Prominent Attributes

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Abstract

Evidence shows that marketers try to make certain attributes "prominent" by influencing which attribute consumers evaluate during decision making. This research asks: How do competitive firms decide which of product attribute to make prominent? We develop a model in which competitive firms price products with two attributes (e.g. styling and performance) after selectively promoting one of them as prominent. The new feature in our model is that firms' strategies regarding attribute prominence affect consumers' context-dependent preferences at the category level and subsequently their evaluation of all products. We find when consumers have limited attention and evaluate multiple attributes, perceived differentiation within an attribute can become diluted, an effect we call the *dilution effect*. This implies that competing symmetric firms may make the same attribute prominent in equilibrium to maintain product differentiation. Only if there is sufficient quality advantages in an attribute do we find equilibria with firms making different attributes prominent. When firms can invest in quality advantages, they may make different attributes prominent to avoid head-to-head competition on quality.

Keywords: *Prominent Attributes, Limited Consumer Attention, Dilution Effect, Context-dependent Preferences, Competitive Strategies, Game Theory.*

1. Introduction

It is common practice in advertising, branding and sales, for a marketer to emphasize only a few of its product's attribute to consumers. For example, automobile manufacturers often only emphasize either performance or styling. Deciding which attribute to pitch to consumers, by making it *prominent*, may not be easy in a competitive context. To illustrate, consider a consumer shopping for a new car. Without much expertise in car buying, he visits the dealer of Brand X who tells him that when buying a car he should pay attention to a car's performance (e.g. quick and responsive versus smooth & comfortable). After evaluating several competing brands he declares that Brand Y is his favorite because he finds it the quickest and most responsive. Afterwards, he sees advertisements for Brand Y, which emphasize the styling of the car – an attribute he had not paid much attention to before now. With part of his attention now drawn to styling, he reconsiders his preferences and realizes that he likes Brand X's styling better than Brand Y's. Before he saw Brand Y's advertisement, his choice was clearly Brand Y. Now, overall, he is relatively indifferent between X and Y, and products now appear *less* differentiated than when he only considered performance.

The above example is a snapshot of consumer decision making in an increasingly crowded product space. Despite the growth in the number of alternatives facing consumers, the amount of time they spend on researching and evaluating products has remained constant, at around 30 minutes per day over the past 50 years (Ott 2011). As a result, consumers may not know about or have time to evaluate all attributes. And, the more attributes a consumer evaluates, the less attention he pays to any particular one. Adding to the marketer's challenge is the fact that bandwidth for communicating with consumers is limited. For instance, most advertisements are limited to 30 seconds, a single page, or a banner. Bandwidth constraints limit the marketer's ability to describe all of its products' attributes. These challenges imply that a marketer must carefully select a few of the product's attributes (for instance, style or performance) to emphasize in the hope of generating sales.

We are not the first to consider the notion of attribute prominence. The consumer psychology literature, most notably, has explored the mechanism by which attribute prominence affects the consumer decision process. That research provides experimental evidence that the manner in which a product is presented has an impact on *how* a consumer evaluates alternatives (e.g. Payne, Bettman, & Johnson 1993). Specifically, the "prominence hypothesis" (Tversky, Sattath, & Slovic 1988) implies that a consumer, who has seen an advertisement emphasizing a certain product attribute, tends to consider that attribute more important when evaluating available alternatives (Wright & Rip 1980, Gardner 1983, and MacKenzie 1986). If a consumer considers only one seller, then deciding which attribute to make prominent is simply a matter of knowing which attribute will most accentuate the attractiveness of that product. This decision may not be so simple, however, in a competitive context because (i) its decision affects consumer's preferences for all products, including rivals' and (ii) each firm can non-cooperatively try to influence the consumer's attention to the same attribute or distinct attributes. Our objective is to study the strategic interactions among competing marketers in their decisions about which attribute to make prominent.

The opening scenario also conveys the fundamental mechanism behind our findings: When consumers have limited attention and evaluate multiple attributes, perceived differentiation within an attribute can become diluted, an effect we call the *dilution effect*. This effect is fundamentally linked to limitations in consumer's attention. When consumers have limited attention, they may not always evaluate and compare alternatives on all attributes (Russo

& Dosher 1983, Bettman, Luce, & Payne, 1998). For new or innovative products (e.g. computer) or complex purchases (e.g. a car) consumers might not even be initially aware of all the relevant attributes. Furthermore, the attributes the consumer pays attention to can be affected by the market environment (Tversky et al. 1988, Shavitt & Fazio 1991), which is influenced by marketers (Gardner 1983, MacKenzie 1986, and Jiang & Punj 2010). As mentioned above, marketers may try to direct a consumer's attention to a specific attribute through advertisements, packaging, or branding. If firms emphasize the same attribute, then a consumer evaluates products only on that attribute. If, however, firms emphasize different attributes, limited attention implies the consumer splits her attention across multiple attributes. In this latter case, a consumer's attention is diluted. As we show, this effect has implications for how a consumer compares available alternatives, which affects the competitive intensity among firms. Consequently, firms will make strategic choices on the prominent attributes to avoid heavier competition.

The principle of product differentiation might suggest that competing firms should always emphasize the distinct attribute in which they excel. This classic notion can be used to explain, for example, an observation in the airline industry. Southwest Airlines consistently promotes the attribute of convenience by emphasizing the simplicity of its services. Virgin America, in contrast, touts the attribute of style, highlighting its chic and sophisticated on-board atmosphere. But there are instances where competing sellers emphasize the same attribute, which seems to be at odds with this principle. Advertising for competing car brands Lexus and Mercedes both highlight the attribute of styling or image. Perhaps more interestingly, advertising for the Apple iPhone 5 and for the Samsung Galaxy S4 both highlighted the photo-taking attribute of their phones despite the fact that the iPhone camera was arguably not as resolute as that of Galaxy (8mp versus 13mp). The central question we ask in this research is *under what market conditions do competing firms emphasize the same attribute, or emphasize distinct attributes, when consumers' limited attention leads to dilution effect*. We are also interested in knowing whether an exogenous quality advantage in one attribute means that a firm should always make it prominent. Finally, we ask how firms' investments in the quality of its attributes interacts with the choice of prominent attributes, under the dilution effect.

We show that limited attention drives firms to emphasize the same attribute. In fact, with limited attention, emphasizing the different attributes can actually decrease consumer's perceived product differentiation, due to the dilution effect. Even if one firm has a quality advantage in one attribute, it may be better to emphasize the same attribute with its competitor to avoid the risk of reducing perceived product differentiation. Only if a firm's quality advantage in a particular attribute is sufficient, does it want to emphasize a different attribute than its competitor.

These results arise from a model in which competitive firms selectively choose an attribute to make prominent. We define an *attribute* as any aspect of a product category that the consumer regards as relevant for determining the level of utility. An attribute need not be a physical feature of the products in that category. Rather, we assume it is a mutually accepted component of the consumer's utility. For example, a consumer may assess the utility of cars based on the attribute of appearance, or driving performance, or both attributes. We further assume that any attribute has a horizontal component. For example, the driving performance of a car has idiosyncratic aspects (e.g. sporty) that some consumers like while others do not. Furthermore, an attribute may have a vertical component that all consumers value in the same way. Continuing with the car example, all consumers prefer that the performance is safe and reliable.

There are two firms, each selling many products, say cars, all of which have two attributes (e.g. performance and appearance). We assume each firm announces exactly one attribute as its prominent attribute. For example, firms will highlight their chosen attribute via advertising, promotion materials, or packaging. Consumers evaluate products by examining random fit parameters for the prominent attributes. We first study the case of symmetric firms when attributes are only horizontally differentiated. In this case, firms choose the same attribute to make prominent in equilibrium. If firms emphasize the same attribute, they ensure that consumers regard that single attribute as the most important one. Given her idiosyncratic tastes on that attribute, the consumer purchases the product that is most satisfying on that attribute. However, if firms emphasize different attributes, a consumer evaluates products on multiple attributes. If the consumer's attention is limited, then she is forced to split her attention across multiple attributes, which attenuates her relative appreciation of any single attribute. With significant likelihood, the consumer will find that a firm who dominates on one attribute will dominate to a lesser degree on both. That is, consumers dilute their attention on any single attribute when evaluating products on multiple attributes. On average, the consumer will view the products as relatively less differentiated. Under the dilution effect, firms must price more competitively.

We then consider the case in which each firm has a quality advantage in exactly one attribute. In this case, firms will make the same attribute prominent in equilibrium if the quality advantage is not too large. That is, one of the firms actually makes its inferior attribute prominent in order to avoid the dilution effect. Only if the quality advantage is sufficiently large will firms benefit from making their best attribute prominent. This situation is considerably different when one firm has the quality advantage in both of its attributes. The better firm wants consumers to evaluate both attributes in order to capture the true differential value from consumers whereas the worse firm prefers that consumers evaluate on a single attribute to avoid the dilution effect. These differing incentives induce a mixed-strategy equilibrium in which the better firm randomly picks an attribute to be prominent in order to keep the other firm from selecting the same attribute. We call such an outcome a "cat & mouse" equilibrium to reflect the idea that the low quality firm is trying to catch the better firm's choice of attribute.

For the above results, we assume that firms decide about prominent attributes for exogenous quality advantages. But, in practice, firms may want to build a quality advantage in an attribute after establishing it as prominent. For instance, the Volvo brand of automobiles, having known for its safe performance, continues to invest in safety and maintain its competitive advantage in performance. Alternatively, as players in the grocery industry have continually highlighted the organic attribute of their produce selections, Whole Foods Markets has reconsidered its investment in organic farmers.[1] In light of the interplay between quality investment and prominent attributes, we extend the model to the case of endogenous quality. We show that this has profound implications on the role of the dilution effect.

With exogenous quality differences, the dilution effect is an incentive for firms to compete on the same attribute – that is both firms making the same attribute prominent. When there is an opportunity to invest in the quality of an attribute, however, making the same attribute prominent creates a prisoner's dilemma. Each firm is tempted to invest in the quality of its prominent attribute in order to gain market share. But, if both firms invest, any quality advantage is competed away. Consequently, firms may prefer to strategically make different attributes

[1] See "Organic Farmers Object to Whole Foods Rating System", *NY Times*, June 12, 2015.

prominent in order to avoid head-to-head competition on quality. As we show, the dilution effect may actually help firms avoid competition on quality.

Our work falls in a recent stream of research in marketing and economics that explores situations in which consumers can make choices with limited information about a product's fit. For instance, Branco, Sun, & Villas-Boas (2012), Bar-Issac, Caruana, & Cuñat (2012a), Bang & Kim (2013), Ke, Shen, & Villas-Boas (2015), and Dukes & Liu (2015) consider consumers who may purchase a product without knowing the full extent of fit. Much of this prior work is focused on the information gathering process of the consumer and the consumer's search trade-offs. We do not study consumer's active search process, but rather try to understand the firms' decisions when they can direct consumer's attention.[2] In our setting, consumers are possibly limited in their evaluation because their limited attention leads firms to strategically make the same attribute prominent, and limitations in firm's ability to communicate all product attributes.

There is a number of works that study firms' advertising content and communication strategies. For instance, Anderson & Renault (2006), Johnson & Myatt (2006), Bar-Isaac, Caruana, & Cuñat (2012b) and Branco, Sun, & Villas-Boas (2015) examine a firm's decision about how much "match" information to reveal to potential customers. That prior work considers content for consumers that is specifically related to the product(s) it sells. Our paper, in contrast, considers content that relates to *all* products in the category, which is not firm-specific. This implies that a firm's advertising strategy affects how consumers evaluate competitive firms as well as its own.[3] None of the above works consider firms' communication strategies under bandwidth constraints. Like our paper, Bhardwaj, Chen, & Godes (2008) and Mayzlin & Shin

[2] Armstrong, Vickers, and Zhou (2009) applies the notion of "prominence" to a classic model of search in which one firm (exogenously determined) receives more attention than other firms.

[3] In this way, our paper connects to Anderson & Renault (2009), which studies a model of comparative advertising in which a firm can reveal to consumers horizontal match information about competitive products. Advertising in our model can be interpreted as revealing partial match information about both products.

(2011) start with the premise that a firm cannot communicate all aspects about its product to consumers, as is typically assumed in models of informative advertising (Butters 1977, Grossman & Shapiro 1984). Bhardwaj et al. (2008) compares "seller-initiated" versus "buyer-initiated" information revelation regarding which subset of attributes the buyer can learn about. Mayzlin & Shin (2011) study a firm's decision about which, if any, product attribute to present to consumers. In both papers, there is asymmetric information about the product's overall quality and the firm's decision about what information to reveal is governed by its impact on consumer beliefs about quality (via signaling). In contrast, in our work there is no uncertainty about quality and the firm's decision about which attribute to announce is determined by competitive interactions when consumer's attention is limited.

Finally, this paper connects to two broad literatures devoted to consumers' context-dependent preferences. First, as noted above, the consumer psychology literature has long suggested that consumers tend to put more attention on an attribute when that attribute is emphasized in a choice context (Tversky et al. 1988 and Shavitt and Fazio 1991), which can be affected by the marketer (Wright & Rip 1980, Gardner 1983, and MacKenzie 1986). A more recent stream of work in economics formalizes the relationship between attention and choice when consumers have context dependent preferences (Kőszegi and Szeidl 2013, Bordolo, Gennaioli, and Shleifer 2013). None of this work, however, formally considers the equilibrium interactions with firms when consumers' limited attention is directed by marketing. Second, there is a literature from economics and from marketing that examines the firm-side implications of context-dependent preferences in an equilibrium framework (Wernerfelt 1995, Kamenica 2008, and Guo & Zhang 2012). Our work, unlike all of the above mentioned research, has two important distinctions. First we concentrate on the case of competitive firms, each of whom contributes to the consumer's choice context. A firm needs to take into account the other firm's attribute strategy when deciding the optimal prominent attribute, even though it maybe suboptimal in monopoly. Second, we model a consumer's context-dependent preferences at the product category level instead of individual products. Specifically, when one firm makes an attribute prominent, the attribute is prominent for the consumer on all products in that category. This implies interesting strategic interactions, which has not yet been examined to the best of our knowledge.

The remainder of the paper is structured as follows. In the next section, we lay out the basic framework of consumer preferences under limited attention and the corresponding purchase rule. Then, in section 3, we examine a number of scenarios to understand the choices regarding which attribute firms make prominent in equilibrium. Next in section 4 we extend our model to allow firms to endogenously decide their quality. Section 5 concludes with an overview and discussion of future research. Proofs of technical results are all relegated to the Appendix.

2. The Model

There are two firms, indexed as $j = 1,2$, each of which produces the same number of *n* product variants with the same price p_j. All products are defined by the same two attributes $k = A, B$. Each attribute has a firm-specific quality component q_{jk}. Any firm can have a quality advantage in one or both attributes. For simplicity we assume that there exists *only* two quality levels in each attribute that firms may produce: low quality q^L and high quality $q^H > q^L$. We also normalize the marginal production cost for both products to be zero. The mass of consumers is normalized to one and each consumer has unity demand. We assume that the quality level is exogenously given in section 3, to allow us to abstract away from the firms' quality decision. In section 4, we expand our analysis of prominent attributes when firms make endogenous quality decision.

The timing of the model is as follows: first, each firm decides its prominent attribute. Second, after observing each other's prominent attribute choice, firms choose their product prices. Third, consumers form their preferences on prominent attribute(s), which are determined by firm's announcement strategies. Finally, consumers evaluate products and purchase one product from a firm.

2.1 Consumer Utility from Evaluation

Each consumer has an idiosyncratic match value for each attribute of each product. If the consumer knows of one prominent attribute she evaluates all products on that one attribute only. If both attributes are prominent, the consumer evaluates all products on both. In this way, we do not explicitly model the consumer's search process or the decision on which attributes or products to evaluate, which allows us to exclusively focus on firms' competitive strategies.

Suppose a consumer has evaluated only attribute k from all products. Her utility of product *i* from firm *j* based on inspecting attribute *k* is given by,

$$u_{ijk} = V - p_j + \theta_k(q_{jk} + \nu_k \varepsilon_{ijk}), \tag{1}$$

where *V* is the intrinsic value of the product, q_{jk} is the quality of firm *j* in attribute *k*, and $\varepsilon_{ijk} \sim N(0,1)$, which captures randomness in consumers' tastes among products on attribute k. The parameter $\varepsilon_{ijk} \sim N(0,1)$ is often referred to as the fit or match value that the consumer draws for product i from firm j on attribute k. We assume the distributions of random match value from two attributes are independent for simplicity. However, allowing correlation between two distributions would not change the model results, and may strengthen our results in some cases. The parameter ν_k captures the degree of heterogeneity of consumer tastes in attribute k. The parameters V, p_j, q_{jk}, and ν_k are common to all consumers whereas the component ε_{ijk} is idiosyncratic to the individual consumer. The factor θ_k is the weight parameter for attribute k depending on firms' prominent attribute decisions.

When consumers evaluate both attributes, we assume the joint utility is additive on the attribute-dependent utility components. In particular, for a consumer evaluating product i on both attributes, her utility is given by

$$u_{ijAB} = \left(V - p_j\right) + \theta_A q_{jA} + \theta_B q_{jB} + \nu_{AB}\varepsilon_{ijAB}, \tag{2}$$

where $\nu_{AB}\varepsilon_{ijAB} = \theta_A \nu_A \varepsilon_{ijA} + \theta_B \nu_B \varepsilon_{ijB}$. Since $\nu_k \varepsilon_{ijk} \sim N(0, \nu_k^2)$, $k = A, B$ and independent, the random fit term $\nu_{AB}\varepsilon_{ijAB} \sim N(0, \nu_{AB}^2)$, where $\varepsilon_{ijAB} \sim N(0,1)$ and $\nu_{AB} = \sqrt{\theta_A^2 \nu_A^2 + \theta_B^2 \nu_B^2}$.

We now discuss the influence of firms' decisions on the value of the weight parameters, θ_A and θ_B. These weight parameters reflect the amount of attention the consumer pays to an attribute. Our approach to modeling consumer's attention is akin to Köszegi and Szeidl (2013) and Bordolo et al. (2013). Consumers' total attention does not change with firms' announcements, which without loss of generality, we normalize the total attention to be 1. We also assume the weight parameter is completely decided by firms' prominent attribute strategies. We make this assumption to capture the influence of firms' marketing activities on consumers' preferences on attributes, which is the key focus of this paper. If both firms make attribute k prominent, consumers treat that attribute as the only important attribute and will devote all their attention to it. Hence in this case, the weight parameter $\theta_k = 1$. On the other hand, when firms announce different attributes, then consumers treat both attributes as equally important and split their attention equally across the two attributes. Therefore, $\theta_A = \theta_B = \frac{1}{2}$: less attention leads to lower appreciation of any single attribute. The symmetric split of attention may, at first, seem to be a restrictive assumption. For instance, consumers may have asymmetric preferences over the two attributes, putting more importance on one over the other. Our main result holds, however, as long as consumers do not fully ignore an attribute and correspondingly allocate some positive attention to both attributes ($0 < \theta_A = 1 - \theta_B < 1$). Therefore, we assume equal weight on each attribute changes for simplicity and without loss of generality.[4]

There is no incomplete information in product evaluation and consumers learn the available match values at no cost. However, the weight parameter, θ is endogenously decided by firms' strategies in choosing prominent attributes, which is the key feature of the context-dependent preferences in this model.

2.2 Consumer Purchase Decisions and Firms' Market Share

The consumer must only decide which product to purchase depending on whether she's evaluated one or two of the products' attributes. When the consumer evaluates only one attribute, say k, then the utility of buying from firm j is then given by

$$u_{jk} = \max\{u_{ijk}\} = V + q_{jk} - p_j + \nu_k \max\{\varepsilon_{ijk}\},$$

where the maximization is over the firm's products $i = 1, \dots, n$. Extreme-Value Theory allows us to conveniently write the purchase probability for firm j when n is large. Specifically, the Fisher-Tippet-Gnedenko theorem (Embrechts et al 1997, Hann & Ferreira 2005) implies that the distribution of $\varepsilon_{jk} = \max\{\varepsilon_{ijk}\}$ approximates the type-I extreme value distribution for n large enough. This permits us to formulate the purchase probability for each firm as a discrete choice logit demand[5] (Anderson et al. 1992), which is derived in the following lemma.

[4] In this model we capture the effect of firms' announcements on consumer preferences on category level. It is also possible that a firm's announcement contains a persuasive element, which may increase the consumer's valuation. This implies that firms will never choose "No announcement" in our framework. However, in a symmetric situation, the competitive effect of persuasion will cancel out and would not affect consumers' decisions in equilibrium.

[5] In the current model we assume the market is fully covered by two firms and there is no outside options. Our results are quantitatively unchanged if consumers have an outside option.

Lemma 1 *If consumers evaluate only attribute k, then the probability of purchasing from firm* $j = 1{,}2$ *is*

$$s_{1k} = 1 - s_{2k} \approx \frac{e^{(q_{1k}-p_1)/\mu_k}}{e^{(q_{1k}-p_1)/\mu_k} + e^{(q_{2k}-p_2)/\mu_k}},$$

where $\mu_k = \nu_k \frac{1}{n\Phi'(a_n)}$, $a_n = \Phi^{-1}(1-\frac{1}{n})$, *and* Φ *is the (standard Normal) cdf for* ε_{ijk}.

If consumers evaluate both attributes A and B, then the probability of purchasing from firm $j = 1{,}2$ *is*

$$s_{1AB} = 1 - s_{2AB} \approx \frac{e^{(\theta_A q_{1A}+\theta_B q_{1B}-p_1)/\mu_{AB}}}{e^{(\theta_A q_{1A}+\theta_B q_{1B}-p_1)/\mu_{AB}} + e^{(\theta_A q_{1A}+\theta_B q_{1B}-p_2)/\mu_{AB}}},$$

where $\mu_{AB} = \frac{1}{2}\sqrt{\mu_A^2 + \mu_B^2}$.

Since there is no closed form solution of a choice model when the match value is normally distributed, Lemma 1 is a useful technical result. Roughly speaking, because we are considering the consumer's optimized choice from a given firm, the relevant distribution is that of the maximized realization, which approximates the type-I, extreme-value, distribution for many well-behaved distributions of $\nu_k\varepsilon_{ijk}$. The purchase probabilities expressed in Lemma 1 exhibit the usual properties of the logit demand. In particular, greater variance in the consumer's maximized value of $\nu_k\varepsilon_{jk}$, as measured by μ_k and proportional to ν_k, implies lower sensitivity to differences in quality and price. Because the variance μ_k is directly proportional to ν_k, we refer only to the parameter μ_k when discussing the degree of horizontal differentiation within attribute *k*. With closed-form expressions for choice probabilities at each firm, we can find the equilibrium of the game played by the two firms.

3. The Prominent Attribute Decision in Equilibrium

In this section, we solve for the prominent attribute decisions of firms in equilibrium.[6] We first assume $\mu_A = \mu_B = \hat{\mu}$, for symmetry.[7] Let $\Delta_k \equiv q_{1k} - q_{2k}$ be the quality advantage for firm 1 in attribute $k = A, B$. We start with the benchmark case in which is no quality differentiation among firms in any attribute, $\Delta_k = 0$. In this case, as demonstrated in Proposition 1 firms make the same attribute prominent in equilibrium. This is a helpful benchmark because it isolates the key mechanism in all of our results regarding firms' prominent attributes decisions.

> **Proposition 1** *If* $\Delta_k = 0$, $\forall k$, *firms choose the same attribute to make prominent in equilibrium.*

To see why announcing the same attribute is an equilibrium, consider a deviation by one firm to announce the other attribute. Doing so implies that the consumer evaluates both products on both attributes. Because a consumer's attention is fixed, she splits her attention across both attributes and therefore places less weight on any particular attribute. The variance of the consumer's random match value is reduced as a result: $\left(\mu_{AB}\varepsilon_{ijAB}\right) = \frac{\hat{\mu}^2}{2} < \hat{\mu}^2 = var\left(\mu\varepsilon_{ijk}\right)$. We call this the *dilution effect*: on average, a reduction in the variance of the match value dilutes perceived firm differentiation by the consumer. To see this another way, suppose a consumer evaluates only attribute *A* and prefers firm 1 ($\max\{u_{i1A}\} > \max\{u_{i2A}\}$). Then, it is likely that the

[6] Because of our interest in competitive interactions in the prominence decision, we do not present the case of monopoly. The monopolist's problem regarding attribute prominence can be mapped to the modeling framework of Johnson & Myatt (2006). Formal details are available from the authors by request.

[7] We also study the case of asymmetrically differentiated attributes ($\mu_A \neq \mu_B$) but do not include the analysis here. It is available upon request.

strength of her preference for firm 1 will shrink when consumers evaluate both attributes.[8] The magnitude of the dilution effect is proportional to $\hat{\mu}^2$. The dilution effect, therefore, has a downward impact on equilibrium prices.[9] Thus, it is not profitable for a firm to announce an attribute different than its rival.

In the remainder of this section we present the main results pertaining to prominent attribute decisions with exogenous qualities. We first examine the case of symmetric quality advantages in which each firm has an advantage in one attribute. Finally, we study the case in which one firm has quality advantage on both attributes.

3.1 Firms with Symmetric Quality Advantage

In this section, we extend the above setting and consider firms with a quality advantage in one attribute. Without loss of generality, assume $q_{1A} = q_{2B} = q^H$, and $q_{1B} = q_{2A} = q^L$, with $q^H > q^L$ so that firm 1 has the quality advantage on attribute *A* and firm 2 on attribute *B*. Lemma 2 shows that if only one attribute is prominent, then firms have opposing preferences for which one it is.

Lemma 2: *Let* $\Delta \equiv \Delta_A = -\Delta_B = q^H - q^L > 0$. *If both firms make attribute A(B) prominent, then firm 1(2)'s corresponding equilibrium price, market share, and profit are higher than firm 2(1).*

When firms announce the same prominent attribute, consumers evaluate products on only that attribute. Therefore, one firm appears to have an absolute quality advantage over the other.

[8] Mathematically, $E[u_{1AB} - u_{2AB} | u_{1A} > u_{2A}] < E[u_{1A} - u_{2A} | u_{1A} > u_{2A}]$.
[9] If consumers' attention is unlimited, it is possible that consumers do not put less weight on each attribute when firms announce different prominent attribute. In this case, the dilution effect does not exist and firms always prefer different attributes.

Naturally, the better quality firm enjoys a larger market share and a higher price. Lemma 2 does not say, however, whether such a quality advantage is sufficient to overturn the mutual benefit of avoiding the dilution effect. Suppose, for example, firm 1 announces attribute *A*. Is it better for firm 2 to announce attribute *B* in order to neutralize the quality advantage? Or should firm 2 announce attribute *A* to maintain the dilution effect? Proposition 2 shows that the answer depends on the relative strength of the quality advantage.

Proposition 2 *There exists a threshold* $\overline{\Delta} > 0$ *such that if* $\Delta < \overline{\Delta}$*, then both firms make the same prominent attribute in equilibrium; otherwise they make different attributes prominent.*

The main intuition of Proposition 2 ($\Delta > 0$) extends that of Proposition 1 ($\Delta = 0$). Regardless of the size of Δ, announcing different attributes dilutes consumer attention into two attributes. If $\Delta < \overline{\Delta}$ is small, therefore, the quality differentiation is not large enough to offset advantages in quality. That is, even though a firm 2 with, say, a quality advantage in attribute *B*, it may prefer to join firm 1 in emphasizing attribute *A* to avoid the dilution effect. Only when $\Delta > \overline{\Delta}$ is large, does firm 2 prefer to accentuate his quality by announcing attribute *B*.

3.2 Firms with Asymmetric Quality Advantages

In this section, we consider the case when one firm has a uniform quality advantage in both attributes. It is first helpful to prove a general result regarding the shape of firms' profits as a function of the degree of horizontal differentiation when firms differ in their relative qualities. Lemma 3 indicates that a firm who is perceived as high quality does not always benefit from greater horizontal differentiation.

Lemma 3 *Suppose that attribute* $k \in \{A, B\}$ *is prominent and that firm 1 has a quality advantage in that attribute:* $\Delta = \Delta_k > 0$ *for some or both* $k \in \{A, B\}$ *. There exists a unique threshold* $\overline{\mu}_\Delta > 0$ *such that the high quality firm's (1's) profit decreases with* μ *when* $\mu < \bar{\mu}_\Delta$ *and increases with* μ *when* $\mu > \bar{\mu}_\Delta$*. The low quality firm's (2's) profit always increases with* μ*.*

Lemma 3 applies to both the symmetric and asymmetric quality advantage cases. It says that, for the firm with the quality advantage in the prominent attribute(s) (the high quality firm), its profit is non-monotonic in μ (or μ_{AB}). While the low quality firm always benefits from a higher variance μ, the high quality firm's profit first decreases in μ and then increases, achieving its lowest at $\mu = \overline{\mu}_\Delta$. High variance increases the horizontal differentiation between two firms and therefore benefits the low quality firm unambiguously. However, if $\mu < \bar{\mu}_\Delta$, higher horizontal differentiation diminishes the high quality firm's quality advantage. To see this, consider a case when the variance $\mu \approx 0$ and Δ is high. Because there is no horizontal differentiation, the high quality firm can charge a price at $\Delta - \varepsilon$ and have all the consumers. But once the variance increases, the low quality firm gains demand due to the random match. The added demand at firm 2 corresponds to a loss in demand at firm 1. Thus, an increase in the variance may reduce the high quality firm's profit. Only if $\mu > \overline{\mu}_\Delta$, does the high quality firm gain from added horizontal differentiation. This logic further implies that the threshold, $\overline{\mu}_\Delta$, is increasing in Δ. Figure 1 graphically illustrates the firms' profit across different μ as derived in Lemma 3.

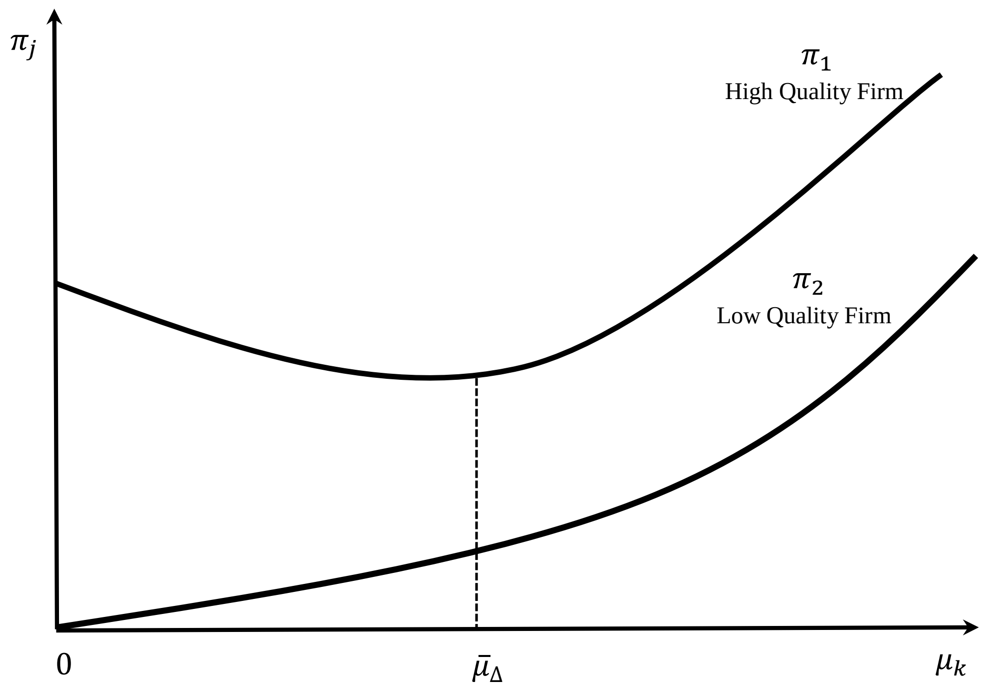

Figure 1: Firms' Profits as Functions of μ

Now assume that $q_{1A} = q_{1B} = q^H$, and $q_{2A} = q_{2B} = q^L$, so that the quality advantage for firm 1 is the same for both attributes *A* and *B*. Firm 2 can have no quality advantage, regardless of the announcement strategy. For the case of asymmetric quality advantage, $\Delta_A = \Delta_B > 0$, Lemma 3 implies that firm 2 prefers the outcome which has the most variation in the match value. In contrast, firm 1 prefers less variance whenever $\mu < \overline{\mu}_\Delta$. This divergence in preferences implies an equilibrium in mixed strategies. As before, both firms are subject to the dilution effect even when one firm has the overall quality advantage. Firm 2's best outcome, therefore, is for consumers to evaluate products on only a single attribute. Firm 1's trade-off, however, is different than firm 2's. As shown in Lemma 3, the advantaged firm's profit can actually decrease in μ, a property that implies a different equilibrium outcome than in the previous cases.

Proposition 3 *Let* $\Delta = \Delta_A = \Delta_B > 0$*, with the corresponding cutoff point* $\overline{\mu}_\Delta$*defined in Lemma 3.*

(a) *If* $\mu < \bar{\mu}_\Delta$*, then the high quality firm (1) wants to announce different attribute while the low quality firm (2) prefers to announce the same attribute. In equilibrium, firms play mixed strategies choosing each attribute with equal probabilities.*

(b) *If* $\mu > \bar{\mu}_\Delta$ *then both firm announce the same attribute.*

Announcing the same attribute is always good for the low quality firm since it increases the variation of the match value; on the other hand, as Lemma 3 indicates, higher variation in the match value may hurt the high quality firm's quality advantage. This leads to the first result. When the μ is large enough, the higher variation (jointly implemented by announcing the same attribute) can compensate for the loss of the quality differentiation. Thus, firms announce the same attribute.

4. Prominent Attribute Strategies under Endogenous Quality

In the previous section, we assumed quality differences were exogenously specified. However, as motivated in the introduction, firms may target certain attributes for quality improvements. This setting can capture situations in which positioning an attribute as prominent is harder to establish and modify relative to a quality improvement. In this section we study the decision of prominent attributes when firms can subsequently invest in quality. While the dilution effect served to intensify price competition, as we show in the case of endogenous quality, the dilution effect may help firms avoid intense competition on quality.

We consider a multi-stage game to accommodate the prominent attribute-quality decisions made by firms. In the first stage, firms simultaneously choose their prominent attributes. In the second stage, after both firms' choices of prominent attributes are known, firms' choose whether to invest in the quality of their prominent attribute. Firms then choose their product prices in stage three. Finally, consumers form their preferences and make their purchase decisions as in section 3. This multi-stage timing reflects settings in which firms make quality decisions after attributes have been well-established as prominent (e.g. the Volvo example of the introduction).

Consumer decisions are the same as the benchmark model given in section 2. Therefore, we only discuss the quality investment decision stage. Each firm is endowed with the same quality level, q^L, in all attributes, but they can spend a cost $c > 0$ in stage 2 to improve the product quality in one attribute to q^H. For simplicity, we assume the binary quality outcomes (H or L), but our results are quantitatively unchanged with continuous quality. Finally, we assume that it is too costly for firms to invest on both attributes so that each firm can only invest on one of them. This allows us to focus on the connection between the choice of the prominent attribute and the decision on quality investment.

We first derive the equilibrium quality decisions for each prominent attribute outcome and then the equilibrium decisions on prominent attributes. A key aspect of this model is that, regardless of the prominent attribute decision in stage one, a firm enjoys a quality advantage only if it invests in quality and the rival firm does not. If both firms invest in quality, then the advantage is competed away in prices. Hence, firms are always worse off when they both invest relative to when they both do not invest. Nevertheless, if investment costs are not too high, then a prisoner's dilemma ensues and investing in quality is a dominant strategy. We are particularly interested in situations in which firms can avoid a prisoner's dilemma by their choice of prominent attributes in stage one. Lemma 4 raises the possibility that this can happen if firms choose different attributes.

Lemma 4

(a) *If firms make the same attribute prominent, then there exists* $\overline{c} > 0$ *such both firms invest in quality when* $c < \bar{c}$.

(b) *If firms make the different attribute prominent, then there exists* $\underline{c} > 0$ *such that both firms do not invest in quality when* $c > \underline{c}$.

If firms make the same attribute prominent, then according to Lemma 4(a), both firms' dominant strategy is to invest in quality as long as c is not too large. Under this scenario firms overinvest on quality due to competitive pressure. If firms choose different attributes, so that consumers evaluate products on both attributes, then as long as c is sufficiently large, firms can avoid the prisoner's dilemma. Overall, Lemma 4 implies that firms may choose to emphasize different attributes as a commitment to avoid competition on quality, but only if $\underline{c} < \bar{c}$.

Lemma 5 *For any* $\mu > 0$, *there exists a threshold* $\widehat{\Delta}(\mu) > 0$ *such that if* $\Delta < \widehat{\Delta}(\mu)$, *then* $\underline{c} < \bar{c}$.

Lemma 5 establishes that when Δ is not too high, there exists a well-defined range of investment costs $c \in (\underline{c}, \overline{c})$ such that, if firms make the same attribute prominent, then they will invest in quality and if firms make different attributes prominent, they will not invest in quality.

For both firms to make different attributes prominent in equilibrium it must be mutually beneficial to avoid the prisoner's dilemma. Recall from Proposition 2 that the dilution effect is stronger than the quality advantage effect when Δ is not too large. As discussed in section 3, this was due to intensified price competition from a lower variance in $\mu_{AB}\varepsilon_{ij}$. Thus, firms would prefer to be in the prisoner's dilemma with quality competition over suffering from a severe dilution effect if $c < \left(1 - \frac{\sqrt{2}}{2}\right)\mu$.

Proposition 4 *When $\Delta < \widehat{\Delta}(\mu)$, firms make different attributes prominent for any $c \in \left(\max\left\{\underline{c}, \left(1 - \frac{\sqrt{2}}{2}\right)\hat{\mu}\right\}, \bar{c}\right)$.*

We are assured the conditions for Proposition 4 occur in our model by Lemmas 4 and 5. The proposition states that, under these conditions, firms announce different attributes in equilibrium. They do this despite and because of the adverse consequences of the dilution effect. While the dilution effect has a downward impact on profits due to more intense price competition, the dilution effect also lowers the incentive to invest in quality when Δ is not too large. This extension shows that, in contrast to Proposition 2, the dilution effect can enable firms from avoiding intense competition. That is, firms can dilute consumer attention by making both attributes prominent so that a unilateral investment in quality in one attribute does not bring a sufficient advantage to induce a prisoner's dilemma.[10]

[10] The dilution effect's impact on quality investment might be illustrated with an anecdote from the grocery industry. Whole Foods Markets (WFM) and conventional grocery stores had increasingly emphasized the attribute related to organic farming. As organic farmers have responded with higher prices, WFM recently introduced a rating system which ranks produce on a variety of "eco-sustainability" attributes, which does not emphasize organics. In view of our model, this helps WFM avoid the high cost of organically farmed foods as consumers' attention is diluted across multiple attributes.

5. Conclusion

The premise of this research is that firms face limitations when communicating to consumers: limited attention and limited bandwidth. These limitations imply that a firm cannot provide a full description of each of its product's attributes and that consumers do not necessarily devote all of their attention to all product attributes before purchase. Consequently, a consumer's preference over available products depends on which attributes marketers choose to highlight, or make prominent. In competitive scenarios, one firm's decision on a prominent attribute affects how a consumer evaluates other firms' products. Our model examined the corresponding strategic interactions between firms deciding which attribute to make prominent. The model points to several results.

First, symmetric firms, which are horizontally differentiated, choose the same attribute to make prominent. With limited consumer attention, firms jointly prefer consumers to devote their attention to few attributes. When consumers evaluate on multiple attributes they perceive less horizontal differentiation because they split their attention across the attributes – a notion we termed the dilution effect. This result was derived from a model of two firms selling products with two attributes. When each firm has a unique quality advantage in exactly one attribute, our model finds that firms may still emphasize the same single attribute. Only if the quality advantages are sufficiently large, will firms emphasize different attributes in equilibrium.

Next we considered the case in which one firm dominates in both attributes. In this case, the two firms' preferences over prominent strategies can diverge. The dominant firm prefers that consumers evaluate products on both attributes so as to accentuate its entire quality advantages. The other firm, in contrast, prefers that consumers consider only one attribute so as not be subject to the dilution effect. This leads to firms playing mixed strategies, with the dominant firm trying to make its choice of prominent attribute unpredictable for the other firm.

Finally, we considered the case in which firms could strategically invest in the quality of their prominent attribute. We showed that firms may face a prisoner's dilemma when investing in quality. That is, under certain circumstances, it is a dominant strategy for the firm to invest to acquire market share and pricing premiums. But when both firms invest, their quality advantages are competed away with prices. The dilution effect provides the right incentives for firms to avoid this prisoner's dilemma by making it relative more attractive to make different attributes prominent. By doing so, each firm does not want to invest in quality, thereby saving themselves from a war on quality.

Our research points to a few areas for future inquiry. Perhaps most obviously is to examine the question of how consumers optimally allocate their attention when it is limited. The notion that consumers pay differential attention across attributes when making a choice has only recently received attention in the economics literature (Kőszegi and Seidl 2013, Bordolo et al. 2013) but is otherwise mum on the manner in which consumers allocate their attention. Examining consumers' attention allocation problem could have important implications for marketers when communicating product information.

In reality consumers may not always be able to evaluate the product's quality before purchase, especially when consumers' attention is limited. In such situations, a firm's attribute choice not only discloses the importance of the attribute, but may also "signal" the quality of the product in this attribute.[11] Incorporating the signaling role of choosing a prominent attribute may lead to insightful findings, but is beyond the scope of this research.

[11] We thank an anonymous reviewer for pointing this out.

The literature on economics of search, while large, typically focuses on product evaluation at a holistic level and has only recently begun to examine the case in which consumers search at the attribute-level. None of this work, however, has studied the case in which attributes differ in terms of the differential attention that consumers pay to certain attributes. Thus, incorporating consumer's judicious decisions regarding where to place their attention would be a fruitful line of inquiry.

In our competitive model, we assumed that firms moved simultaneously in their announcements of attributes. But consumers encounter some firms before others and even some firms have a timing advantage in moving more quickly than rivals. It is reasonable to anticipate that sequential exposure might have differential context biases on consumer's attention. For instance, if recency effects are significant, then moving last would lend that marketer an advantage in directing consumer's attention where it wants. In contrast, primacy effects would do the opposite. The presence of either form of bias would have implications on how firms strategically time their announcement of a prominent attribute.

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Appendix

This appendix contains the proofs of all results in the main text.

Proof of Lemma 1

Fix $j = 1,2$ and, with a slight abuse of notation, $k = A, B, AB$. Consider the sequence $\{\nu_k \varepsilon_{ijk}\}_{i=1}^{n}$, which is an i.i.d. sample of size n from $N(0, \nu_k^2)$, the normal distribution with zero mean and variance ν_k^2. Define $M_{jk} \equiv \max_{1 \leq i \leq n}\{\nu_k \varepsilon_{ijk}\}$ as the maximum random-match value obtained from when evaluating all n products from firm j on attribute(s) k. (Note that $i^*(j,k) = \text{argmax}_i\{\nu_k \varepsilon_{ijk}\}$ is the consumer's most preferred of the n products offered by firm j.) De Haan & Ferreira (2006) show that the Fisher-Tippet-Gnedenko Theorem implies that the distribution of M_{jk} approximates that of a type-I, extreme-value random variable with a cdf:

$$\Pr[M_{jk} < x] \cong e^{-e^{-n\Phi'(a_n)x/\nu_k}},$$

where $a_n = \Phi^{-1}\left(1 - \frac{1}{n}\right)$. Thus, for all k, the consumer's choice of a firm approximates a logit choice framework. Specifically, suppose a consumer evaluates products on one attribute, $k = A$ or B, only. In this case, the consumer chooses firm 1 if and only if $u_{1k} > u_{2k}$, or equivalently:

$$M_{1k} > \frac{(q_{2k} - q_{1k}) - (p_2 - p_1)}{\mu_k} + M_{2k},\ k = A, B,\ \mu_k = \nu_k \left[\frac{a_n}{n\Phi'(a_n)}\right]$$

which occurs with probability $s_{1k} = 1 - s_{2k}$, which is expressed in the first part of the statement of the lemma. Suppose a consumer evaluates products on both attributes so that $\mu_k = \mu_{AB} = \frac{\sqrt{2}}{2}\mu$. The consumer choses firm 1 if and only if $u_{1AB} > u_{2AB}$, which is equivalent to:

$$M_{1AB} > \frac{(q_{2A} - q_{1A}) + (q_{2B} - q_{1B}) - (p_2 - p_1)}{2\mu/\sqrt{2}} + M_{2AB},$$

which occurs with probability $s_{1AB} = 1 - s_{2AB}$. ■

Proof of Proposition 1

Based on Lemma 1, we know that firm j's profit when announcing k $(k = A, B, AB)$ is equal to $\pi_{jk} = p_j s_{jk}$ where s_{jk} is given in Lemma 1. The first order condition gives us $\frac{\partial \pi_{jk}}{\partial p_j} = s_{jk} + p_j \frac{\partial s_{jk}}{\partial p_j} = s_{jk} - \frac{1}{\mu_k} s_{jk} s_{-jk} p_j = 0$, where $-j \neq j$. Therefore we know $p_j = \mu_k \frac{1}{s_{-jk}}$, and the profit is equal to $\pi_{jk} = \mu_k \frac{s_{jk}}{s_{-jk}}$. The condition $\Delta = 0$ directly implies that in equilibrium $s_{jk} = s_{-jk} = \frac{1}{2}$. Therefore $\pi_{jk} = \pi_{-jk} = \mu_k$. Since $\mu_A = \mu_B > \mu_{AB}$, firms will prefer to announce the same attribute. ■

Proof of Lemma 2

The consumer's utility from firm j when evaluating only attribute k is $u_{jk} = V - p_j + q_{jk} + \mu_k \max\{\varepsilon_{ijk}\}$, where $\max\{\varepsilon_{ijk}\}$ is distributed as a type I extreme value distribution by Lemma 1. Anderson et al (1992) established that there exists a unique price equilibrium in a same setting. Moreover, Anderson et al (1992) show that as the quality difference Δ increases, the equilibrium price of the high quality firm increases as well. Hence, $\Delta_k > 0$ leads to $p_1 > p_2$. From the market share expressions in Lemma 1 we deduce that $s_{1k} > s_{2k}$ for $p_1 > p_2$. Correspondingly, profit for firm 1 is larger than for firm 2. ■

Proof of Proposition 2

When firms announce different attributes, the consumer's utility when buying from firm j is $u_{jAB} = (V - p_j) + q_{jA} + q_{jB} + \mu_{AB}\varepsilon_{jAB}$. In the symmetric equilibrium both firms have equal market share, prices, $p_1 = p_2 = \mu_{AB}/2$ and profit, $\pi_{1AB} = \pi_{2AB} = \mu_{AB}$. Suppose, without loss of generality, firm 1 makes attribute A prominent. Let π_{1A} and π_{2A} be firms' profits when both make attribute *A* the prominent attribute. From Lemma 2 we know that $\pi_{1A} > \pi_{2A}$ and that $\frac{\partial \pi_{2A}}{\partial \Delta} < 0$ for $\Delta > 0$. We also know that $\lim_{\Delta \to 0} \pi_{2A} \to \mu_A > \mu_{AB}$ (from Proposition 1). Furthermore, $\lim_{\Delta \to \infty} s_{2k} = 0$ (from Lemma 1), so that $\lim_{\Delta \to \infty} \pi_{2A} \to 0$. Because π_{2A} is strictly monotonically decreasing in Δ, there exists a $\overline{\Delta} > 0$ such that $\pi_{2A} > \pi_{2AB}$ whenever $\Delta < \overline{\Delta}$. Under this condition, it is beneficial for firm 2 to make attribute *A* prominent as well, rather than attribute *B*. But when $\Delta > \overline{\Delta}$, we have $\pi_{2A} < \pi_{2AB}$ so that firm 2 will choose to make attribute *B* prominent. Similarly, $\pi_{1AB} > \pi_{1B}$, which means firm 1 prefers to make attribute *A* prominent when firm 2 makes *B* prominent. Thus, firms make different attributes prominent in equilibrium whenever $\Delta > \overline{\Delta}$. ■

For the results in section 3.3, we need an intermediate result.

> **Lemma A** *Suppose that attribute $k \in \{A, B\}$ is prominent and that firm 1 has a quality advantage in that attribute: $\Delta = \Delta_k > 0$ for some or both $\in \{A, B\}$. Then $f(\mu) \equiv (q^H - p_1) - (q^L - p_2)$ is strictly decreasing in μ.*

Proof of Lemma A

Taking the derivative: $\frac{df(\mu)}{d\mu} = -\frac{dp_1}{d\mu} + \frac{dp_2}{d\mu}$. From firms' first order condition we know that

$$\mu - p_1 s_2 = 0$$

$$\mu - p_2 s_1 = 0.$$

In this same setting, Anderson et al. (1992) showed that as quality increases, the equilibrium price increases as well so that $p_1 > p_2$. Furthermore, from the first order condition we have $p_1 > p_2 \Leftrightarrow s_1 > s_2$.

Taking the total derivative with respect to μ on both first order conditions we have

$$s_2\frac{dp_1}{d\mu}+p_1\frac{ds_2}{d\mu}=1 \tag{A3}$$
$$s_1\frac{dp_2}{d\mu}+p_2\frac{ds_1}{d\mu}=1 \tag{A4}$$

We also know that

$$\frac{ds_1}{d\mu}=\frac{\partial s_1}{\partial p_1}\frac{dp_1}{d\mu}+\frac{\partial s_1}{\partial p_2}\frac{dp_2}{d\mu}+\frac{\partial s_1}{\partial \mu},$$

$$\frac{ds_2}{d\mu}=\frac{\partial s_2}{\partial p_1}\frac{dp_1}{d\mu}+\frac{\partial s_2}{\partial p_2}\frac{dp_2}{d\mu}+\frac{\partial s_2}{\partial \mu}.$$

Substitute $\frac{ds_1}{d\mu}$ and $\frac{ds_2}{d\mu}$ into (A3) and (A4) we have

$$\begin{bmatrix} s_2+p_1\frac{\partial s_2}{\partial p_1} & p_1\frac{\partial s_2}{\partial p_2} \\ p_2\frac{\partial s_1}{\partial p_1} & s_1+p_2\frac{\partial s_1}{\partial p_2} \end{bmatrix}\begin{bmatrix} \frac{dp_1}{d\mu} \\ \frac{dp_2}{d\mu} \end{bmatrix}=\begin{bmatrix} 1-p_1\frac{ds_2}{d\mu} \\ 1-p_2\frac{ds_1}{d\mu} \end{bmatrix}.$$

And

$$\begin{bmatrix} \frac{dp_1}{d\mu} \\ \frac{dp_2}{d\mu} \end{bmatrix}=\begin{bmatrix} s_2+p_1\frac{\partial s_2}{\partial p_1} & p_1\frac{\partial s_2}{\partial p_2} \\ p_2\frac{\partial s_1}{\partial p_1} & s_1+p_2\frac{\partial s_1}{\partial p_2} \end{bmatrix}^{-1}\begin{bmatrix} 1-p_1\frac{ds_2}{d\mu} \\ 1-p_2\frac{ds_1}{d\mu} \end{bmatrix}=\frac{\begin{bmatrix} s_1+p_2\frac{\partial s_1}{\partial p_2} & -p_1\frac{\partial s_2}{\partial p_2} \\ -p_2\frac{\partial s_1}{\partial p_1} & s_2+p_1\frac{\partial s_2}{\partial p_1} \end{bmatrix}\begin{bmatrix} 1-p_1\frac{ds_2}{d\mu} \\ 1-p_2\frac{ds_1}{d\mu} \end{bmatrix}}{\left|\left(s_1+p_2\frac{\partial s_1}{\partial p_2}\right)\left(s_2+p_1\frac{\partial s_2}{\partial p_1}\right)-p_1\frac{\partial s_2}{\partial p_2}p_2\frac{\partial s_1}{\partial p_1}\right|}.$$

We also know

$$\frac{\partial s_1}{\partial p_1}=-\frac{1}{\mu}s_1s_2;\ \frac{\partial s_1}{\partial p_2}=\frac{1}{\mu}s_1s_2;\ \frac{\partial s_1}{\partial \mu}=-\frac{1}{\mu^2}s_1s_2f(\mu)<0;$$

$$\frac{\partial s_2}{\partial p_1}=\frac{1}{\mu}s_1s_2;\ \frac{\partial s_2}{\partial p_2}=-\frac{1}{\mu}s_1s_2;\ \frac{\partial s_2}{\partial \mu}=\frac{1}{\mu^2}s_1s_2f(\mu)>0.$$

Therefore,

$$\left(s_1+p_2\frac{\partial s_1}{\partial p_2}\right)\left(s_2+p_1\frac{\partial s_2}{\partial p_1}\right)-p_1\frac{\partial s_2}{\partial p_2}p_2\frac{\partial s_1}{\partial p_1}=s_1s_2+s_1p_1\frac{1}{\mu}s_1s_2+s_2p_2\frac{1}{\mu}s_1s_2>0.$$

Define $G\equiv s_1s_2+s_1p_1\frac{1}{\mu}s_1s_2+s_2p_2\frac{1}{\mu}s_1s_2$. Then

$$\frac{dp_1}{d\mu}=\frac{\left[\left(s_1+p_2\frac{\partial s_1}{\partial p_2}\right)\left(1-p_1\frac{ds_2}{d\mu}\right)-p_1\frac{\partial s_2}{\partial p_2}\left(1-p_2\frac{ds_1}{d\mu}\right)\right]}{G};\text{ and}$$

$$\frac{dp_2}{d\mu}=\frac{\left[-p_2\frac{\partial s_1}{\partial p_1}\left(1-p_1\frac{ds_1}{d\mu}\right)+\left(s_2+p_1\frac{\partial s_2}{\partial p_1}\right)\left(1-p_2\frac{ds_2}{d\mu}\right)\right]}{G}.$$

Finally, we can write

$$-\frac{dp_1}{d\mu}+\frac{dp_2}{d\mu}=\frac{\left[-\left(s_1+p_2\frac{\partial s_1}{\partial p_2}\right)\left(1-p_1\frac{ds_2}{d\mu}\right)+p_1\frac{\partial s_2}{\partial p_2}\left(1-p_2\frac{ds_1}{d\mu}\right)-p_2\frac{\partial s_1}{\partial p_1}\left(1-p_1\frac{ds_1}{d\mu}\right)+\left(s_2+p_1\frac{\partial s_2}{\partial p_1}\right)\left(1-p_2\frac{ds_2}{d\mu}\right)\right]}{G}=$$

$$=\frac{\left[-s_1\left(1-p_1\frac{ds_2}{d\mu}\right)+s_2\left(1-p_2\frac{ds_1}{d\mu}\right)\right]}{G}=\frac{\left[-s_1+s_2+s_1p_1\frac{ds_2}{d\mu}-s_2p_2\frac{ds_1}{d\mu}\right]}{G}<0. \quad \blacksquare$$

Proof of Lemma 3

Suppose both firms make attribute A prominent. We establish the result by signing the derivatives of firms' profit functions with respect to μ. From Lemma 1 we can write derive the following derivatives:

$$\frac{\partial s_1}{\partial p_1}=-\frac{\frac{1}{\mu}e^{(q^H-p_1)/\mu}}{e^{(q^H-p_1)/\mu}+e^{(q^L-p_2)/\mu}}+\frac{\frac{1}{\mu}(e^{q^H-p_1/\mu})^2}{\left[e^{(q^H-p_1)/\mu}+e^{(q^L-p_2)/\mu}\right]^2}=-\frac{1}{\mu}s_1(1-s_1)=-\frac{1}{\mu}s_1s_2;$$

$$\frac{\partial s_1}{\partial p_2}=\frac{\frac{1}{\mu}e^{(q^H-p_1)/\mu}*e^{(q^L-p_2)/\mu}}{\left[e^{(q^H-p_1)/\mu}+e^{(q^L-p_2)/\mu}\right]^2}=\frac{1}{\mu}s_1s_2;$$

$$\frac{\partial s_1}{\partial \mu}=\frac{-\left[e^{(q^H-p_1)/\mu}\right]\frac{q^H-p_1}{\mu^2}}{e^{(q^H-p_1)/\mu}+e^{(q^L-p_2)/\mu}}+\frac{e^{(q^H-p_1)/\mu}*\left[\frac{q^H-p_1}{\mu^2}e^{\frac{q^H-p_1}{\mu}}+\frac{q^L-p_2}{\mu^2}e^{\frac{q^L-p_2}{\mu}}\right]}{[e^{(q^H-p_1)/\mu}+e^{(q^L-p_2)/\mu}]^2};$$

$$=-\frac{q^H-p_1}{\mu^2}s_1+s_1\left(\frac{q^H-p_1}{\mu^2}s_1+\frac{q^L-p_2}{\mu^2}s_2\right)=-\frac{q^H-p_1}{\mu^2}s_1s_2+\frac{q^L-p_2}{\mu^2}s_1s_2;$$

$$=\frac{1}{\mu^2}s_1s_2[-(q^H-p_1)+q^L-p_2];$$

The firm 1's profit is given by $\pi_1=p_1s_1$. Therefore,

$$\frac{d\pi_1}{d\mu}=\frac{\partial\pi_1}{\partial p_1}\frac{dp_1}{d\mu}+\frac{\partial\pi_1}{\partial p_2}\frac{dp_2}{d\mu}+\frac{\partial\pi_1}{\partial\mu}=\frac{\partial\pi_1}{\partial p_2}\frac{dp_2}{d\mu}+\frac{\partial\pi_1}{\partial\mu}.$$

We know $\frac{\partial\pi_1}{\partial p_2}=p_1\frac{\partial s_1}{\partial p_2}=p_1\frac{1}{\mu}s_1s_2$, and similarly, $\frac{d\pi_1}{d\mu}=p_1\frac{\partial s_1}{\partial\mu}$.

In addition, we know the first order conditions for profit maximization in firms' prices p_1^*, p_2^*, which simultaneously solve the following equations.

$$F_1(p_1, p_2; \mu) \equiv \frac{\partial \pi_1}{\partial p_1} = s_1 + p_1 \frac{\partial s_1}{\partial p_1} = s_1 - p_1 \tfrac{1}{\mu} s_1 s_2 = \mu - p_1 s_2 = 0, \tag{A1}$$

$$F_2(p_2, p_1; \mu) \equiv \frac{\partial \pi_2}{\partial p_2} = s_2 + p_2 \frac{\partial s_2}{\partial p_2} = s_2 - p_2 \tfrac{1}{\mu} s_1 s_2 = \mu - p_2 s_1 = 0. \tag{A2}$$

We can assess the signs of $\partial \pi_j / \partial \mu$, for $j = 1,2$, by taking the total derivatives:

$$\frac{\partial F_1}{\partial p_1}\frac{dp_1}{d\mu} + \frac{\partial F_1}{\partial p_2}\frac{dp_2}{d\mu} + \frac{\partial F_1}{\partial \mu} = 0 \qquad \text{and} \qquad \frac{\partial F_2}{\partial p_1}\frac{dp_1}{d\mu} + \frac{\partial F_2}{\partial p_2}\frac{dp_2}{d\mu} + \frac{\partial F_2}{\partial \mu} = 0.$$

Rearranging we have

$$\frac{dp_1}{d\mu} = \frac{-\frac{\partial F_2}{\partial p_2}\frac{\partial F_1}{\partial \mu} + \frac{\partial F_1}{\partial p_2}\frac{\partial F_2}{\partial \mu}}{\frac{\partial F_1}{\partial p_1}\frac{\partial F_2}{\partial p_2} - \frac{\partial F_1}{\partial p_2}\frac{\partial F_2}{\partial p_1}}; \qquad \text{and} \qquad \frac{dp_2}{d\mu} = \frac{\frac{\partial F_2}{\partial p_1}\frac{\partial F_1}{\partial \mu} - \frac{\partial F_1}{\partial p_1}\frac{\partial F_2}{\partial \mu}}{\frac{\partial F_1}{\partial p_1}\frac{\partial F_2}{\partial p_2} - \frac{\partial F_1}{\partial p_2}\frac{\partial F_2}{\partial p_1}}.$$

We know from (A1) that

$$\frac{\partial F_1}{\partial p_1} = -s_2 - p_1 \frac{\partial s_2}{\partial p_1} = -s_2 - p_1 \tfrac{1}{\mu} s_1 s_2 = -1,$$

$$\frac{\partial F_1}{\partial p_2} = -p_1 \frac{\partial s_2}{\partial p_2} = p_1 \tfrac{1}{\mu} s_1 s_2 = s_1,$$

and

$$\frac{\partial F_1}{\partial \mu} = 1 - p_1 \frac{\partial s_2}{\partial \mu} = 1 + p_1 \frac{\partial s_1}{\partial \mu} = 1 + \frac{1}{\mu} s_1 [(q_{1A}^H - p_1) - (q_{2A}^H - p_2)].$$

Similarly, from (A2) we have

$$\frac{\partial F_2}{\partial p_1} = s_2, \frac{\partial F_2}{\partial p_2} = -1, \text{ and } \frac{\partial F_2}{\partial \mu} = 1 - \frac{1}{\mu} s_2 [-(q_{2A}^H - p_2) + q_{1A}^H - p_1].$$

It can be verified that $\frac{\partial F_1}{\partial p_1}\frac{\partial F_2}{\partial p_2} - \frac{\partial F_1}{\partial p_2}\frac{\partial F_2}{\partial p_1} = 1 - s_1 s_2 > 0$, since $s_j < 1/2$, $j = 1,2$. Hence,

$$\frac{dp_2}{d\mu} = \frac{\frac{\partial F_2}{\partial p_1}\frac{\partial F_1}{\partial \mu_A} - \frac{\partial F_1}{\partial p_1}\frac{\partial F_2}{\partial \mu_A}}{\frac{\partial F_1}{\partial p_1}\frac{\partial F_2}{\partial p_2} - \frac{\partial F_1}{\partial p_2}\frac{\partial F_2}{\partial p_1}} = \frac{s_2\frac{\partial F_1}{\partial \mu} + \frac{\partial F_2}{\partial \mu}}{1 - s_1 s_2}$$

We can thus write,

$$\frac{d\pi_1}{d\mu} = \frac{\partial \pi_1}{\partial p_2}\frac{dp_2}{d\mu} + \frac{\partial \pi_1}{\partial \mu} = p_1\frac{\partial s_1}{\partial p_2}\frac{s_2\frac{\partial F_1}{\partial \mu} + \frac{\partial F_2}{\partial \mu}}{1 - s_1 s_2} + p_1\frac{\partial s_1}{\partial \mu} = p_1\frac{\partial s_1}{\partial p_2}\frac{s_2\left(1 + p_1\frac{\partial s_1}{\partial \mu}\right) + \left(1 - p_2\frac{\partial s_1}{\partial \mu}\right)}{1 - s_1 s_2} + p_1\frac{\partial s_1}{\partial \mu}$$

$$= p_1\left\{\frac{\partial s_1}{\partial p_2}\frac{s_2 + 1}{1 - s_1 s_2} + \frac{\partial s_1}{\partial \mu}\left[\frac{(s_2 p_1 - p_2)\frac{\partial s_1}{\partial p_2}}{1 - s_1 s_2} + 1\right]\right\} = p_1\left\{\frac{1}{\mu}s_1 s_2\frac{1 + s_2}{1 - s_1 s_2} + \frac{\partial s_1}{\partial \mu}\left[\frac{(s_2 p_1 - p_2)\frac{1}{\mu}s_1 s_2}{1 - s_1 s_2} + 1\right]\right\}.$$

$$= p_1\left[\frac{1}{\mu}s_1 s_2\frac{1 + s_2}{1 - s_1 s_2} + \frac{\partial s_1}{\partial \mu}\left(\frac{s_1 s_2 - s_2 + 1 - s_1 s_2}{1 - s_1 s_2}\right)\right] = p_1\left[\frac{1}{\mu}s_1 s_2\frac{1 + s_2}{1 - s_1 s_2} + \frac{\partial s_1}{\partial \mu}\frac{s_1}{1 - s_1 s_2}\right]$$

$$= p_1\left\{\frac{1}{\mu}s_1 s_2\frac{1 + s_2}{1 - s_1 s_2} + \frac{1}{\mu^2}s_1 s_2\left[-(q^H - p_1) + q^L - p_2\right]\frac{s_1}{1 - s_1 s_2}\right\}$$

$$= \frac{p_1 s_1 s_2}{\mu(1 - s_1 s_2)}\left\{1 + s_2 - \frac{s_1}{\mu}\left[(q^H - p_1) - q^L - p_2\right]\right\}.$$

As we proved in Lemma A, $|(q^H - p_1) - q^L + p_2|$ is strictly decreasing in μ. Therefore, we can define $\overline{\mu}_\Delta$ as the unique value of μ that makes the curly-bracketed expression zero. Furthermore, since – $(q^H - p_1) + q^L - p_2 < 0$ when $q^H > q^L$, we have $d\pi_1/d\mu < 0$ whenever , $\mu < \bar{\mu}_\Delta$ and $d\pi_1/d\mu > 0$ whenever $\mu > \bar{\mu}_\Delta$. A similar derivation can show that $d\pi_2/d\mu > 0$ for all μ. ■

Proof of Proposition 3

We first show when $\mu > \bar{\mu}_\Delta$, both firm announce the same attribute in equilibrium. It follows from Lemma 3 that when $\mu > \bar{\mu}_\Delta$, both firms' profits are monotonically increasing in μ. Making different attributes prominent, however, strictly lowers μ since $\mu_{AB} = \frac{\sqrt{2}}{2}\mu$. Hence no firm has the incentive to deviate to a different attribute.

When $\mu \leq \bar{\mu}_\Delta$, firm 1s' profit is monotonically decreasing in μ, while firm 2's profit is increasing in μ. Therefore firm 1 prefers to have both attributes prominent, while firm 2 prefers to have the same prominent attribute. We can prove by contradiction that there does not exist a pure strategy equilibrium in this situation. Assuming firm 1 makes attribute *A* prominent in equilibrium, then the best strategy for firm 2 is to make attribute *A* prominent as well. Given firm 2 picks attribute *A*, the best strategy for firm 1 is to make attribute *B* prominent. Therefore there does not exist any pure strategy equilibrium. To characterize the mixed strategy equilibrium, we refer to the payoff matrix:

		Firm 2	
	Attribute Choice	A	B
Firm 1	A	π_{1A}, π_{2A}	π_{1AB}, π_{2AB}
	B	π_{1AB}, π_{2AB}	π_{1B}, π_{2B}

We notice $\pi_{1A} = \pi_{1B} > \pi_{2A} = \pi_{2B}$, and $\pi_{1AB} > \pi_{1A}$ and $\pi_{2AB} < \pi_{2A}$. In this case the mixed strategy for both firms is choosing each attribute with equal probabilities. ■

Proof of Lemma 4

For (a), when both firms make attribute *k* prominent, each firm's payoff when deciding on the investment is given by the following 2 by 2 matrix:

		Firm 2	
	Invest on Quality	Y	N
Firm 1	Y	$\pi_{1k}^{HH} - c, \pi_{2k}^{HH} - c$	$\pi_{1k}^{HL} - c, \pi_{2k}^{HL}$
	N	$\pi_{1k}^{LH}, \pi_{2k}^{LH} - c$	$\pi_{1k}^{LL}, \pi_{2k}^{LL}$

In this payoff matrix, π_{1k}^{HL} represents firm 1's profit when the prominent attribute is *k* and firm 1 invests in quality (so the quality is q^H) and firm 2 does not invest (so the quality remains at q^L).

We can see that when both firms improve the quality, they have the same quality q^H and therefore share the market equally with payoff $\pi_{1k}^{HH} = \pi_{2k}^{HH} = \hat{\mu}$. Similarly we know that $\pi_{1k}^{LL} = \pi_{2k}^{LL} = \hat{\mu}$. We can also see $\pi_{2k}^{LH} = \pi_{1k}^{HL} = \bar{\pi}_k > \hat{\mu} > \pi_{1k}^{LH} = \pi_{2k}^{HL} = \underline{\pi}_k$ based on Lemma 2. Let's define $\bar{c} = \min\{|\bar{\pi}_k - \hat{\mu}|; |\underline{\pi}_k - \hat{\mu}|\} > 0$. Then when $c < \bar{c}$, $\hat{\mu} - c > \underline{\pi}_k$, and $\bar{\pi}_k - c > \hat{\mu}$. Firm 1's dominant strategy is to invest on quality regardless firm 2' strategy, and vice versa for firm 2. Therefore, the equilibrium has both firms investing on quality.

The proof for (b) is quite similar to (a), when both firms make different attributes prominent, each firm's payoff when deciding on the investment is given by the following 2 by 2 matrix:

		Firm 2	
	Invest on Quality	Y	N
Firm 1	Y	$\pi_{1AB}^{HH} - c, \pi_{2AB}^{HH} - c$	$\pi_{1AB}^{HL} - c, \pi_{2AB}^{HL}$
	N	$\pi_{1AB}^{LH}, \pi_{2AB}^{LH} - c$	$\pi_{1AB}^{LL}, \pi_{2AB}^{LL}$

We know the payoff $\pi_{1AB}^{HH} = \pi_{2k}^{HH} = \frac{\sqrt{2}}{2}\hat{\mu}$. Similarly we know that $\pi_{1AB}^{LL} = \pi_{2AB}^{LL} = \frac{\sqrt{2}}{2}\hat{\mu}$. We can also see $\pi_{2AB}^{LH} = \pi_{1AB}^{HL} = \bar{\pi}_{AB} > \frac{\sqrt{2}}{2}\hat{\mu} > \pi_{1k}^{LH} = \pi_{2k}^{HL} = \underline{\pi}_{AB}$ based on Lemma 2. Let's define $\underline{c} = \max\{|\bar{\pi}_{AB} - \frac{\sqrt{2}}{2}\hat{\mu}|; |\underline{\pi}_{AB} - \frac{\sqrt{2}}{2}\hat{\mu}|\} > 0$. We can see that when $c > \underline{c}$ firm 1's dominant strategy is to not invest on quality, and vice versa for firm 2. The equilibrium has neither firm investing in quality. ■

Proof of Lemma 5

We prove this lemma by first showing that $\bar{c} = \min\{|\bar{\pi}_k - \hat{\mu}|; |\underline{\pi}_k - \hat{\mu}|\} = \hat{\mu} - \underline{\pi}_k$, and $\underline{c} = \max\{|\bar{\pi}_{AB} - \frac{\sqrt{2}}{2}\hat{\mu}|; |\underline{\pi}_{AB} - \frac{\sqrt{2}}{2}\hat{\mu}|\} = (\bar{\pi}_{AB} - \frac{\sqrt{2}}{2}\hat{\mu})$. Intuitively it means the marginal increase in profit for the firm who improves the quality is larger than the corresponding decrease in profit for the low quality firm. Then we show there always exist a $\widehat{\Delta}(\mu) > 0$ such that when $\Delta < \widehat{\Delta}(\mu)$ we have $\underline{c} < \bar{c}$.

By assumption, when both firms do not invest, they have the same quality q^L. So $\Delta_k = 0$, $\forall k = A, B$. Now assume firm 1 invests to increase its product quality to q^H in attribute *k* so that $\Delta \equiv \Delta_k > 0$. We know that $\pi_{1k}^{HL} = \bar{\pi}_k > \hat{\mu} > \pi_{2k}^{HL} = \underline{\pi}_k$. What we want to show is $\pi_{1k}^{HL} - \hat{\mu} > \hat{\mu} - \pi_{2k}^{HL}$, or equivalently $\frac{d\pi_1}{d\Delta} + \frac{d\pi_2}{d\Delta} > 0$. To show this, we again refer to the firm's first order condition that:

$$F_1(p_1, p_2; \Delta) \equiv \mu - p_1 s_2 = 0, \qquad \text{(A3)}$$

$$F_2(p_2, p_1; \Delta) \equiv \mu - p_2 s_1 = 0. \qquad \text{(A4)}$$

Taking the total derivatives and rearranging we have:

$$\frac{dp_1}{d\Delta} = \frac{-\frac{\partial F_2}{\partial p_2}\frac{\partial F_1}{\partial \Delta} + \frac{\partial F_1}{\partial p_2}\frac{\partial F_2}{\partial \Delta}}{\frac{\partial F_1}{\partial p_1}\frac{\partial F_2}{\partial p_2} - \frac{\partial F_1}{\partial p_2}\frac{\partial F_2}{\partial p_1}}; \qquad \text{and} \qquad \frac{dp_2}{d\Delta} = \frac{\frac{\partial F_2}{\partial p_1}\frac{\partial F_1}{\partial \Delta} - \frac{\partial F_1}{\partial p_1}\frac{\partial F_2}{\partial \Delta}}{\frac{\partial F_1}{\partial p_1}\frac{\partial F_2}{\partial p_2} - \frac{\partial F_1}{\partial p_2}\frac{\partial F_2}{\partial p_1}}.$$

We know from (A3) and (A4) that

$$\frac{\partial F_1}{\partial p_1} = -1, \frac{\partial F_1}{\partial p_2} = s_1, \text{ and } \frac{\partial F_1}{\partial \Delta} = -p_1\frac{\partial s_2}{\partial \Delta} = p_1\frac{1}{\mu}s_1 s_2 = s_1;$$

$$\frac{\partial F_2}{\partial p_1} = s_2, \frac{\partial F_2}{\partial p_2} = -1, \text{ and } \frac{\partial F_2}{\partial \Delta} = -s_2$$

Therefore,

$$\frac{dp_1}{d\Delta} = \frac{-\frac{\partial F_2}{\partial p_2}\frac{\partial F_1}{\partial \Delta} + \frac{\partial F_1}{\partial p_2}\frac{\partial F_2}{\partial \Delta}}{\frac{\partial F_1}{\partial p_1}\frac{\partial F_2}{\partial p_2} - \frac{\partial F_1}{\partial p_2}\frac{\partial F_2}{\partial p_1}} = \frac{s_1^2}{1 - s_1 s_2}; \qquad \text{and} \qquad \frac{dp_2}{d\Delta} = \frac{\frac{\partial F_2}{\partial p_1}\frac{\partial F_1}{\partial \Delta} - \frac{\partial F_1}{\partial p_1}\frac{\partial F_2}{\partial \Delta}}{\frac{\partial F_1}{\partial p_1}\frac{\partial F_2}{\partial p_2} - \frac{\partial F_1}{\partial p_2}\frac{\partial F_2}{\partial p_1}} = \frac{-s_2^2}{1 - s_1 s_2}.$$

We know that

$$\frac{d\pi_1}{d\Delta} = \frac{\partial \pi_1}{\partial p_2}\frac{dp_2}{d\Delta} + \frac{\partial \pi_1}{\partial \Delta} = p_1\frac{\partial s_1}{\partial p_2}\frac{dp_2}{d\Delta} + p_1\frac{\partial s_1}{\partial \Delta} = p_1\frac{1}{\mu}s_1 s_2\left(\frac{-s_2^2}{1 - s_1 s_2} + 1\right) = p_1\frac{1}{\mu}s_1 s_2\frac{s_1}{1 - s_1 s_2} > 0;$$

$\frac{d\pi_2}{d\Delta} = \frac{\partial\pi_2}{\partial p_1}\frac{dp_1}{d\Delta} + \frac{\partial\pi_2}{\partial\Delta} = p_2\frac{\partial s_2}{\partial p_1}\frac{dp_1}{d\Delta} + p_2\frac{\partial s_2}{\partial\Delta} = p_2\frac{1}{\mu}s_1 s_2\left(\frac{s_1^2}{1-s_1 s_2} - 1\right) = p_2\frac{1}{\mu}s_1 s_2\frac{-s_2}{1-s_1 s_2} < 0;$

Since $p_1 > p_2$ when $\Delta > 0$, we can see that $\frac{d\pi_1}{d\Delta} + \frac{d\pi_2}{d\Delta} > 0$. Therefore we know that $\pi_{1k}^{HL} - \hat{\mu} > \hat{\mu} - \pi_{2k}^{HL}$, which gives us $\bar{c} = \hat{\mu} - \underline{\pi}_k$. The proof for $\underline{c} = (\bar{\pi}_{AB} - \frac{\sqrt{2}}{2}\hat{\mu})$ is very similar and therefore omitted.

Next we show that there always exists a $\widehat{\Delta}(\mu) > 0$ for any given μ such that when $\Delta < \widehat{\Delta}(\mu)$, we have $\underline{c} < \bar{c}$. We first notice that $\bar{\pi}_{AB} < \bar{\pi}_k$ but $\underline{\pi}_{AB} > \underline{\pi}_k$. The reason for this result is when firms make both attribute prominent, the quality advantage is diluted. To see this, consider the following example: if firm 1 invests on quality in attribute A, and firm 2 does not match the investment. The consumer's utility of buying from firm 1 is $u_{1AB} = V - p_1 + \frac{1}{2}q^H + \frac{1}{2}q^L + \frac{\sqrt{2}}{2}\hat{\mu}\max\{\varepsilon_{i1AB}\}$, and from firm 2 is $u_{2AB} = V - p_1 + q^L + \frac{\sqrt{2}}{2}\hat{\mu}\max\{\varepsilon_{i2AB}\}$. The overall vertical difference due to the quality different becomes $\frac{1}{2}\Delta$, which is lower than the difference Δ when firms make the same attribute prominent. Because of that, $\hat{\mu} - \underline{\pi}_k > \hat{\mu} - \underline{\pi}_{AB}$, and $\bar{\pi}_{AB} - \frac{\sqrt{2}}{2}\hat{\mu} < \bar{\pi}_k - \frac{\sqrt{2}}{2}\hat{\mu}$. We also notice that $\lim_{\Delta\to 0}\hat{\mu} - \underline{\pi}_{AB} = \hat{\mu} - \frac{\sqrt{2}}{2}\hat{\mu} = \lim_{\Delta\to 0}\bar{\pi}_k - \frac{\sqrt{2}}{2}\hat{\mu}$, which gives us $\hat{\mu} - \underline{\pi}_k > \bar{\pi}_{AB} - \frac{\sqrt{2}}{2}\hat{\mu}$ when $\Delta \to 0$. Because of the continuity of profit functions, there must exist a threshold $\widehat{\Delta}(\mu) > 0$ such that when $\Delta < \widehat{\Delta}(\mu)$, we have $\underline{c} < \bar{c}$. ■

Proof of Proposition 4

When $\Delta < \widehat{\Delta}(\mu)$, from Lemma 5 we know that $\underline{c} < \bar{c}$, which leads to the payoff for firms to be $\frac{\sqrt{2}}{2}\hat{\mu}$ if making both attributes prominent, and $\hat{\mu} - c$ if making a single attribute *k* prominent. Hence when $c > \hat{\mu} - \frac{\sqrt{2}}{2}\hat{\mu}$ we have $\frac{\sqrt{2}}{2}\hat{\mu} > \hat{\mu} - c$, which gives us the required conditions for firms to make different attributes prominent in equilibrium. Figure A1 graphically illustrates $\underline{c}$, $\bar{c}$ and $\left(1 - \frac{\sqrt{2}}{2}\right)\hat{\mu}$ by simulation. ■

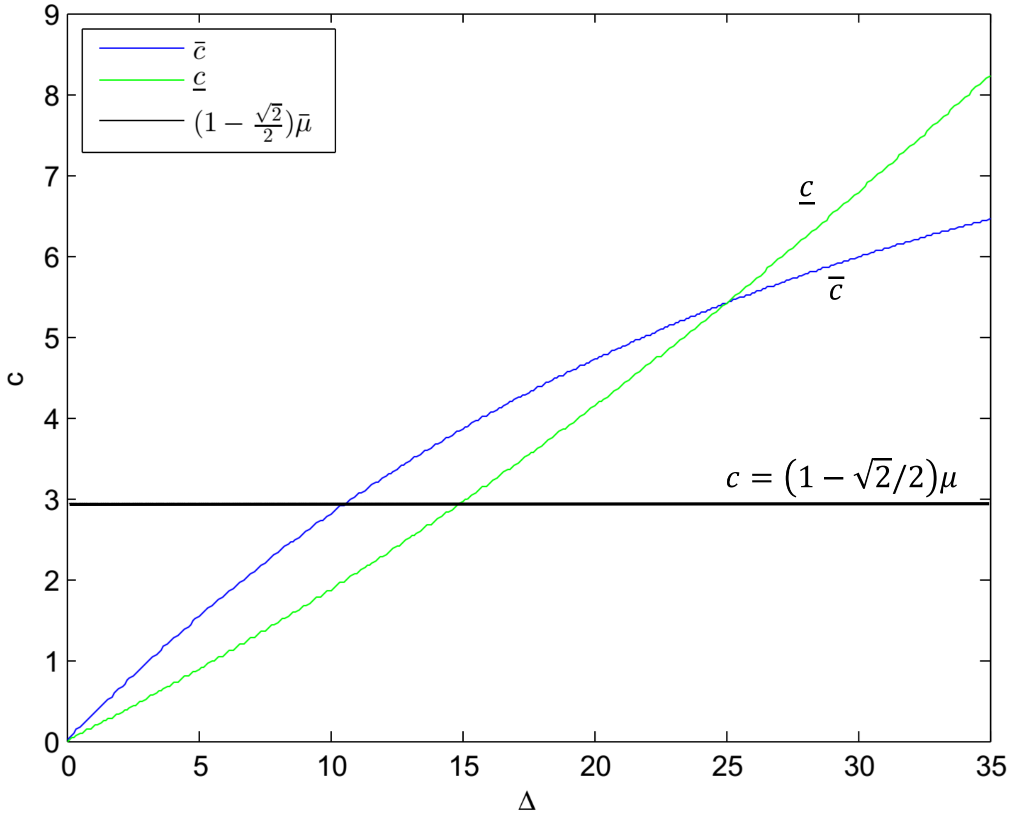

Figure A1: Illustration of $\underline{c}$, $\bar{c}$ and $\left(1-\frac{\sqrt{2}}{2}\right)\hat{\mu}$ when $\hat{\mu}=10$ and $\Delta \in [0,35]$