, we find that, while targeting is more efficient, competing firms may be worse off than under "blind" targeting as they have strong incentives to target the same customers.

Keywords: word-of-mouth, game theory, competition, CRM.

1 Introduction

Direct marketing (DM) is increasingly used by firms as customer databases become more available. Internet commerce and services assisted by computer databases (e.g. telecom subscriptions) allow marketers to identify individual consumers and provide customized offers to them (Chen and Iyer 2002). As a result, customer relationship management (CRM) has grown to be a dynamic field in marketing (Villanueva and Hanssens 2007).

Current DM methods however, generally ignore word-of-mouth (WOM) communication between customers (Villanueva, Yoo, and Hanssens 2008). Although marketers have always known that WOM has a very powerful influence on customer decisions (Bass 1969, Herr, Kardes, and Kim 1991, Van den Bulte and Joshi 2007), until recently, they did not have much information about the individual communication patterns of consumers. As a result, while aggregate models have always considered the effect of WOM (see e.g. the famous Bass-model), marketing tools built for DM techniques have generally ignored the structure of customer communication networks and were designed to treat individual customers as independent consumption units.

Increasingly, however, modern customer databases allow the reconstruction of consumer communication patterns. For instance, social networking is one of the revolutionary new applications of the Internet where members not only reveal certain aspects of their preferences (e.g. their taste for particular movies and music) but also reveal their friends. The largest such sites, MySpace and Facebook have over 200 million members and have become powerful advertising media. A dozen other similar sites have also emerged and managed to post exorbitant valuations reaching $billion figures. Successful social networking sites exist in many other countries, often attracting a high proportion of the national population.[1] Social networking is but one context where consumers' connection patterns can be identified. Telecom data provide similar possibilities. Each

[1] Cyworld gathers approximately 25% of the South Korean population among its members and iWiW has about the same rate of penetration in Hungary.

call (or a critical number of calls) between consumers can be considered to represent a connection between them and the so-defined graph is a reasonable proxy for the communication network of the consumers. Similarly, e-mail services and blogging sites can all reconstruct their members' communication paths.

The availability of consumer network data has not gone unnoticed by marketers. A small industry is being born to exploit large datasets on consumer connections[2]. Firms such as Xtract and Idiro promise their customers to do social network analysis on large datasets to reveal the network structure of their membership base and help optimize marketing campaigns. For legal reasons, often, this can only be done on the aggregate. For example, in many countries, telephone operators are allowed to analyze their customers' calling patterns but are not allowed to exploit information related to individuals' calling partners. Also, advertisers who send text messages to potential consumers may buy a *telephone number range* to target their messages to, essentially only deciding about the number of subscriber identifiers in the range. Finally, in many cases, advertisers buy a fixed number of banner placements to be shown to users of a social network portal, potentially agreeing in a maximum number of banner displays per network member. Nevertheless, under some conditions (e.g. under customer opt-in) social network analysis can reveal customer groups who form tight sub-networks. Similarly, marketers can identify influential members of their customer networks. The goal is to increase the efficiency of marketing campaigns by targeting members who are likely to spread product information further in the customer base.

These opportunities are particularly interesting for social network sites and other Web2.0 media who struggle to monetize their membership networks with third-party advertisers in order to justify their hefty valuations. It is also becoming a central issue for telecom companies who explore the potential of their subscriber network for advertising. How could direct marketers use information on the network if they wanted to take advantage of WOM across consumers? In

[2]A detailed description of recent developments in "network marketing" can be found in "What's a friend worth?" *Business Week*, June 1, 2009, pp32.

particular, how would the DM campaign change as a function of the structure of the network? How would the presence of competition change the behavior of direct marketers? Our goal is to provide insights with respect to these questions both for a monopolist marketer and, more importantly, under competition.

To address this issue, we build an analytic model in which firms target a population of consumers with direct messages. Demand is conditional on consumers being informed about firms' offerings. Information may be received from firms but can also spread via WOM communication on the exogenous connection patterns across consumers. On the supply side, we vary firms' knowledge about the network connections, the level of competition between them and the cost of sending a DM message. We are interested in the effect of these market characteristics on the optimal targeting strategies of firms.

We are particularly interested in the effect of the network structure on firms' behavior. For the case of *partial network knowledge* (i.e. when individual connections cannot be exploited) we show a monotone relationship between the "density" of connections in the network and the optimal DM strategies of firms. We argue that the notion of "density" (defined precisely in the sequel) is a useful and robust measure to characterize the relevant "network structure" for DM on aggregate network data. In an extension, we also consider the complex case of *full network knowledge*, where individual connections can be exploited by the direct marketer. Under this scenario, we argue that it is essential to understand the "community structure" within the network, that is, identify clusters of consumers that are tightly connected between themselves, while being relatively isolated from members of other communities. Here, we find that the distribution of communities has a dramatic impact on firms' strategies: the more communities consumers are organized into, the more the network provides ground for asymmetric outcomes.

Another set of important results relate to the nature of competition between direct marketers. Interestingly, under partial network knowledge by firms, we find that intense competition between identical direct marketers may lead to highly asymmetric equilibria even in a simultaneous-move game and this is more so the more intensely firms compete with one another. In turn, this feature of the competitive interaction leads to strong first-mover advantages in a sequential game. For the case of full network knowledge, we find that competition may end up being fiercer than in the case of partial network knowledge. As both firms tend to target highly connected consumers, they tend to coordinate on the same individuals and end up increasing competition compared to the "blind" targeting case. Similarly to the "blind" marketing case, under pure strategies, increased competition tends to result in more asymmetric equilibria.

The rest of the paper is organized as follows. First, we briefly summarize the related literature. Next, we present the base model. Section 4 describes the analytic results for the case of partial network knowledge. In Section 5, we explore the case when firms may also use individuals' location in the network. Section 5 also looks at more complex models of WOM by considering directed networks, individual influence weights, indirect social influence and endogenous network growth. We end the paper with concluding remarks and directions for future research. To ease exposition, all proofs have been relegated to an Appendix.

2 Related literature

Our work builds on three broad literature streams: (i) the new product diffusion literature, (ii) the direct marketing or CRM literature and, (iii) the network data mining literature. Table 1 summarizes how our work differs from these.

New product diffusion models in marketing (see, e.g. Bass (1969)) have considered WOM communication as a fundamental driver of the product adoption process. The basic assumption that WOM has a strong impact on consumers' adoptions has been confirmed by many studies on consumer behavior (Borgida and Nisbett 1977, Grewal, Cline, and Davies 2003, Herr et al. 1991) and the effects of WOM have also been explored recently in the context of the Web (see, e.g.

Stream	References	Method	Model of WOM	Competition	Targeting
Diffusion of Innovations	Bass (1969)	Empirical	Aggregate	No	Random
	Goldenberg, Libai, and Muller (2001)	Analytical	Grid	No	Random
	Van den Bulte and Lilien (2003)	Empirical	Network	No	Mixed
CRM / Direct Marketing	Chen and Iyer (2002)	Analytical	No WOM	Yes	Deterministic
	Villanueva and Hanssens (2007)	Empirical	Aggregate	No	Random
Network Data Mining	Richardson and Domingos (2002)	Mixed	Network	No	Random and deterministic
	Kempe, Kleinberg, and Tardos (2003)	Mixed	Network	No	Random and deterministic
	This study	Analytical	Network	Yes	Random and deterministic

Table 1: Summary of Related Literature Streams

Godes and Mayzlin (2004)). However, WOM has traditionally been treated only at the aggregate level, often reduced to a single parameter. Only recently have diffusion researchers started to explore the role of underlying connections between consumers: Goldenberg et al. (2001) simulate the diffusion process on a grid while Van den Bulte and Lilien (2003) study adoption on a real network of consumers. Our work follows along these lines by assuming WOM communication on a consumer network. However, our focus is the direct marketers' problem of designing an optimal communication campaign. More importantly, we also study competition in an analytic framework.

Exploiting the availability of customer databases, the DM and CRM literatures have focused on understanding how best target (acquire or retain) the firm's (potential) customers. So called, "consumer addressability" (Blattberg and Deighton 1991, Chen and Iyer 2002), has boosted research in CRM in recent years (Kumar and Reinartz 2006, Villanueva and Hanssens 2007). This literature, however, typically neglected WOM communication across customers - largely due to the lack of information on customer connections. While Villanueva et al. (2008) successfully incorporate the estimation of (aggregate) WOM effects into the CRM framework, they still ignore the underlying consumer network. The present paper extends the DM and CRM literatures by adding the explicit consideration of communication between consumers (WOM) along an exogenous network of connections. In this respect, this paper builds on the theoretical foundations of informative WOM presented in Goldenberg et al. (2001).

Our work is closest in spirit to the research in the network data mining area of computer science (Gruhl, Liben-Nowell, Guha, and Tomkins 2004, Hill, Provost, and Volinsky 2006, Kempe et al. 2003, Richardson and Domingos 2002). As our paper, this research aims at solving the firm's DM problem while explicitly taking into account the underlying consumer network structure. However, just as the CRM and diffusion literatures do, this stream also ignores competition. We intend to close this gap with the objective of generating strategic insights for competing direct marketers. Furthermore, we consider several scenarios depending on how granular firms' information about the social network is.

3 Model

We assume that there are 2 symmetric firms promoting their product or service to n consumers who form a social network, denoted $G(V,E)$. In our notation, V represents the set of nodes in the network and E is the set of undirected relationships. Thus, we assume that a connection between two consumers allows the communication between them in both directions.[3] Consumers differ in their connection patterns but otherwise are assumed to be identical. Word-of-mouth (WOM) on the network is assumed to be purely informative and is modeled as follows: if consumer $v \in V$ is targeted with a message, both v and all the neighbors of v in G will be affected by the message. For instance, in the network of Figure 1, if a message reaches Node 1, then Nodes 1, 2, and 3 will all be aware of the product advertised by the message.

The above representation of WOM seems limited as its reach only extends to a single neighbor (i.e. there are no indirect effects of social influence). However, previous work on social networks (Kempe et al. 2003) demonstrates that many WOM influence models can be reduced to

[3] When E contains only undirected relationships, we also refer to $G(V,E)$ as *undirected*.

the present representation. In Section 5, we relax both of the above assumptions concerning WOM and look at directed graphs while generalizing to the Cascade Model of Goldenberg et al. (2001).

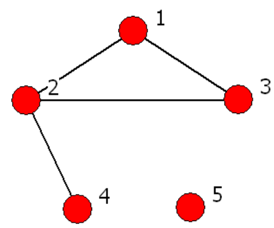

Figure 1: A simple network of five consumers.

On the supply-side, we assume that firms are risk neutral and use direct marketing (DM) to reach consumers. The cost of sending a DM message is $c < 1$ for both firms. Throughout the analysis presented in the next section, we assume that all the consumers who are in the market V can be directly targeted by the firms. (In an extension, we discuss the scenario when this is not the case and the consumer network may grow as a result of the advertising campaign.)

Without loss of generality, assume that the cost of production for firms is 0. Further, let consumers' preferences for the two competing brands be distributed the following way. With respect to their products marketed, half of the consumers prefer Firm 1 over Firm 2 while the other half prefer Firm 2 over Firm 1. Any consumer in the market has w_h valuation for her favorite and $w_l \leq w_h$ for her least preferred brand. After product information spreads in the market (through DM and WOM), the prices offered to a particular consumer emerge as a result of the two- or three-party Nash bargaining (see e.g. Iyer and Villas-Boas (2003)) involving the customer and the firms that have reached him/her. Consumers who are informed about only one of the products divide the surplus equally with the corresponding manufacturer, paying half of their respective valuations for the product, $w_h/2$ or $w_l/2$. However, when both firms reach a consumer, the prices are first competed down to the Bertrand equilibrium, and the surplus is then shared equally between the consumer and her more preferred manufacturer. The preferred firm thus makes sales at the price of $(w_h - w_l)/2$. Considering that any consumer prefers Firm 1 with probability 1/2, under this setup, when the DM message of Firm i reaches consumer v, Firm i's expected net surplus from this consumer is:

$$r_i(v) = \begin{cases} (w_h + w_l)/4 & \text{if } v \text{ has not been reached by Firm } -i, \\ (w_h - w_l)/4 & \text{if } v \text{ has been reached by both firms,} \end{cases}$$

where $i = 1, 2$ and $-i$ denotes the competing firm.

Let $w_h + w_l = 4$ and denote the amount $(w_h - w_l)/4$ by p. Then Firm i's expected net surplus from consumer v simplifies to

$$r_i(v) = \begin{cases} 1 & \text{if } v \text{ has not been reached by Firm } -i, \\ p & \text{if } v \text{ has been reached by both firms.} \end{cases}$$

It is interesting to take a closer look at this expression. Note that, from a given consumer, the quantity $0 \leq 1 - p \leq 1$ describes the (in expectation) foregone surplus of a firm when its competitor also reaches the consumer. Thus, $1 - p$ can be directly associated to the intensity of competition. When $w_h = w_l$, the firms lack advantage in any segment of the market, ending up in a perfect price competition. This scenario is characterized by $1 - p = 1$. On the contrary, when $w_l = 0$, the two halves of the market are only interested in their preferred brand. Firms can thus ignore their competitor and price (to "their" respective segments) as monopolists. In this case, $1 - p = 0$. Finally, as the relative advantage of a firm in "its" market segment, $w_h - w_l$ decreases, the quantity $1 - p = 2w_l/(w_h + w_l)$ increases, indicating more intense competition.

We start by assuming that the structure of the network is common knowledge across firms but firms do not know where individuals are positioned in the network. Clearly, the best that firms can do under such conditions is randomly selecting the targets of their advertising messages. In an extension, we also analyze the unlikely case when individual network connections are identified and exploited by firms. Further, throughout this paper we assume that firms are unaware of the identity of consumers targeted by their competitor.

In the random-targeting scenario, the decision variable of each firm is thus the number of customers to target with DM. (In the scenario when individual network connections are identified and exploited by firms, the number of messages *as well as* their target constitute firms' strategy profiles.) Given the chosen DM intesity, k, the targeted k consumers are selected uniformly at random and independently from the competitor. Under competition, we require that firms' choices of k be best responses to one another. Further, to eliminate degenerate equilibria, we only consider Pareto optimal outcomes.

3.1 Network Structure and Random Targeting

A central issue we explore is how the network structure influences the optimal DM campaign by firms in a competitive setting. It is therefore crucial to characterize the construct of "network structure" in a useful way. To do so, consider first a monopolist direct marketer. Let v be a node in the network and let $d(v)$ denote its *degree*, the number of other consumers who may influence the purchase decision of v if reached by the advertising firm. Let k be the action chosen by the monopolist, i.e., the number of advertising messages sent. The node v is reached if and only if either it is a target of an advertising message or at least one of its neighbors are targeted. Since there are $d(v)$ such neighbors, the probability that neither of these events happens is exactly

$$\Pr_{|S|=k}[v \text{ is not reached}] = \frac{\binom{n-\mathrm{d}(v)-1}{k}}{\binom{n}{k}},$$

where $S \subseteq V$ is the set of targeted consumers, chosen uniformly at random. For the network displayed in Figure 1, Table 2 shows the probabilities of reaching different nodes for all possible intensities of the DM campaign conducted by the monopolist. Even without a detailed analysis, it is clear from the table that the impact of DM is different based on the connectivity (degree) of the nodes in the network. For instance, at any campaign intensity, the most connected node in the network, Node 2, has the highest chance to be reached and the opposite is true for Node 5, the isolated node.

Number of DM messages	Chance of reaching				
(k)	**Node 1**	**Node 2**	**Node 3**	**Node 4**	**Node 5**
0	0%	0%	0%	0%	0%
1	60%	80 %	60%	40%	20%
2	90%	100%	90%	70%	40%
3	100%	100%	100%	90%	60%
4	100%	100%	100%	100%	80%
5	100%	100%	100%	100%	100%

Table 2: Probabilities corresponding to reaching the nodes in the network in Figure 1 via random targeting by the monopolist.

In case v is reached, the surplus from v is exactly 1. Thus, the expected surplus from v is

$$\underset{|S|=k}{\mathrm{E}}[r_1(v)] = 1 - \frac{\binom{n-\mathrm{d}(v)-1}{k}}{\binom{n}{k}}.$$

Under our simple model of social influence, the likelihood that a certain consumer is aware of a product, and hence the expected surplus from that consumer is earned, is determined by the level of advertising intensity and the *local* network connectivity of the consumer. As such, to characterize a social network, we can use its *degree distribution*, i.e., the frequency of node degrees in the entire network. However, this is not practical for either managerial, or comparative statics purposes. Therefore, to characterize the intensity of WOM, we use an alternative, 1-dimensional measure, the *density* of the network.

Definition. *For an undirected network $G(V,E)$, the* density *of the network is*

$$D(G) = \frac{2 \cdot |E|}{|V| \cdot (|V|-1)}.$$

The social networks that we look at can be directly associated to the intensity of WOM: the more influential relationships there are in a network, the higher is its density. We illustrate the importance of considering the density of the network on Figure 2. First, we compare the optimal

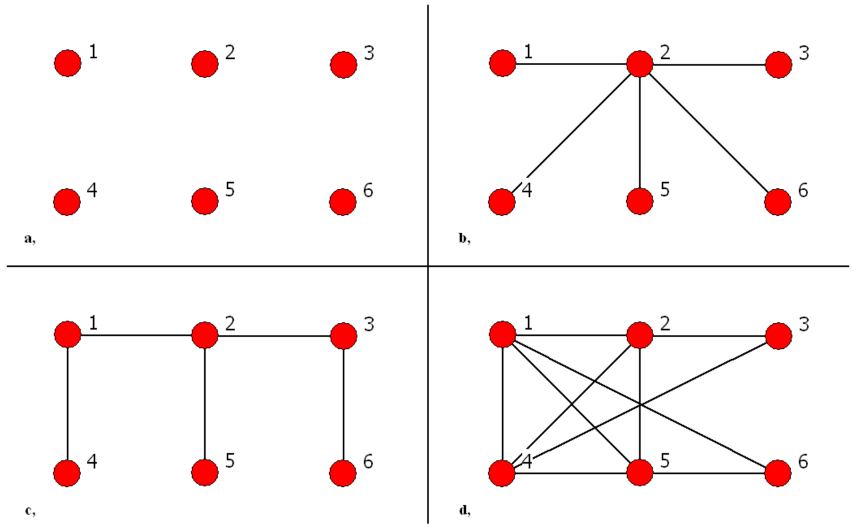

Figure 2: Four simple consumer networks of three different densities. The densities of Networks *a, b, c* and *d* are 0, 0.33, 0.33, 0.67, respectively.

strategies of a monopolist direct marketer over the four networks with densities of 0, 0.33, 0.33, 0.67, respectively. Assume that the cost of sending a message is $c = 0.3$. In Network *a*, there is no WOM, so the monopolist marketer is best off saturating the entire network by sending six messages (making 4.2 units of profit). In the other three networks, sending the last message does not lead to extra surplus since the last consumer to target is already reached by the firm via WOM. In Network *b*, the monopolist marketer sends five messages and in Network *c*, it sends four, making about the same profits (4.5 and 4.6 units, respectively) in the two cases. In Network *d*, the impact of WOM is very strong as the first message already reaches 4.33 consumers in expectation. The optimal strategy of the monopolist is accordingly lower - it is enough to send two messages to reach the optimal profits of 5 units.

Now let us look at a competitive set-up. To this end, consider the best response of Firm 2 when $p = 0.1$, $c = 0.3$ and when Firm 1 sends one DM message into the network at random. For Network *a*, the one message sent by Firm 1 reaches only one node, so Firm 2 does not need to worry about the drop in demand caused by its competitor - its best response is to saturate the entire network by sending six messages. For Networks b and c, the one message of Firm 1 reaches 2.67 nodes in expectation, decreasing the maximum achievable surplus for Firm 2 to $2.67 \cdot p + 3.33 = 3.60$. This drop in demand, along with WOM improving the efficiency of the DM campaign makes Firm 2 send only four messages in both cases. Finally, in Network d, the maximum demand is $4.33 \cdot p + 1.67 = 2.10$ and so the best response of Firm 2 is to send only two messages.[4]

In sum, we find that, under both monopoly and competitive scenarios, firms' optimal strategies essentially depend on the density of the underlying social network. In particular, firms' strategies (and payoffs) are roughly equal for Networks b and c, which are different but have identical density. Consequently, in our context, network density - as defined above - is a good qualitative descriptor of a network's structure.

4 Random targeting on the network

In this section we solve the above model, first considering a monopolist direct marketer, followed by the case of a duopoly.

4.1 Benchmark: Monopoly

Under partial network knowledge, the monopolist chooses a number k, selects k customers uniformly at random and sends a DM message to each of them.

Proposition 1. *For any network $G(V,E)$ and for any $c > 0$, the optimal number of messages to be sent by the monopolist is*

$$k^{M*}(c) = \arg\max_{k \in \mathbb{N}} \left\{ \left[\sum_{v \in V} 1 - \frac{\binom{n-\mathrm{d}(v)-1}{k}}{\binom{n}{k}} \right] - k \cdot c \right\}.$$

Furthermore, if $E \neq \emptyset$ then the maximum of the above profit function is either unique, or is taken at two neighboring values of k.

[4] For completeness, we provide the full payoff matrix and best response structure for all four networks in the Appendix, in Table 3.

Note that the case $E = \emptyset$ corresponds exactly to the case when there is no WOM, resulting in $d(v) = 0$ for each node v. It easily follows that in that case, the monopolist either saturates the entire network with DM messages (when $c < 1$) or completely abstains from the campaign (when $c > 1$).[5] As such, without WOM, in our model, direct marketing is similar to broadcast advertising. In reality, most DM efforts would exhibit decreasing returns leading to an optimal number of messages smaller than the size of the potential market. This can happen for a variety of reasons, the most common being a convex cost function. However, on the types of networks that we consider (i.e. when each customer is readily identified in the customer database) the cost of a DM message is constant. Therefore, our main point is that the existence of WOM creates decreasing returns for the DM campaign *over and above* other reasons that have been traditionally considered.[6] In this sense, Proposition 1 shows that marketing on a network is qualitatively different from traditional direct marketing.

Ignoring WOM has a severe impact on DM campaign efficiency. For an illustration, consider the network on Figure 3. In this network, we have 5 different types of consumers with respect to the number of other consumers who may influence their purchase decision. The most well-connected individuals are at the bottom of the figure, followed by the "cloud" of consumers in the top. The third-most-connected individuals are in between these two regions in the network. Finally on the two sides of the figure we can find consumers with only one or zero relationships.

Assuming $c = 0.3$ for the cost of sending a message, the optimal profit of the monopolist is reached at 129 messages (targeting only 43% of the consumers). Ignoring WOM, the optimal strategy is blanket advertising (sending 300 messages), which results in an expected 7.5% drop in profits.

Next, we turn to comparative statics. Independently from the network structure, each addi-

[5] For $c = 1$, the monopolist is indifferent among actions between 0 and n.

[6] We note that this property of the profit function also follows from Theorem 2.2 of Kempe et al. (2003). We refer the reader to the proof of Proposition 1 for details.

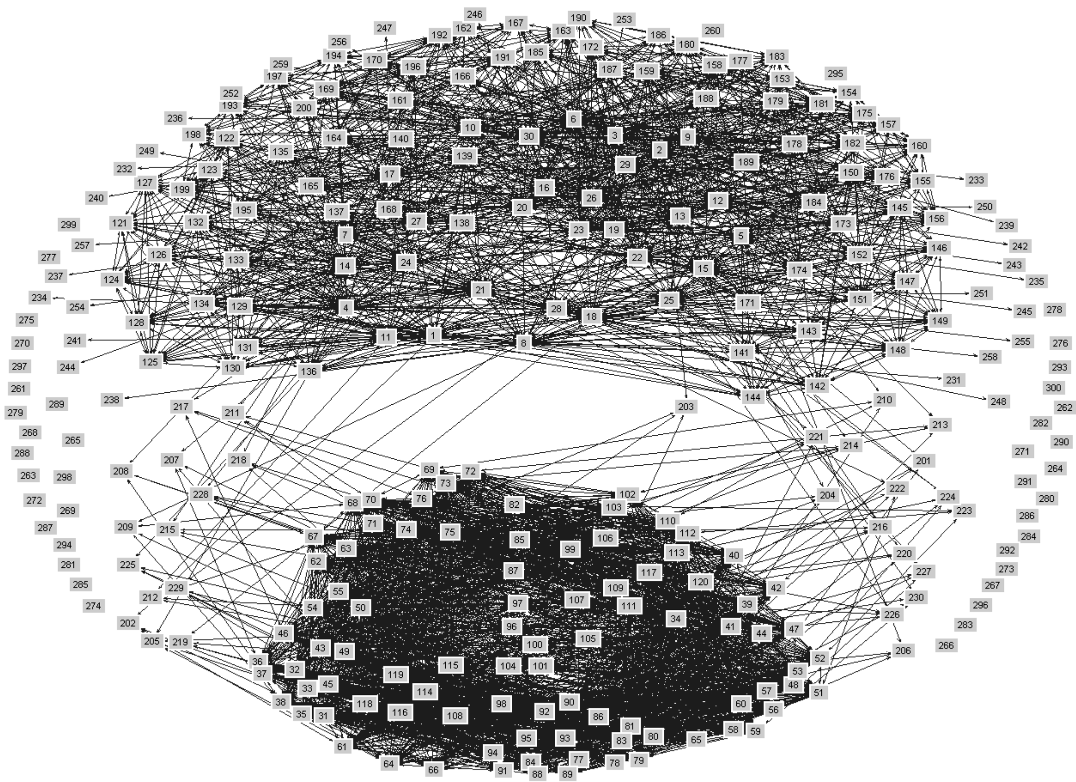

Figure 3: Example of a consumer network with 300 nodes.

tional message by the monopolist decreases the expected benefit of the next message because the probability that the target has been already reached through her neighbor via WOM is higher. On the other hand, the cost of each message is the same. At higher cost the monopolist will thus send fewer messages.

Corollary 1. *The optimal action for the monopolist, k^{M*} is (weakly) decreasing in c, and thus Π^{M*} is decreasing and (weakly) convex in c.*

To assess the impact of the network structure, Figure 4 displays the average level of optimal DM intensities for randomly chosen networks over 300 nodes as a function of their density. For each possible number of edges in the network, the optimal DM intensities were averaged over 50 randomly drawn networks. The results illustrate a trend similarly intuitive to those presented above: as density (the intensity of WOM) increases, the intensity (and thus the cost) of the optimal advertising campaign is reduced.

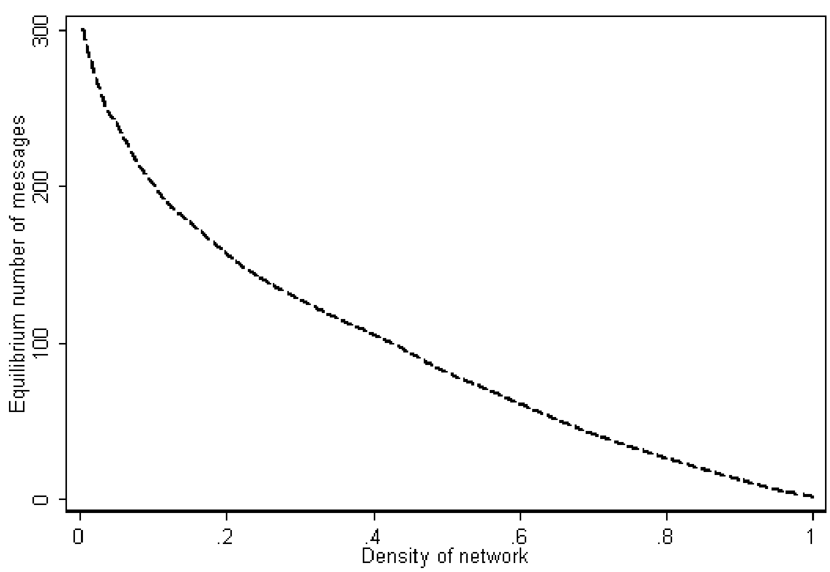

Figure 4: Optimal strategy of the monopolist on random networks when the cost of sending a DM message is 0.3. Results are averaged over 50 draws of networks.

4.2 Simultaneous-Move Duopoly

Similarly to the monopolist, under partial network knowledge, competing firms choose the number of messages to send, k_i and randomly target the network's members.

Proposition 2. *For any network $G(V,E)$, any $c > 0$ and any $0 \leq p \leq 1$, the game of random targeting under duopoly has at least one pure-strategy equilibrium (k_1^*, k_2^*). Furthermore, if*

$$\sum_{v \in V} \left[\frac{\mathrm{d}(v)+1}{n}\right] \cdot \left[p + (1-p) \cdot \frac{\binom{n-\mathrm{d}(v)-1}{k^{M*}(c)}}{\binom{n}{k^{M*}(c)}}\right] > c$$

then this equilibrium is not trivial in the sense that $k_1^*, k_2^* > 0$.

Corollary 2. *For any network $G(V,E)$ and any p^*, $0 < p^* < 1$ there exists $c^* > 0$ such that for any p, $p^* < p \leq 1$ and any c, $0 \leq c < c^*$, the game of random targeting has a unique symmetric equilibrium (where both firms send out the same number of advertising messages).*

Note that in the case when there is no WOM ($E = \emptyset$), the only Pareto optimal equilibria of the game are (n,n) when $p > c$ and $(n,0)$ or $(0,n)$ when $p \leq c$.[7] Thus, in our model, even in the competitive case, the lack of WOM essentially makes the firms decide between engaging into a mass marketing campaign and staying out. Similarly to the monopoly case, the chosen advertising strategies are strikingly different when the marketer takes into account the effects of WOM.

When WOM is present, asymmetric equilibria still may exist. However, as Corollary 2 shows, for any network and any intensity of competition, there is a cost range in which the unique equilibrium is symmetric. The intuition behind the co-existence of qualitatively different equilibria stems from the presence of two strategic effects and is particularly interesting. First, there is a "saturation effect" resulting from WOM: sending additional messages has decreasing returns for the sender. This effect is present even under monopoly and affects both competing firms the same

[7] For $p < c$, there may be a symmetric internal equilibrium, but that equilibrium is not Pareto optimal since both firms make zero profit, unlike in the mentioned asymmetric equilibria. For $p = c$, the same holds for equilibria of the form (n, k_2) and (k_1, n) for any $0 \leq k_1, k_2 \leq n$.

way. The other effect is a "substitution effect". When Firm i saturates the network, the *other* firm has a lower incentive to send messages than in the case when Firm i sends less messages. This is because the other firm is then likely to hit consumers who are also attained by its opponent, getting a surplus of only $p < 1$. This effect - if strong - can lead to asymmetric equilibria. The qualitative nature of the outcome depends on which effect is stronger.

To gauge the profit implications of WOM, we again look at the network displayed in Figure 3, when $p = 0.1$ and $c = 0.3$. We look at three scenarios: when the competitor chooses the action of the symmetric equilibrium, the highest action in the most asymmetric equilibrium and the lowest action in the most asymmetric equilibrium. Against these DM intensities, the optimal profits are reached at 61, 47 and 77 messages, respectively (never targeting more than 25.67% of the consumers). Compared to these optimal levels, blanket advertising (sending 300 messages) results in negative profits against the competitors playing at least the action of the symmetric equilibrium and an expected 89% drop in profits when competing with a firm taking the lowest action in the most asymmetric equilibrium. In sum, ignoring the network structure leads to sub-optimal decisions by the direct marketer and the corresponding losses are larger the more intense is the competition between the firms.

Let us now turn to comparative statics. With respect to the level of competition, $1 - p$, and the cost of sending a DM message, c, we can say the following.

Proposition 3. *(Villas-Boas 1997)*[8] *Let (k_1^*, k_2^*) be an equilibrium under the parameter combination (p, c). Let $(p', -c') > (p, -c)$ under the standard component-wise order and let (k_1', k_2') be any equilibrium under (p', c'). Then $(k_1', k_2') \not< (k_1^*, k_2^*)$.*

Proposition 3 says that, when either the competition becomes more intense or the cost of DM rises, then, in any equilibrium, either both of the firms send exactly as many DM messages as

[8]This is an application of Theorem 3 in the cited article. The sharpness of the two inequalities are reversed but essentially the same proof applies to our case.

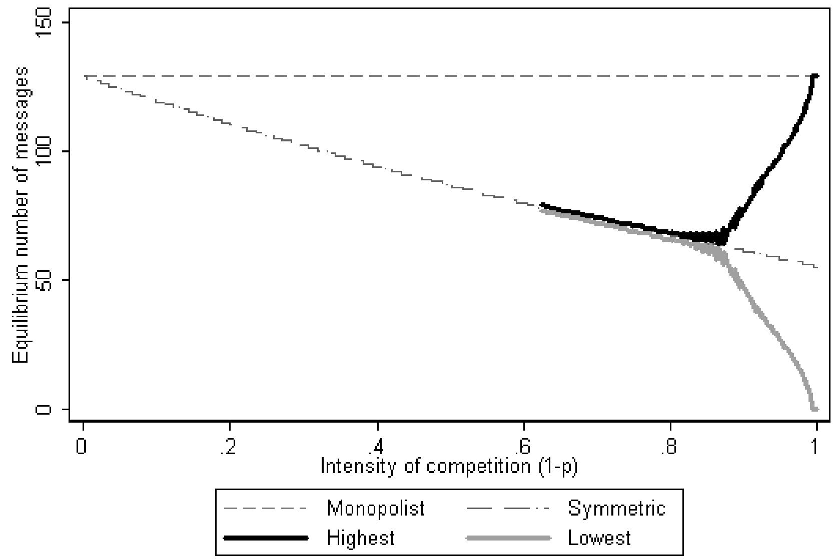

Figure 5: The actions of the two competing firms in the symmetric and in the most asymmetric equilibrium as a function of the level of competition. The network is that displayed in Figure 3 and the cost of sending a message is 0.3.

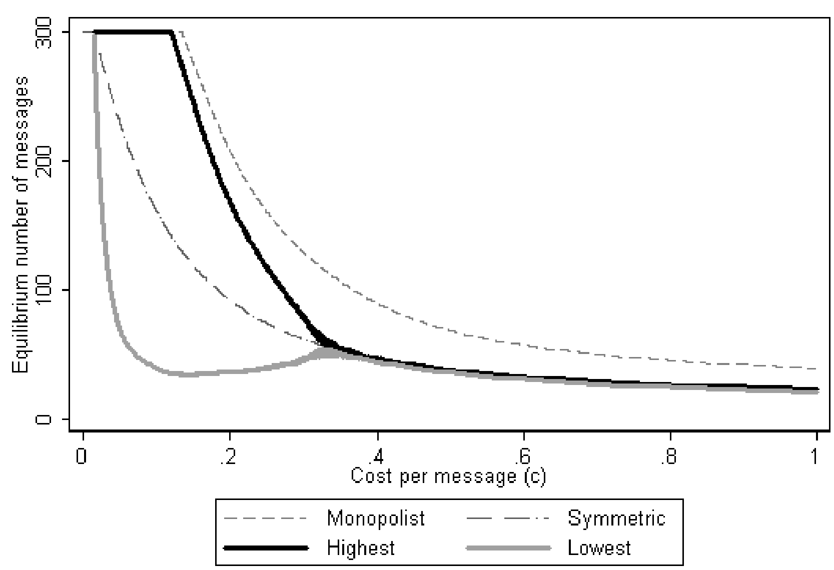

Figure 6: The actions of the two competing firms in the symmetric and in the most asymmetric equilibrium as a function of the cost of sending a DM message. The network is that displayed in Figure 3 and after every consumer reached by both firms, the expected surplus is 0.1 for each firm.

before (e.g., when the change in the exogenous parameters is small), or at least one of the firms decreases its DM intensity. Figures 5 and 6 illustrate the evolution of the equilibria as a function of the level of competition and the cost per message. As expected, at larger cost and more intense competition, firms send less messages in the symmetric equilibrium. When equilibria become asymmetric, the rise in one of the DM intensities happens simultaneously to the drop in the other.

It is interesting to analyze the difference in how the intensity of competition and the cost of sending a message influence the nature of the equilibria. First, we discuss the impact of competition on the outcome of the game. In Figure 5, we observe the largest asymmetry in the equilibria at the lowest values of p. This happens because as p decreases, the substitution effect becomes stronger while the saturation effect remains essentially the same. Firms will thus have less incentives to simultaneously reach nodes in the network, resulting in the emergence of asymmetric equilibria.

Turning our attention to the cost of sending a message, we look at Figure 6. We see that the increase in the cost of sending a message first increases, then decreases the asymmetry of the equilibria. This happens because c changes the strength of both the substitution and the saturation effects significantly. This phenomenon, best explained on Figure 3, is also related to the network structure. In the network in Figure 3, we can see regions of different connectivity. Nodes with the highest degrees (39) are at the bottom of the image, followed by the group of nodes (of degree 19) on the top. Nodes of degree 4 are placed between these two clouds. Finally, on the two sides of the figure there are nodes of degree 1 and degree 0. In Figure 6, we see that for $c < 0.012$, as Corollary 2 says, the unique equilibrium is symmetric. In particular, both firms apply blanket advertising as at such low cost levels, it is profitable to reach every node in the network. As c grows, however, the substitution effect increases and reaching the weakly connected nodes is no longer profitable for both firms. Assuming that Firm 1 takes the higher action (i.e., it never sends less messages than Firm 2), this means that Firm 2 decreases its DM campaign level while Firm 1 still conducts blanket advertising. It is worth noting that despite this, Firm 2 still sends at least 34 messages which guarantees a close to 1 chance of reaching all the *well-connected* consumers in the network. Further increasing the cost of sending a message, it has a greater impact on the saturation effect. At $c \approx 0.12$, the cost becomes too high even for one firm to profit from reaching all isolated and weakly connected consumers. Hence, Firm 1 starts to decrease its DM intensity. Simultaneously, this allows Firm 2 to reach more consumers who are unaware of Firm 1's product. Therefore, as c grows further, Firm 2 slowly increases its advertising intensity up to the point, around $c \approx 0.33$, above which both firms essentially focus only on the well-connected consumers in the network, resulting in only minor asymmetries in the equilibria.

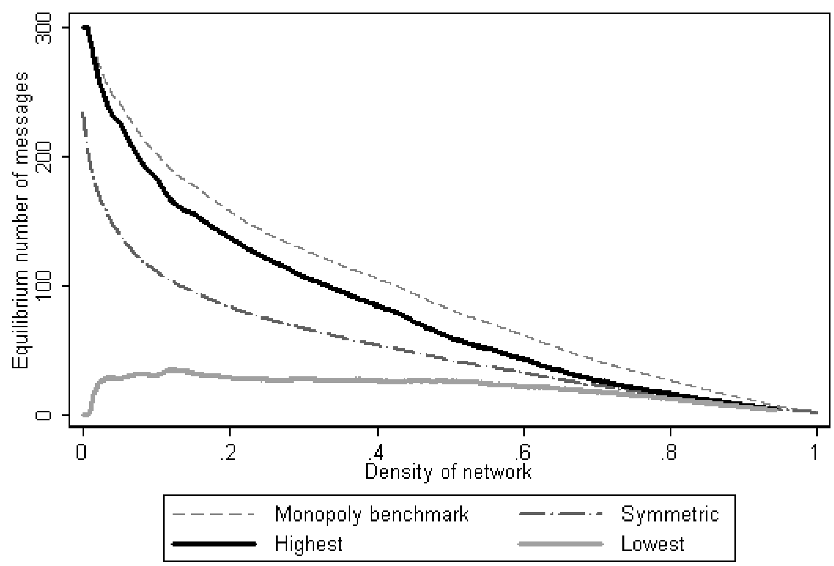

Figure 7: Optimal strategies in the symmetric and the most asymmetric equilibria on random networks when the expected per-firm surplus after every consumer reached by both firms is 0.1 and the cost of sending a DM message is 0.3. Results are averaged over 50 draws of networks.

Lastly, we again analyze how the equilibria depend on network density by examining equilibria over random networks. Figure 7 displays the average level of optimal DM intensities for randomly chosen networks over 300 nodes. For each possible number of edges in the network, the optimal DM intensities were averaged over 50 randomly drawn networks. The results illustrate two things. First, the general trend is that as the density of the network grows, DM levels in the most asymmetric equilibrium get closer to the DM level in the symmetric equilibrium. Second, as the network density grows, at least one of the firms sends less DM. This second phenomenon is similar to Proposition 3 and shows how network density can be thought of as a parameter of the game.

4.3 Stackelberg Duopoly

In the Stackelberg version of the random targeting game, first Firm 1 chooses the number of consumers to target, k_1, and sends out the messages randomly. Firm 2 observes k_1 but not the actual targets of the message. Then Firm 2 chooses the number of messages to send, k_2, and sends them out independently at random.

Proposition 4. *For any network $G(V,E)$, any $c > 0$ and any $0 \leq p \leq 1$, the Stackelberg game of random targeting under duopoly has at least one pure-strategy equilibrium. Let (k_1^{S*}, k_2^{S*}) denote the Stackelberg equilibrium where Firm 1 plays the highest action. Let further (k_1^*, k_2^*) denote the most asymmetric equilibrium in the simultaneous-move game defined in Section 4.2. Then $k_1^{S*} \geq k_1^*$ and $k_2^{S*} \leq k_2^*$. Furthermore, if*

$$\sum_{v \in V} \left[p + (1-p) \cdot \frac{\binom{n-d(v)-1}{k^{M*}(c/p)}}{\binom{n}{k^{M*}(c/p)}} \right] - (n + k^{M*}(c)) \cdot c > \Pi_1^D(k^{M*}(c), \mathrm{BR}_2(k^{M*}(c), p, c), p, c),$$

then $k_1^{S*} > k_1^{M*}(c)$.

Proposition 4 provides two basic insights. First, it states that first-mover advantages always result in equilibria that are at least as asymmetric as any equilibrium in the simultaneous-move game. This implies that if a firm has the possibility to commit to saturating most of the network, it can drive out its competitor of the vast part of the market. The most well-connected consumers, however, are exceptions to this: the second-mover firm often sends out a few messages with the idea of almost certainly reaching the nodes in the network that have the highest degrees.

The second take-away from Proposition 4 is that, at intense competition, the strategy of driving out the competitor may result in equilibria where the first-mover sends more DM messages than the monopolist at the same cost. The intuition behind this is the following. On the one hand, at the optimal level of DM for the monopolist, sending the next message does not bring more surplus than c (in expectation). On the other hand, sending further messages in a Stackelberg setting may force the second-mover to send out less messages, thereby increasing the possible surplus from many nodes by $1-p$ units. In other words, the substitution effect becomes more dominant in the Stackelberg game, in some instances justifying even blanket advertising on networks where the monopolist sends fewer messages.

Next, we turn to comparative statics. Figures 8 and 9 illustrate the evolution of the equilibria over the network shown in Figure 3, at $c = 0.3$ and $p = 0.1$, respectively. Beyond the phenomena detailed above, they also show that for low values of competition and per-message cost, both the simultaneous-move and the Stackelberg equilibria are symmetric.

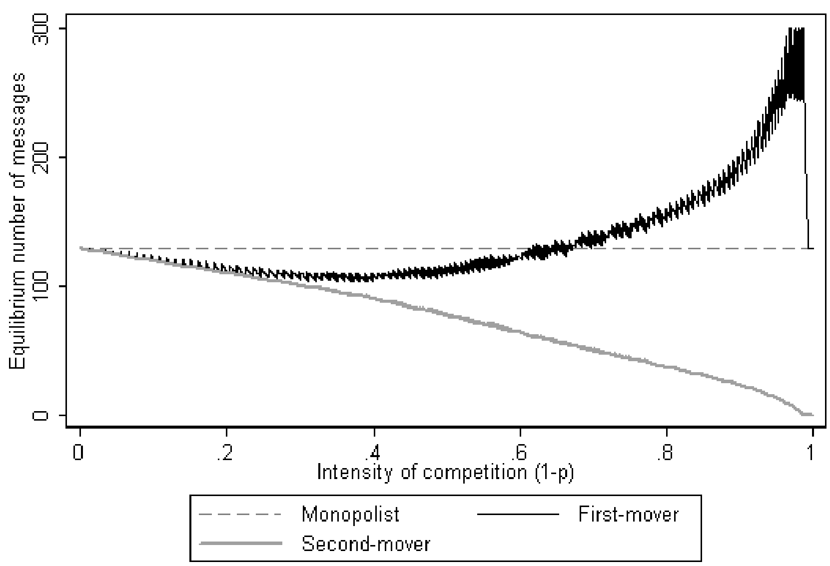

Figure 8: The equilibrium actions of the two competing firms in the Stackelberg game as a function of the level of competition (per-firm surplus per customer they both reach). The network is that displayed in Figure 3 and the cost of sending a message is 0.3.

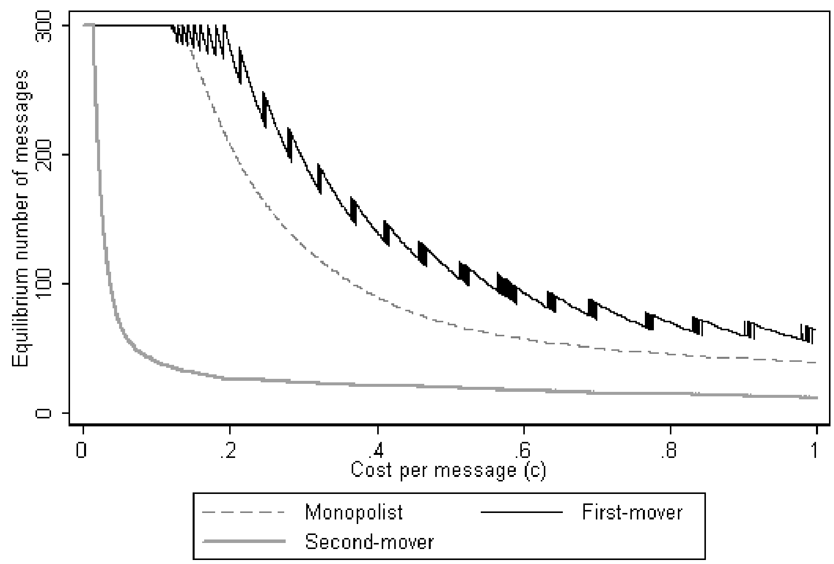

Figure 9: The equilibrium actions of the two competing firms in the Stackelberg game as a function of the cost of sending a DM message. The network is that displayed in Figure 3 and after every consumer reached by both firms, the expected surplus is 0.1 for each firm.

Finally, we analyze the impact of network structure on the outcome of the Stackelberg game. Figure 10 displays the average level of optimal DM intensities in the Stackelberg game for randomly chosen networks over 300 nodes. For each possible number of edges in the network, the optimal DM intensities were averaged over 50 randomly drawn networks. The results illustrate similar effects to those concluded in the previous subsection. First, the general trend is that as the density of the network grows, DM levels in the Stackelberg equilibrium get closer to each other. Second, as the network density grows, the equilibrium number of messages show a generally decreasing trend. Finally, we may observe that for the examined parameter values, the first-mover always sends at least as many DM messages as the monopolist, forcing the second-mover to target only less than 5% of the consumers.

In sum, we find three important differences between the simultaneous-move and the Stackelberg games of random targeting. First, asymmetric equilibria are more prevailing in the Stackelberg game. Second, it is only for the simultaneous-move game that symmetric and asymmetric

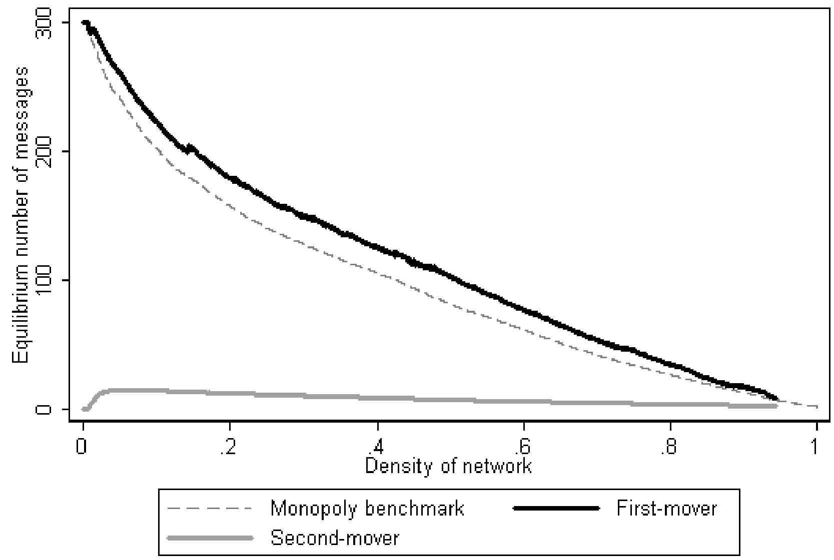

Figure 10: Optimal strategies in the Stackelberg equilibria on random networks when the expected per-firm surplus after every consumer reached by both firms is 0.1 and the cost of sending a DM message is 0.3. Results are averaged over 50 draws of networks.

equilibria co-exist: in the Stackelberg game, the symmetric equilibrium is only present for parameter ranges where first-mover advantages don't exist. Finally, asymmetries in the equilibria tend to be larger for the Stackelberg game and the first-mover, in order to drive out its competitor of the market, sometimes sends more DM than the optimal DM level of a monopolist.

Comparing our findings across the three scenarios discussed, we can summarize the most important characteristics of direct marketing on a network. First, we concluded that it is important for firms to take into account the existence of WOM in order to avoid wasteful DM costs. This is especially important for firms when competition is present. Second, we found that in the presence of WOM on a network, competing firms may end up in asymmetric equilibria even in a simultaneous-move game. In turn, the Stackelberg game of random targeting admits very strong first-mover advantages. This is an important strategic insight for competing DM firms. Finally, we showed that the network structure has a dramatic impact on the outcome both for the monopolist and for competing firms: the more dense is the social network, the less messages firms have to send, and the less asymmetric the competitive equilibria become. In the next section, we explore three directions in which our basic model can be extended.

5 Extensions

First, we analyze more complex models of social influence. Second, we consider the case when firms can only target a certain segment of the market but may also sell their products to consumers from the rest of the market if they are reached via WOM. Finally, we investigate the scenario when individual network connections may be identified and exploited by firms.

5.1 Refined Models of Social Influence

To focus on strategic insights, so far we have used a very simplistic model of social influence. In this subsection we show how the results apply for more general models of WOM. We extend our basic model in three directions. First, we allow social influence to be asymmetric. This is equivalent to looking at a directed network. Second, we step away from the deterministic mechanism governing contagion and we allow WOM to be random. Finally, we lift the restriction on the range of WOM (i.e., that information may travel a distance of only one link). Considering all three extensions at once, we show how our results above provide insights for the Cascade Model of Goldenberg et al. (2001).

Let us outline the Cascade Model of Goldenberg et al. (2001), as presented in Kempe et al. (2003). We assume that the network is directed, and that to any edge $e(v_1, v_2) \in E$, $0 < p_{v_1,v_2} \leq 1$ denotes the probability of the "activation" of $e(v_1, v_2)$. We allow for indirect social influence in the following way. The DM campaign unfolds in discrete steps. At step 0, some nodes S_i^0 are initially targeted by Firm i for $i = 1, 2$. Further, when a node v is aware of product i already, we call v "activated" by Firm i. When node v first becomes activated by Firm i in step t, it gets a single chance to activate all of its neighbors. For each neighbor w, v succeeds with probability

$p_{v,w}$ independently of the history so far. If w has multiple active neighbors, their attempts are sequenced in an arbitrary order. If v succeeds, w will be activated by i in step $t+1$; but whether or not v succeeds, it cannot make any further attempts to activate w by i in subsequent rounds. Let A_v denote the set of neighbors that v successfully informs about product i. We assume that, if Firm $-i$ activates v later, v will inform exactly its neighbors in A_v about product $-i$. The process runs until no further activations are possible. We finally assume that all price negotiations and purchase transactions take place after the social influence process had completed.

It is easy to see that the above process is equivalent to first making all the random draws along the edges, then activating the initial sets S_i^0 and finally unfolding the process along the activated edges. Let ω be a random draw, and let the activated edges form the set E' over the nodes V. Then we can reduce the network $G(V,E')$ to an instance $G'(V,E'')$ in a way that for every pair of nodes $v,w \in V$, $e(v,w) \in E''$ if and only if there is a path in $G(V,E')$ from v to w consisting entirely of activated edges. Clearly, this can be done for any ω, and the resulting game will be played according to our model of social influence from Section 3. To treat directed relationships (in each such instance), instead of simply counting the related consumers, for every node v, we now look at the *in-degree* of the node, that is, the number of consumers who may influence the purchase decision of v. In this way, for any instance determined by some ω, the key properties of the game in section 3 also hold. In particular, the expected surplus is a sum of increasing and concave functions. Therefore, it is still increasing and (weakly) concave in k, and so the expected profit is also weakly concave in k. Further, the strategies of the two firms are still strategic substitutes, and thus, applying our technique from the proof of Proposition 2, we can prove the following.

Proposition 5. *For any network $G(V,E)$, any $c > 0$ and any $0 \le p \le 1$, with the Cascade Model describing social influence, the game of random targeting has at least one pure-strategy equilibrium (k_1^*, k_2^*).*

Also, we can extend Proposition 3 to this general case.

Corollary 3. *Let (k_1^*, k_2^*) be an equilibrium under the parameter combination (p, c) from Proposition 5. Let $(p', -c') > (p, -c)$ under the standard component-wise order and let (k_1', k_2') be any equilibrium under (p', c'). Then $(k_1', k_2') \not< (k_1^*, k_2^*)$.*

In sum, the stylized model of social interactions presented in Section 3 and its more complex generalizations analyzed above provide the same qualitative insights. Competitive outcomes are determined by the relationship of the saturation effect and the substitution effect and thus may be asymmetric but tend to be less so when WOM is stronger. Consequently, blanket advertising results in drops in expected profits and these profit drops are more significant when WOM is stronger and when competition is present.

5.2 Endogenous Network Size

In this subsection, we challenge the assumption that the size of the consumer network $G(V, E)$ is fixed. It is indeed very plausible that consumers who are not in the database of the firms (and who are hence unreachable by DM) would join the market upon receiving WOM information from consumers reached by one or both of the advertising firms. Let V' denote these consumers who can not be directly targeted. Naturally, we assume that $V' \cap V = \emptyset$. We assume that members of V' have the same demand as those consumers in V. Thus, when the DM message of Firm i reaches consumer $w \in V'$, Firm i's expected net surplus from this consumer is:

$$r_i(w) = \begin{cases} 1 & \text{if } w \text{ has not been reached by firm } -i, \\ p & \text{if } w \text{ has been reached by both firms,} \end{cases}$$

where $0 \leq p \leq 1$, $i = 1, 2$ and $-i$ denotes the competing firm. We assume the original model of social influence which means that consumers in V' do not further propagate the information about the advertised products. W.l.o.g., we will therefore assume that there are no edges in the network connecting members of V'.

Proposition 6. *For any network $G'(V \cup V', E)$ with $V \cap V' = \emptyset$ and $(V' \times V') \cap E = \emptyset$, any $c > 0$ and any $0 \leq p \leq 1$, with the model of social influence described in Section 3, the game where the two* *competing firms may target members of V at random, has at least one pure-strategy equilibrium* (k_1', k_2'). *Further, for any equilibrium* (k_1^*, k_2^*) *in the simultaneous-move random targeting game played on* $G(V,E)$ *with the same parameters p and c, we have* $(k_1', k_2') \not< (k_1^*, k_2^*)$.

Proposition 6 says that when there are potential consumers who may only be reached via WOM, typically at least one of the firms conducts a more intense DM campaign. Nevertheless, it is possible that the asymmetry of the equilibrium grows upon considering the consumers in V'. For instance, the possibility to get extra surplus from DM messages could make the firm choosing the equilibrium DM level to send even more messages. When p is small enough, this may lead the other firm to send even less DM than before.

Another interesting extension could be to consider a DM game of multiple periods on G' with the following twist. The nodes in V' reached by a firm in one period become directly targetable in the next period. This would make firms to consider a non-empty edge set over V' and the potential benefits of bringing in new *connections* to their network, to increase the efficiency of later campaigns. It is beyond the scope of this paper to conduct the analyis of the so-arising network dynamics but we note that since the main characteristics of the game (the saturation and substitution effects) remain the same, the result of Villas-Boas (1997) likely applies for this more complicated model formulation too.

5.3 Targeting under full network knowledge

In our basic model, we assumed that firms cannot assess the network position of individual consumers. As mentioned before, this is the relevant case because even if individual network information is available, the usage of such information is usually illegal. Nevertheless, it is interesting to compare this scenario to random targeting. In this subsection, we examine firms' behavior when they are aware of the position of individuals in the network and are also allowed to use this information. We maintain the assumption however, that competing firms are unaware of the identity of consumers targeted by their competitor. Also, we keep our earlier notation: c denotes the cost of sending a DM message and p is the expected per-firm net surplus from a consumer reached by both firms.

Focusing on the monopolist's problem, Kempe et al. (2003) provide a deep investigation of this question. Their negative result is that, even for a monopolist, it is a computationally very hard problem to decide whether a given set of nodes is optimal to target, which further implies that it is computationally very hard to find the optimal set to target. Whereas stated for the more general influence model of Goldenberg et al. (2001), their proof applies to our model of social influence as well.[9] On the other hand, Kempe et al. (2003) also describe a $(1-1/e-\varepsilon)$-approximation for the influence maximization problem.[10] This essentially means that there is a computationally feasible algorithm, which can achieve at least 63% of the available producer surplus in the network at the optimal cost (thus, for low values of c, reaching about 63% of the possible profit). Implementing several heuristics over a collaboration network, the results of Kempe et al. (2003) further indicate that the magnitude by which the deterministic approximation algorithm outperforms random targeting decreases as WOM becomes more intense.

For our purpose, the take-away from this research is that a monopolist direct marketer can do significantly better under full network knowledge than under random targeting. To study competition and overcome issues of computational complexity, herein we look at a special case of the problem outlined in Section 3. In particular, we only look at networks where every connected component contains at least one "opinion leader", a node representing an individual who can spread the message to every member in their connected component. This stylized model captures the general idea that social networks are composed of *communities* and within each community there are only a few central individuals. To save space, herein we only analyze the simultaneous-move

[9]The proof is a reduction from the NP-complete Set Cover problem. We refer the interested reader to Kempe et al. (2003).

[10]The ε term comes from the fact that they need to run simulations to determine the degree of indirect influences and they can thus only work with approximate influence rates.

duopoly in detail. This is not a major restriction however, since the optimal strategy of the monopolist is simply the strategy of either firm for $p = 1$ in the simultaneous-move game, while the only equilibrium of the Stackelberg game is exactly the most asymmetric equilibrium in the simultaneous-move game.[11]

5.3.1 Simultaneous-Move Duopoly

As every member in each connected component can be reached by targeting an opinion leader in the component, the optimal strategies of firms depend on the respective sizes of the connected components in the network. Large enough connected components are saturated through targeting one opinion leader in the group, while too small connected components are ignored.

Assume that the firms follow deterministic strategies. That is, when carrying out the advertising campaign, Firm i selects a set $S_i \subseteq V$ and sends a DM message to each member of S_i.[12] For our stylized networks, we can characterize all the stable competitive outcomes of the deterministic DM targeting game.

Proposition 7. *Let $G(V,E)$ be a network such that for any connected component $G'(V',E')$ of G, there is a node $v \in V'$ such that v can influence the entire set of V'. Let further $0 < p \leq 1$, $0 < c \leq 1$. Then, in any Nash equilibrium in the simultaneous-move game of deterministic targeting, the strategies of the firms have the following properties:*

- *For every connected component of size at least c/p, both Firm 1 and Firm 2 target an opinion leader in the component, respectively (which opinion leaders do not have to be the same individual).*
- *For every connected component of size less than c/p but at least c, only one of the firms*

[11] Further details on these statements may be obtained from the authors.

[12] In the Stackelberg version of the full information targeting game, first, Firm 1 chooses the set $S_1 \subseteq V$ of consumers to target and sends out DM to each member of S_1. Firm 2 observes S_1, then chooses the set $S_2 \subseteq V$ and sends a DM message to every consumer in S_2.

targets an opinion leader while the other firm does not send advertising messages to any individual in the component.

- *For every connected component of size less than c, neither of the firms send advertising messages to any individual in the component.*

Note that for $c \leq p$, the entire network gets saturated in (the only) equilibrium. The difference from a mass marketing campaign by both firms is that here firms may enjoy this full coverage at a lower cost.

Proposition 7 states that there may be many equilibria depending on the intensity of competition, the cost of sending a message and the network structure. These equilibria are similar in the sense that all the large components of the network are saturated by both firms. We also know that, in equilibrium, the small components are divided between the two competitors so that each small component is saturated by only one of the firms. In case that both firms take (quasi-) equal shares from the small (but large enough to be considered by a monopolist) components, the corresponding equilibrium is (quasi-) symmetric. On the other hand, if one firm targets all the small components, we arrive to the most asymmetric equilibrium for the given network. Thus, in this special case we know exactly how the network structure impacts the symmetry of the equilibria. Naturally, the level of competition and the cost of sending a message impact the equilibrium structure through marking the thresholds above which a given connected component is beneficial to target for only 1 or even 2 firms, respectively. This also means that when competition is mild and the cost per message is low, the unique equilibrium is symmetric and that the possibility for asymmetry arises as the cost per message increases and as the competition gets more intense. Qualitatively, this is similar to the case of random targeting (see Figures 5 and 6). Network density also plays a role similar to that in random targeting - when all components are large, network density is also large. In this case, the unique equilibrium is symmetric but when there are more smaller components, asymmetric equilibria like those displayed in Figure 7 are the typical outcome of the game.

Comparing random versus deterministic targeting, we can first of all say that, *ceteris paribus*, a firm is always better off using all the information in the network. The following result states this.

Proposition 8. *For any network $G(V,E)$, $0 \leq p \leq 1$, $0 < c$, let (k_1,k_2) be any equilibrium of the random targeting game. Then there are deterministic strategies $S_i \in V$, $i = 1,2$ such that unilaterally deviating to a deterministic targeting of S_i is weakly profitable for either of the firms.*

On the other hand, we are interested in comparing the competitive outcomes when the random targeting is exogenously *imposed* on the competing firms (say, they are not allowed to use information on personal connections) to targeting under full network knowledge. We find that in many cases, the firm sending less messages in asymmetric equilibria is better off in the random-random equilibrium than in the deterministic-deterministic equilibrium (although this is not true for the firm which sends out more advertising messages). For example, consider a network with $n = 100$ nodes that group into 10 equal-sized groups such that within each group, any consumer can influence the purchase decision of every other but across groups, no influential relationships are present. Using standard network terminology, we refer to these groups as *clique*s. Let further the cost be $c = 0.7$ and the parameter describing competition $p = 0.05$. As the size of any clique is less than $c/p = 14$, in any equilibrium of Proposition 7, every clique is targeted by only 1 of the firms. In an extreme equilibrium, Firm 2 does not send any messages and thus makes 0 profit, while Firm 1 saturates the network by sending 1 message to each clique, extracting 93 units of profit. On the other hand, under the random-targeting scenario where Firm 1 takes the highest and Firm 2 the lowest equilibrium action, they send 15 and 12 messages, making 13.89 and 7.99 units of profit, respectively. Thus, while introducing full network knowledge increases the efficiency of targeting, it may also significantly increase the intensity of competition. The effect of full network knowledge in the DM game is that it gives firms very strong incentives to target the same consumers. As a result, in competitive environments with low p, allowing both firms to exploit full network knowledge may lock one of the firms out of the network.

6 Concluding remarks

Recent development of sophisticated customer databases provide marketers with an environment of high consumer addressability, in which direct marketers have to identify the most profitable customers to target. Since word-of-mouth communication is well-known to be a powerful driver of consumer behavior, consumers' connection patterns are a key determinant of their profitability.

Many of today's customer databases keep records of communication among the members of the customer base. However, most of the existing customer relationship management and direct marketing methods ignore word-of-mouth. Moreover, the few papers that account for word-of-mouth do not consider competitive interactions (Domingos and Richardson 2001, Gruhl et al. 2004, Hill et al. 2006, Kempe et al. 2003, Richardson and Domingos 2002). In this paper, our goal was to analyze the strategic impact of taking into account the underlying network structure among consumers who communicate to one another. Considering that exploiting personal network connections is often illegal, our main focus was on the case when marketers know the general structure of the network but are unaware (or not allowed to use) individuals' position in the network. In this regard, we characterized the problem of random targeting and described the optimal behavior of the monopolist. We then solved for the competitive equilibria both in the simultaneous-move game and under the Stackelberg setting. We found that, depending on the level of competition, the cost of the campaign and the density of the consumer network, these equilibria may be asymmetric. In particular, under intense competition and sufficiently high DM cost, the game of direct marketing admits asymmetric equilibria: when a firm "fills" the network, the other is best off essentially staying out of the market. However, as the density of the consumer network increases, the firms may reach more consumers at lower cost and these asymmetries vanish.

In three extensions, we looked at variations of the above game. First, we examined more complex models of social influence. Second, we considered how the strategies of firms change when a segment of the market may only be addressed via WOM and not via DM. Finally, we analyzed the case in which firms are allowed to target their DM deterministically, based on the structure of personal networks. For this scenario of *full network knowledge*, we looked at stylized consumer networks in which there are opinion leaders in every "consumer community". In all three cases, we found that the qualitative structure of the competitive equilibria is essentially the same as in our basic model. Further, comparing the random targeting game to that played under full network knowledge, we found that introducing full information may intensify the competition, leaving the firm sending less messages worse off than when personal connections are not allowed to be incorporated in the strategic decisions of the players.

Our paper has several limitations. First, our model does not capture previously considered models of persuasive information. In terms of the underlying social network model, despite our generalizations, we cannot handle the threshold models of social influence (Granovetter 1978, Valente 1995), where the separately insufficient influential power of two or more neighbors may add up to successfully activate a node. Whereas the results concerning comparative statics are likely to be the same for most network instances under that scenario as well,[13] our proof techniques used herein are insufficient to identify the conditions characterizing such networks. On the other hand, it is easy to construct counterexamples, e.g. a clique where no-one is opinion-leader but any two people aware of the product can increase the demand of the entire network.

Another way to look at persuasive information is to consider changes in the demand share of firms due to their advertising activities or WOM. A recent paper on combative advertising (Chen, Joshi, Raju, and Zhang 2009) examines the effects of advertising when all consumers have full information about all products, the category demand is fixed and advertising only changes preferences. We believe that combining persuasive advertising with WOM is a promising direction for

[13] In fact, for the case when activation thresholds are drawn at random for the entire population, the results of Kempe et al. (2003) suggest that our results also apply.

future research.

Second, we have only examined a duopoly. While we expect the fundamental insights to be similar, generalizing our results to markets with more than 2 competitors would be of interest. Finally, there may be alternative characterizations possible for the scenario when firms are allowed and able to use information on the position of individuals in the network. Despite these limitations, we believe that the techniques and the insights presented herein are going to provide a good first step in the development of a new generation of direct marketing and customer relationship management tools.

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Appendix

PROOF OF PROPOSITION 1: Let v be a node in the network and let k be the number of advertising messages sent by the monopolist. As shown in Section 3.1, the expected surplus from v is

$$\mathop{\mathrm{E}}_{|S|=k}[r_1(v)] = 1 - \frac{\binom{n-\mathrm{d}(v)-1}{k}}{\binom{n}{k}}.$$

Summing this up for every node in the network and taking into account the cost of sending messages, we arrive to the following formula for the profit of the monopolist:

$$\Pi^M(k,c) = \left[\sum_{v\in V} 1 - \frac{\binom{n-\mathrm{d}(v)-1}{k}}{\binom{n}{k}}\right] - k\cdot c.$$

To see the rest of the claim, we are going to prove that $\Pi^M(k,c)$ is (weakly) concave in k. Whereas it is probably shorter to do this applying basic calculus, we choose a different proof technique as it reveals more general underlying mechanisms that may help gaining a further insight to the problem discussed in this article.

First we are going to prove the submodularity[14] of the surplus function corresponding to the deterministic selection of the set of targeted consumers. Next, we carry out an averaging argument to see that the submodularity also holds for the surplus from randomized targeting. This property of the surplus function then in turn implies the claims.

For notational convenience, for any set $S \subseteq V$, let $r(v,S)$ be the surplus of the monopolist from node v when exactly the nodes in S are targeted. Let then the total surplus of the monopolist under deterministic selection be $W(S) = \sum\limits_{v\in V} r(v,S)$ and furthermore the expected surplus from selecting k nodes at random be

$$R(k) = \mathop{\mathrm{E}}_{|S|=k}[W(S)] = \frac{\sum\limits_{|S|=k} W(S)}{\binom{n}{k}}. \tag{1}$$

[14]The submodularity is understood over the lattice spanned by the power set of V.

The following lemma states the submodularity of the surplus under deterministic targeting.

Lemma 1. *(Kempe et al. 2003)*[15] *$W(\cdot)$ is submodular, i.e.*

$$W(S \cup \{v\}) - W(S) \geq W(T \cup \{v\}) - W(T)$$

for all nodes v and all pairs of sets $S, T \subseteq V$ such that $S \subseteq T$.

PROOF: For any pair of sets S, T with $S \subseteq T \subseteq V$ and any given nodes $x, v \in V$, we have the following:

1. If x is aware of the product after the firm targets all members of S, then it is also aware of the message after the firm targets all members of T. Thus, targeting v does not generate any additional surplus on x.

2. If x is unaware of the product after the firm targets S (or T, respectively) then

   - If x becomes aware of the product after the firm targets v, then targeting v generates an additional surplus of 1 for Firm 1 on x.
   - If x remains unaware of the product after the firm targets v, then targeting v does not generate any additional surplus for Firm 1 on x.

Thus, $r(x, S \cup \{v\}) - r(x, S) \geq r(x, T \cup \{v\}) - r(x, T)$. The claim follows. □

The next step is to show how the submodularity of $W(\cdot)$ leads to the "diminishing returns" property of $R(\cdot)$. For a given k, let now sum up all the possible submodularity inequalities of the form

$$W(X_{k-1}) - W(X_{k-2}) \geq W(Y_k) - W(Y_{k-1}),$$

where $X_{k-1} = X_{k-2} \cup \{v\}, Y_k = Y_{k-1} \cup \{v\}, |X_{k-2}| = k-2, |Y_{k-1}| = k-1$ and $X_{k-2} \subset Y_{k-1}$. To describe such an equation, we have to select a $(k-2)$-element set X_{k-2} ($\binom{n}{k-2}$ possibilities), a

[15] In Kempe et al. (2003), this claim is proven, in a more general setting, through a slightly more involved technique.

node v outside this set ($n-k+2$ possibilities) and a node u such that $\{u\} = Y_{k-1} \setminus X_{k-2}$ ($n-k+1$ possibilities). Thus, there are

$$\binom{n}{k-2}(n-k+2)(n-k+1) = k(k-1)\binom{n}{k}$$

such submodularity inequalities.

Because of the symmetry of these inequalities, for every k-element set S_k, $W(S_k)$ appears

$$\frac{k(k-1)\binom{n}{k}}{\binom{n}{k}} = k(k-1) \text{ times on the right-hand side.}$$

Similarly, for every $(k-1)$-element set S_{k-1}, $W(S_{k-1})$ appears

$$\frac{k(k-1)\binom{n}{k}}{\binom{n}{k-1}} = (k-1)(n-k+1) \text{ times on both sides (with opposite signs).}$$

Finally, for every $(k-2)$-element set S_{k-2}, $W(S_{k-2})$ appears

$$\frac{k(k-1)\binom{n}{k}}{\binom{n}{k-2}} = (n-k+1)(n-k+2) \text{ times on the left-hand side.}$$

Thus, substituting (1) into the sum of the submodularity inequalities, we get

$$(k-1)(n-k+1)\binom{n}{k-1}\cdot R(k-1) - (n-k+1)(n-k+2)\binom{n}{k-2}\cdot R(k-2) \geq$$
$$k(k-1)\binom{n}{k}\cdot R(k) - (k-1)(n-k+1)\binom{n}{k-1}\cdot R(k-1),$$

which simplifies to $R(k-1)-R(k-2) \geq R(k)-R(k-1)$. The (weak) concavity of $R(\cdot)$ and that of $\Pi^M(\cdot,c)$ follows. Further, if $E \neq \emptyset$ and $R(k-1)-R(k-2) > 0$ then in fact there is at least one sharp submodularity inequality in the above proof and so $R(k-1)-R(k-2) > R(k)-R(k-1)$. Since $c > 0$, this implies for any $k \geq 0$ that if $E \neq \emptyset$ and if $\Pi^M(k,c) = \Pi^M(k+1,c)$ then $\Pi^M(k+1,c) > \Pi^M(k+2,c)$, concluding the proof of Proposition 1. □

PROOF OF COROLLARY 1: It is straightforward to check that $\Pi^M(k,c)$ is decreasing in c and has decreasing differences in (k,c). As c is taking values from the convex interval $(0,\infty)$, the convexity of $\Pi^M(k,c)$ follows. □

PROOF OF PROPOSITION 2: Let v be a node in the network and k_1 and k_2 be the actions chosen by the two firms, respectively. By either firm, node v is reached if and only if it is either a target of an advertising message sent by the given firm, or if one of its neighbors are targeted. The corresponding probabilities can be calculated as in the proof of Proposition 1, yielding the following formula for the expected surplus of Firm i from node v:

$$\mathop{\mathrm{E}}_{|S_i|=k_i,|S_{-i}|=k_{-i}}[r_i(v)] = \left[1-\frac{\binom{n-\mathrm{d}(v)-1}{k_i}}{\binom{n}{k_i}}\right]\cdot\left[\frac{\binom{n-\mathrm{d}(v)-1}{k_{-i}}}{\binom{n}{k_{-i}}}+p\cdot\left(1-\frac{\binom{n-\mathrm{d}(v)-1}{k_{-i}}}{\binom{n}{k_{-i}}}\right)\right],$$

where $S_i, S_{-i} \subseteq V$ are the sets of targeted consumers by Firms i and $-i$ respectively, chosen independently uniformly at random. Summing this up for every node in the network and taking into account the cost of sending messages, we arrive to the following formula for the profit of the firms competing in a duopoly:

$$\Pi_i^D(k_i,k_{-i},p,c) = \sum_{v\in V}\left[1-\frac{\binom{n-\mathrm{d}(v)-1}{k_i}}{\binom{n}{k_i}}\right]\cdot\left[p+(1-p)\cdot\frac{\binom{n-\mathrm{d}(v)-1}{k_{-i}}}{\binom{n}{k_{-i}}}\right]-k_i\cdot c.$$

To see the first half of the proposition, observe that in the above sum, the strategies of the two firms are interacting only through, for every node v, the probabilities that v is reached by the firms, respectively. As these probabilities are (weakly) increasing in the corresponding firm's actions, we get that $\Pi_i^D(k_i,k_{-i},p,c)$ has decreasing differences in (k_i,k_{-i}). That is, the strategies of the competing firms are strategic substitutes.

Let us now formulate a game by reversing the actions of Firm 2, i.e., consider the game where the payoffs to the strategies $(k_1, n-k_2)$ are the payoffs from the random targeting game to the strategies (k_1,k_2). By the above, the reversed game is supermodular. Thus, by Topkis (1998), its equilibria form a non-empty complete lattice, with a greatest and a least element (the former being Pareto optimal). Since the mapping of the strategy spaces is one-to-one, this proves the existence of a pure-strategy equilibrium in the game of random targeting.

To see the final statement of Proposition 2, w.l.o.g., let us show that if the condition is met then $k_1^* \neq 0$. Consider $k' = \mathrm{BR}_2(0,p,c)$. It is obvious that $k' = k^{M*}(c)$, and based on the submodularity of the game, we have $\mathrm{BR}_1(\mathrm{BR}_2(0,p,c),p,c) = \mathrm{BR}_1(k^{M*}(c),p,c)$. Now, if

$$\sum_{v \in V} \left[\frac{\mathrm{d}(v)+1}{n}\right] \cdot \left[p + (1-p) \cdot \frac{\binom{n-\mathrm{d}(v)-1}{k^{M*}(c)}}{\binom{n}{k^{M*}(c)}}\right] > c,$$

then $\mathrm{BR}_1(k^{M*}(c),p,c) > 0$ (since the left-hand-side of the inequality is exactly the surplus of Firm 1 from sending 1 message at random) and so any equilibrium k_1^* is different from 0. This concludes the proof of Proposition 2. □

PROOF OF COROLLARY 2: Let the expected surplus for selecting k_i nodes when the competitor selects k_{-i} nodes (the two firms selecting their targets independently, uniformly at random) be

$$R_i(k_1,k_2,p) = \frac{\sum\limits_{|S_1|=k_1} \sum\limits_{|S_2|=k_2} W_i(S_1,S_2,p)}{\binom{n}{k_1} \cdot \binom{n}{k_2}}.$$

Further, for any set $S_2 \subseteq V$, let

$$R_1'(k_1,S_2,p) = \frac{\sum\limits_{|S_1|=k_1} W_1(S_1,S_2,p)}{\binom{n}{k_1}}.$$

From Proposition 1 it follows that there is a k^* such that for the surplus of a monopolist, $R(k^*) - R(k^*-1) > 0$ but for any $z \in \mathbb{N}$, $R(k^*+z+1) - R(k^*+z) = 0$. Moreover, we also have that for any $0 < k < k^*$, $R(k) - R(k-1) \geq R(k^*) - R(k^*-1)$.

Clearly, for any firm, independently from the competitor's behavior, the surplus from a yet unreached node when reached is at least p (p or 1). Thus, with the above claims we get that for any $0 < k_1 \leq k^*$ and any set $S_2 \subseteq V$,

$$R_1'(k_1,S_2,p) - R_1'(k_1-1,S_2,p) \geq p \cdot (R(k^*) - R(k^*-1)),$$

which, averaging over $|S_2| = k_2$, yields that for any k_2,

$$R_1(k_1,k_2,p) - R_1(k_1-1,k_2,p) \geq p \cdot (R(k^*) - R(k^*-1)).$$

Similarly, for any $k_1 > k^*$ and any set k_2, we have

$$R_1(k_1,k_2,p) - R_1(k_1-1,k_2,p) = 0.$$

Fix now any $p^* > 0$ and let $c^* = p^* \cdot (R(k^*) - R(k^*-1))$. Clearly, for any p, $p^* < p \leq 1$ and any c, $0 \leq c < c^*$, the inequality $c < p \cdot (R(k^*) - R(k^*-1))$ holds and so the state (k^*,k^*) is the unique equilibrium of the random targeting game. $\square$

PROOF OF PROPOSITION 3: Our proof is essentially an application of Theorem 3 in Villas-Boas (1997). For the reason that a few conditions slightly differ in our case, we here re-prove the theorem, applying it to our context. Throughout this proof, we implicitly rely on the transitivity and reflexivity of the standard component-wise order over $\mathbb{N} \times \mathbb{N}$.

Let $T_1, T_2 : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \times \mathbb{N}$ be the mappings defined as
$T_1(k_1,k_2) = (\mathrm{BR}_2(k_1,p',c'), \mathrm{BR}_1(k_2,p',c'))$ and $T_2(k_1,k_2) = (\mathrm{BR}_2(k_1,p,c), \mathrm{BR}_1(k_2,p,c))$. Applying considerations detailed in the proofs of Results 1 and 2 we can easily see the following.

Observation 1. *T_1 is weakly higher than T_2, i.e., for any $k_1,k_2 \in \mathbb{N}$, $T_1(k_1,k_2) \geq T_2(k_1,k_2)$ where the order is the standard component-wise order.*

Furthermore, by the statement of Proposition 3, we know that (k_1^*,k_2^*) is a fixed point of T_2 and (k_1',k_2') is a fixed point of T_1. Finally, T_1 is a weakly decreasing mapping, i.e., for any $(k_1,k_2) \geq (l_1,l_2)$, we have $T_1(l_1,l_2) \geq T_1(k_1,k_2)$.

Suppose now that $(k_1^*,k_2^*) > (k_1',k_2')$. Because T_1 is weakly higher than T_2, we have

$$T_1(k_1^*,k_2^*) \geq T_2(k_1^*,k_2^*) = (k_1^*,k_2^*) > (k_1',k_2') = T_1(k_1',k_2').$$

But, because T_1 is weakly decreasing, we have $T_1(k_1',k_2') \geq T_1(k_1^*,k_2^*)$, a contradiction. $\square$

PROOF OF PROPOSITION 4: Since the action sets of both players are finite, the existence of the Stackelberg equilibrium is trivial. Now assume that $k_1^{S*} < k_1^*$. Then $k_2^{S*} = \mathrm{BR}_2(k_1^{S*}(p,c)) \geq$

$\mathrm{BR}_2(k_1^*, p, c) = k_2^*$, and so, using the formula presented in the proof of Proposition 2 describing the profits of the two firms, in particular that Firm 1's profits decrease in the action of Firm 2, we get that

$$\Pi_1^D(k_1^{S*}, k_2^{S*}, p, c) \leq \Pi_1^D(k_1^{S*}, k_2^*, p, c) \leq \Pi_1^D(\mathrm{BR}_1(k_2^*, p, c), k_2^*, p, c) = \Pi_1^D(k_1^*, k_2^*, p, c),$$

a contradiction. Thus, $k_1^{S*} \geq k_1^*$, and so $k_2^{S*} = \mathrm{BR}_2(k_1^{S*}(p,c)) \leq \mathrm{BR}_2(k_1^*(p,c)) = k_2^*$ easily follows.

To see the second claim, let us examine what happens when Firm 1 sends DM to all consumers in the network. In this case, Firm 2 only receives p surplus after each consumer reached. Normalizing Firm 2's problem to have the reduced form presented in Section 3, we get that $\mathrm{BR}_2(n, p, c) = k^{M*}(c/p)$. Thus, the profits of Firm 1 in this case would be

$$\sum_{v \in V} \left[p + (1-p) \cdot \frac{\binom{n-d(v)-1}{k^{M*}(c/p)}}{\binom{n}{k^{M*}(c/p)}} \right] - n \cdot c.$$

Comparing this level to the profits obtained when sending $k^{M*}(c)$ messages, we find that Firm 1 prefers saturating the entire market over sending only $k^{M*}(c)$ messages. Since the cost of the latter action is less than that of sending n messages, we also get that

$$R(k^{M*}(c), \mathrm{BR}_2(k^{M*}(c), p, c), p) \leq R(n, k^{M*}(c/p), p).$$

Now, using that $R(\cdot, k_2, p)$ is concave in its first parameter (which follows exactly as the weak concavity of $R(\cdot)$ in the proof of Proposition 1), we may conclude that for any $0 \leq k \leq k^{M*}$, we have $R(k, \mathrm{BR}_2(k, p, c), p) \leq R(k, \mathrm{BR}_2(k^{M*}(c), p, c), p) \leq R(k^{M*}(c), \mathrm{BR}_2(k^{M*}(c), p, c), p)$. By the condition stated in the Proposition, it then follows that Firm 1 prefers the action n over every action no larger than $k^{M*}(c)$. Hence, $k_1^{S*}(p, c) > k^{M*}(c)$. □

PROOF OF PROPOSITION 5 AND COROLLARY 3: After applying the reduction detailed in Section 5, we get a game that is a convex combination of games of the set-up discussed in Section 3. All the properties of this game that we used in the proofs of Proposition 2 and Proposition 3 hold. The same proof techniques thus carry through. □

PROOF OF PROPOSITION 6: Let $w \in V'$ and k_1 and k_2 be the actions chosen by the two firms, respectively. By either firm, node w is reached if and only if one of its neighbors (in V by our assumption) are targeted. The corresponding probabilities can be calculated as in the proof of Proposition 1, yielding the following formula for the expected surplus of Firm i from node w:

$$\mathop{\mathrm{E}}_{|S_i|=k_i,|S_{-i}|=k_{-i}}[r_i(w)] = \left[1 - \frac{\binom{n-\mathrm{d}(w)}{k_i}}{\binom{n}{k_i}}\right] \cdot \left[\frac{\binom{n-\mathrm{d}(w)}{k_{-i}}}{\binom{n}{k_{-i}}} + p \cdot \left(1 - \frac{\binom{n-\mathrm{d}(w)}{k_{-i}}}{\binom{n}{k_{-i}}}\right)\right],$$

where $S_i, S_{-i} \subseteq V$ are the sets of targeted consumers by Firms i and $-i$ respectively, chosen independently uniformly at random. Summing this up for every node in V' and the expected surplus for members of V (exactly as in the proof of Proposition 2) and taking into account the cost of sending messages, we arrive to the following formula for the profit of the firms competing in a duopoly with endogenously determined market size:

$$\begin{aligned}\Pi_i^D(k_i,k_{-i},p,c) \quad = \quad & \sum_{v\in V}\left[1 - \frac{\binom{n-\mathrm{d}(v)-1}{k_i}}{\binom{n}{k_i}}\right] \cdot \left[p + (1-p) \cdot \frac{\binom{n-\mathrm{d}(v)-1}{k_{-i}}}{\binom{n}{k_{-i}}}\right] + \\ & \sum_{w\in V'}\left[1 - \frac{\binom{n-\mathrm{d}(w)}{k_i}}{\binom{n}{k_i}}\right] \cdot \left[p + (1-p) \cdot \frac{\binom{n-\mathrm{d}(w)}{k_{-i}}}{\binom{n}{k_{-i}}}\right] - k_i \cdot c.\end{aligned}$$

Just as in the proof of Proposition 2, the strategies of the two competing firms are strategic substitutes. The existence of the pure-strategy equilibrium follows exactly as in the proof of Proposition 2.

Now, to see the second claim, we use the same technique as in the proof of Proposition 3. Let $T_1, T_2 : \mathbb{N}\times\mathbb{N} \to \mathbb{N}\times\mathbb{N}$ be the mappings defined as
$T_1(k_1,k_2) = (\mathrm{BR}_2(k_1,p',c'), \mathrm{BR}_1(k_2,p',c'))$ on G' and $T_2(k_1,k_2) = (\mathrm{BR}_2(k_1,p,c), \mathrm{BR}_1(k_2,p,c))$ on G. Since the expected return to the same DM campaign cannot be lower on G' than on G, it is easy to see the following.

Observation 2. *T_1 is weakly higher than T_2, i.e., for any $k_1,k_2 \in \mathbb{N}$, $T_1(k_1,k_2) \geq T_2(k_1,k_2)$ where the order is the standard component-wise order.*

Furthermore, by the statement of the proposition, we know that (k_1^*, k_2^*) is a fixed point of T_2 and (k_1', k_2') is a fixed point of T_1. Finally, T_1 is a weakly decreasing mapping, i.e., for any $(k_1, k_2) \geq (l_1, l_2)$, we have $T_1(l_1, l_2) \geq T_1(k_1, k_2)$.

Suppose now that $(k_1^*, k_2^*) > (k_1', k_2')$. Because T_1 is weakly higher than T_2, we have

$$T_1(k_1^*, k_2^*) \geq T_2(k_1^*, k_2^*) = (k_1^*, k_2^*) > (k_1', k_2') = T_1(k_1', k_2').$$

But, because T_1 is weakly decreasing, we have $T_1(k_1', k_2') \geq T_1(k_1^*, k_2^*)$, a contradiction. □

PROOF OF PROPOSITION 7: Clearly, if v is a node to saturate the connected component V' then for either firm, any strategy S in which any node of V' is saturated is weakly dominated by the strategy $S' = S \setminus V' \cup \{v\}$. The rest of the claims are implied by the structure of the competition. □

PROOF OF PROPOSITION 8: W.l.o.g., we show a weakly profitable deviation for Firm 1. Let the targeted set S_1^* be

$$S_1^* = \arg \max_{|S_1|=k_1} \frac{\sum\limits_{|S_2|=k_2} W_1(S_1, S_2, p)}{\binom{n}{k_2}}.$$

Then for the surplus from deterministically targeting S_1^* versus the random targeting of k_2 nodes, here denoted by $R_1''(S_1^*, k_2, p)$, we have

$$R_1''(S_1^*, k_2, p) = \max_{|S_1|=k_1} \frac{\sum\limits_{|S_2|=k_2} W_1(S_1, S_2, p)}{\binom{n}{k_2}} \geq \frac{\sum\limits_{|S_1|=k_1} \sum\limits_{|S_2|=k_2} W_1(S_1, S_2, p)}{\binom{n}{k_1} \cdot \binom{n}{k_2}} = R_1(k_1, k_2, p),$$

where the inequality describes the relationship of the maximum versus the average surplus for the targeted sets of nodes having sizes k_1 and k_2, respectively. This concludes the proof.

Note that except for a few trivial examples (where k_1 is either zero or leads to a complete saturation of the network), the above inequality is typically sharp, resulting in a profitable deviation for the firm choosing to target consumers deterministically. □

Profits of the two competing firms ($\Pi_1^D(\mathbf{k_1},\mathbf{k_2},\mathbf{p},\mathbf{c})$, $\Pi_2^D(\mathbf{k_1},\mathbf{k_2},\mathbf{p},\mathbf{c})$) on the networks of Figure 2 at $\mathbf{p}=\mathbf{0.1}$ and $\mathbf{c}=\mathbf{0.3}$.							
Sent messages by Firm 2 ($\mathbf{k_2}$)	0	1	2	3	4	5	6
Sent messages by Firm 1 ($\mathbf{k_1}$)				**Network *a***			
0	0.00, 0.00	0.00, 0.70	0.00, 1.40	0.00, 2.10	0.00, 2.80	**0.00**, 3.50	**0.00**, **4.20**
1	0.70, 0.00	0.55, 0.55	0.40, 1.10	0.25, 1.65	0.10, 2.20	-0.05, 2.75	-0.20, **3.30**
2	1.40, 0.00	1.10, 0.40	0.80, 0.80	0.50, 1.20	0.20, 1.60	-0.10, 2.00	-0.40, **2.40**
3	2.10, 0.00	1.65, 0.25	1.20, 0.50	0.75, 0.75	0.30, 1.00	-0.15, 1.25	-0.60, **1.50**
4	2.80, 0.00	2.20, 0.10	1.60, 0.20	1.00, 0.30	0.40, 0.40	-0.20, 0.50	-0.80, **0.60**
5	3.50, **0.00**	2.75, -0.05	2.00, -0.10	1.25, -0.15	0.50, -0.20	-0.25, -0.25	-1.00, -0.30
6	**4.20**, **0.00**	**3.30**, -0.20	**2.40**, -0.40	**1.50**, -0.60	**0.60**, -0.80	-0.30, -1.00	-1.20, -1.20
Sent messages by Firm 1 ($\mathbf{k_1}$)				**Network *b***			
0	0.00, 0.00	0.00, 2.37	0.00, 3.40	0.00, 4.10	0.00, 4.47	**0.00**, **4.50**	**0.00**, 4.20
1	2.37, 0.00	0.97, 0.97	0.57, 1.60	0.27, 2.00	**0.07**, **2.17**	-0.03, 2.10	-0.03, 1.80
2	3.40, 0.00	1.60, 0.57	0.88, 0.88	**0.34**, 1.04	-0.02, **1.05**	-0.20, 0.90	-0.20, 0.60
3	4.10, 0.00	2.00, 0.27	1.04, **0.34**	0.32, 0.32	-0.16, 0.21	-0.40, 0.00	-0.40, -0.30
4	4.47, 0.00	**2.17**, **0.07**	**1.05**, -0.02	0.21, -0.16	-0.35, -0.35	-0.63, -0.60	-0.63, -0.90
5	**4.50**, **0.00**	2.10, -0.03	0.90, -0.20	0.00, -0.40	-0.60, -0.63	-0.90, -0.90	-0.90, -1.20
6	4.20, **0.00**	1.80, -0.03	0.60, -0.20	-0.30, -0.40	-0.90, -0.63	-1.20, -0.90	-1.20, -1.20
Sent messages by Firm 1 ($\mathbf{k_1}$)				**Network *c***			
0	0.00,0.00	0.00,2.37	0.00,3.73	0.00,4.40	0.00,**4.60**	**0.00**,4.50	**0.00**,4.20
1	2.37,0.00	1.22,1.22	0.55,1.91	0.19,2.23	**0.03**,**2.26**	-0.03,2.10	-0.03,1.80
2	3.73,0.00	1.91,0.55	0.83,0.83	**0.23**,**0.90**	-0.06,0.81	-0.17,0.60	-0.17,0.30
3	4.40,0.00	2.23,0.19	**0.90**,**0.23**	0.15,0.15	-0.23,-0.03	-0.37,-0.27	-0.37,-0.57
4	**4.60**,0.00	**2.26**,**0.03**	0.81,-0.06	-0.03,-0.23	-0.45,-0.45	-0.62,-0.72	-0.62,-1.02
5	4.50,**0.00**	2.10,-0.03	0.60,-0.17	-0.27,-0.37	-0.72,-0.62	-0.90,-0.90	-0.90,-1.20
6	4.20,**0.00**	1.80,-0.03	0.30,-0.17	-0.57,-0.37	-1.02,-0.62	-1.20,-0.90	-1.20,-1.20
Sent messages by Firm 1 ($\mathbf{k_1}$)				**Network *d***			
0	0.00, 0.00	0.00, 4.03	0.00, **5.00**	0.00, **5.00**	0.00, 4.80	0.00, 4.50	0.00, 4.20
1	4.03, 0.00	1.08, 1.08	**0.31**, **1.28**	**0.18**, 1.15	**0.13**, 0.90	**0.13**, 0.60	**0.13**, 0.30
2	**5.00**, 0.00	**1.28**, **0.31**	0.25, 0.25	0.03, 0.03	-0.04, -0.24	-0.04, -0.54	-0.04, -0.84
3	**5.00**, 0.00	1.15, **0.18**	0.03, 0.03	-0.22, -0.22	-0.31, -0.51	-0.31, -0.81	-0.31, -1.11
4	4.80, 0.00	0.90, **0.13**	-0.24, -0.04	-0.51, -0.31	-0.60, -0.60	-0.60, -0.90	-0.60, -1.20
5	4.50, 0.00	0.60, **0.13**	-0.54, -0.04	-0.81, -0.31	-0.90, -0.60	-0.90, -0.90	-0.90, -1.20
6	4.20, 0.00	0.30, **0.13**	-0.84, -0.04	-1.11, -0.31	-1.20, -0.60	-1.20, -0.90	-1.20, -1.20

Table 3: Payoff matrices of the four networks displayed on Figure 2 in a simulatenous-move duopoly competition where $p = 0.1$ and $c = 0.3$,