Competitive Poaching in Sponsored Search Advertising and Strategic Impact on Traditional Advertising

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Abstract

An important decision for a firm is how to allocate its advertising budget among different types of advertising. Most traditional channels of advertising, such as advertising on television and in print, serve the purpose of building consumer awareness and desire about the firm's products. With recent developments in technology, sponsored search (or paid search) advertising at search engines in response to a keyword searched by a user has become an important part of most firms' advertising efforts. An advantage of sponsored search advertising is that, since the firm advertises in response to a consumer-initiated search, it is a highly targeted form of communication and the sales-conversion rate is typically higher than in traditional advertising. However, a consumer would search for a specific product or brand only if she is already aware of the same due to previous awareness-generating traditional advertising efforts. Moreover, competing firms can use sponsored search to free-ride on the awareness-building efforts of other firms by directly advertising on their keywords and therefore "poaching" their customers. Anecdotal evidence shows that this is a frequent occurrence. In other words, traditional advertising builds awareness, while sponsored search is a form of technology-enabled communication that helps to target consumers in a later stage of the purchase process, which induces competitors to poach these potential customers.

Using a game theory model, we study the implications of these tradeoffs on the advertising decisions of competing firms, and on the design of the sponsored search auction by the search engine. We find that symmetric firms may follow asymmetric advertising strategies, with one firm focusing on traditional advertising and the other firm focusing on sponsored search with poaching. Interestingly, the search engine benefits from handicapping poaching, i.e., it benefits from *discouraging* competition in its own auctions. This explains why search engines such as Google, Yahoo! and Bing use "keyword relevance" scores to under-weight the bids of firms bidding on competitors' keywords. We also obtain various other interesting insights on the interplay between sponsored search advertising and traditional advertising.

1 Introduction

Online advertising is the fastest growing channel of advertising, likely to exceed 25% of the total US advertising expense, by 2015.[1] This rapid growth in online advertising is impressive given that television advertising, which firms have been using for decades, has a market share of about 35%. On aggregate, firms allocate nearly half of the online advertising spend to sponsored search advertising.[2] There are several unique advantages of sponsored search advertising. First, advertisers can target users of a specific age group, location, . . . , when they search for certain keywords on a search engine. Second, sponsored search is easily accessible to most firms; a firm with an advertising budget of as little as $5 can advertise on sponsored search, and setting up a campaign can be done in less than five minutes from a personal computer without having to contact any marketing agency. Third, the advertiser pays only when a user visits its website, which makes the effectiveness and value of sponsored search advertising easy to measure.

Given its unique advantages and spectacular growth, sponsored search advertising has received a lot of attention from researchers and practitioners. While firms are dedicating progressively larger fractions of their advertising budget to sponsored search advertising at the expense of traditional advertising, the strategic implications of the interactions between these two types of advertising have not been carefully researched.

A widely employed marketing framework is the awareness-interest-desire-action (AIDA) model that sequentially captures the various stages of a typical consumer's decision process before finally purchasing a product. Traditional channels of advertising, such as television, newspapers, radio and billboards are directed more towards the initial stages of the AIDA model. They are initiated by the firm and are highly effective in creating awareness and getting customers interested in a firm's brand or the product category. However, sponsored search is located more towards the last stages of the AIDA model and influences the purchase action. Sponsored search effectively targets the customers who are already aware of the product and have shown some interest or desire in the product by searching for an associated keyword at a search engine. In the context of the AIDA model, traditional advertising can be interpreted as "upstream advertising," while sponsored search can be interpreted as "downstream advertising." Thus, traditional awareness-

[1] http://www.onlinemarketing-trends.com/2011/06/18-us-media-ad-spending-to-go-digital.html

[2] http://searchengineland.com/emarketer-among-online-ads-search-to-gain-most-new-dollars-in-2011-80707

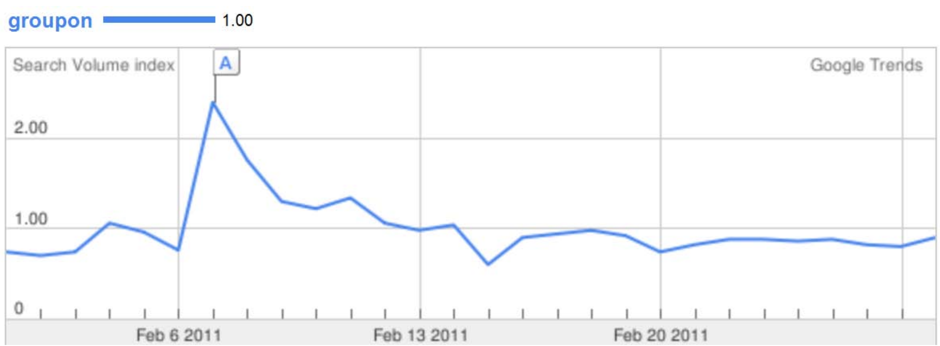
generating advertising and sponsored search advertising are inter-related and play complementary roles in successfully consummating the sale of a product.

In a strategic market with competing firms, creating awareness has benefits and perils especially when the awareness created through traditional advertising for one brand can be exploited by sponsored search advertising by a competing firm. Competitors, instead of allocating their advertising budget to create awareness about their own products, can advertise in sponsored search on the keywords of a firm in the same industry that is creating awareness by investing in traditional advertising, trying to steal the latter's potential customers. We refer to this as "poaching" in sponsored search. In fact, since the competitors are not spending to create awareness, they can bid more aggressively on sponsored search keywords (typically sold through position auctions run by the search engine) and thus can even enjoy an advantage over the firm that has attracted the customers in the first place. We provide anecdotal examples where such poaching is evident.

Figure 1 shows the effect of Super Bowl advertising on the search volumes of the advertising firms' keywords. The shoe company "Skechers" advertised its "Shape Ups" model during Super Bowl 2011. Also, the social coupon firm "Groupon" and the online florist "Teleflora" had their own ads during Super Bowl 2011. It can be easily seen that the advertising has created considerable awareness resulting in heavy keyword search on the internet. Such traffic reaches a peak the day after the Super Bowl. While these firms spent millions of dollars for their Super Bowl commercials, we see that their competitors, at the same time, are poaching on their keywords as depicted in Figure 2. "Reebok" is poaching on the keyword "Shape Ups," while "LivingSocial" is poaching on the keyword "Groupon." Other online florists are poaching on the keyword "Teleflora." These are only a few of many instances of poaching, which is happening with increasing frequency on the internet. In summary, poaching is when a firm creates awareness resulting in pertinent keyword search on the internet, and competing firms aggressively bid on these keywords and display their products.

Poaching in sponsored search has two important aspects. First, the poaching firm is *free riding* on the awareness that is created by its competitor through traditional advertising. This is often accomplished by advertising on the competitor's specific keyword or on a more general "category keyword." Second, the poaching firm is *stealing* potential customers from its competitor. When a customer sees iPad's television commercial and searches for the keyword "iPad" with the ultimate

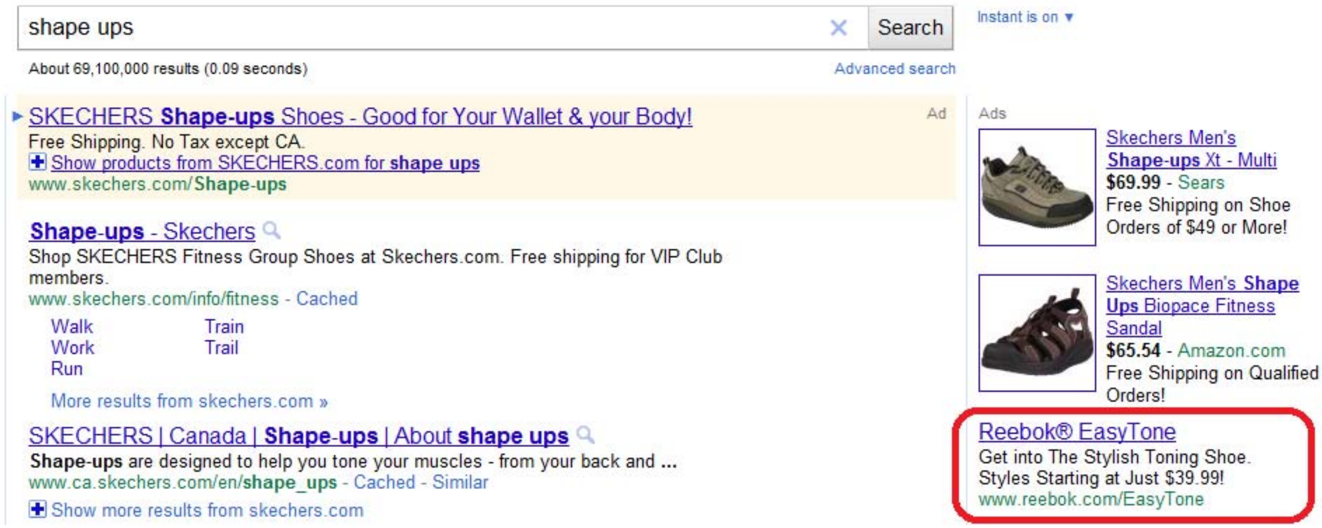
(a) Skechers, Shape Ups



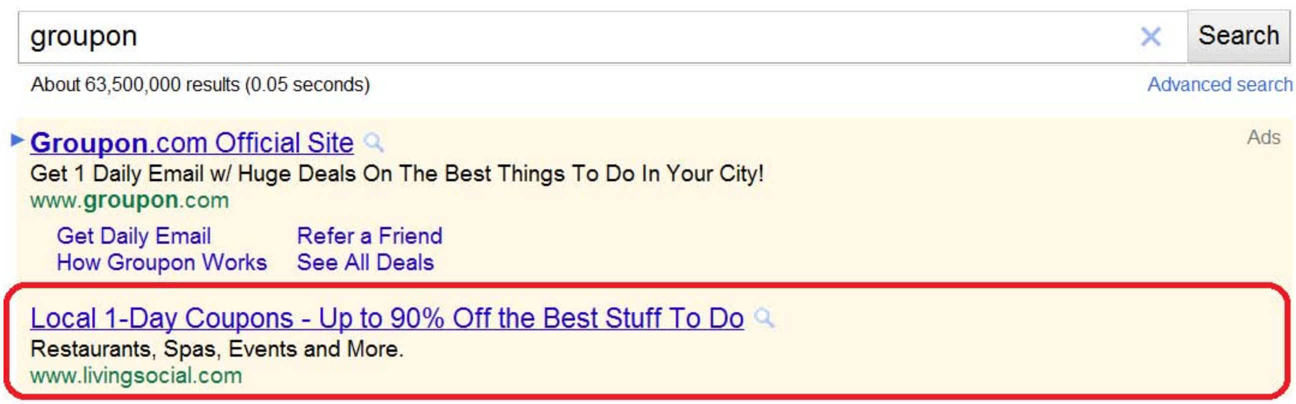
(b) Groupon

(c) Teleflora

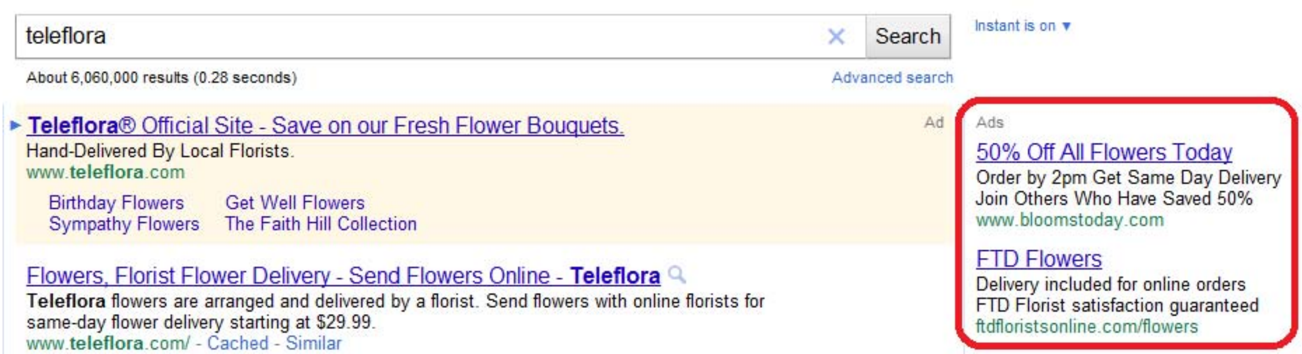
Figure 1: The effect of Super Bowl advertising on the search volume of the firms' keywords: Super Bowl was held on February 6, 2011.



(a) Shape Ups



(b) Groupon



(c) Teleflora

Figure 2: Poaching

goal of purchasing it, she is a potential customer for Apple. If Samsung can place an ad for its own tablet, Galaxy Tab, on the search results page for the keyword "iPad," it will be an extremely effective targeting strategy through which Samsung can try to convince potential customers of Apple to buy Samsung's Galaxy Tab. Thus, Samsung has a strong incentive to bid on the keyword "iPad," effectively freeriding on the awareness created by Apple for iPad. Of course, if Samsung invests in awareness-generating advertising leading to corresponding keyword searches for the Galaxy Tab, Apple can also advertise on Samsung's keywords. In other words, competing firms can practice mutual poaching.

Such poaching behavior has implications not only for the competing firms' strategies on the sponsored search and traditional advertising channels, they also strategically affect the search engine's auction strategy. In this paper, we use examine these issues in an analytical framework. We address three broad questions. First, under what conditions will poaching arise and be beneficial for a firm? Second, what are the effects of poaching on competing firms' decisions for budget allocation among traditional and sponsored search advertising? Third, what are the consequences of poaching for the search engine and what should be the search engine's best response?

We first consider the case in which there are two identical competing firms. We find the existence of an asymmetric equilibrium in which one firm mostly advertises on traditional channels and creates awareness, while its competitor poaches on its keyword in sponsored search. This is because poaching increases the competition in sponsored search auctions which increases the advertising prices of the keywords. This increases the per-customer cost to the firm from sponsored search, which makes sponsored search a less desired option, and incentivizes the firms to move a larger share of their money to traditional advertising. However, poaching remains a profitable strategy for one firm, which leads to the asymmetric budget allocation. Interestingly, although the competition in sponsored search increases with poaching, the search engine's revenue may decrease because of the incentive of firms to move some of their advertising budget to traditional advertising. As a result, the search engine may benefit from *discouraging competition* in its own keyword auctions by making poaching harder for the firms. This offers an explanation for why major search engines such as Google, Yahoo! and Bing use "keyword relevance scores" to under-weight the bids of firms bidding on competitors' keywords.

We extend our analysis to the case of asymmetric firms with different advertising budgets.

When the difference between budgets is large enough, the firm with the smaller advertising budget has a greater incentive to poach as compared to the firm with the larger budget (because the latter conducts more traditional advertising and drives more traffic towards its keywords). Interestingly, with asymmetric firms, the search engine may in fact benefit from poaching—its revenue is maximized when the poaching is *controlled* but not prohibited. By employing appropriate keyword relevance scores, the search engine continues to under-weight the bid of the poaching firm only to the extent that it still prefers to poach. At this point, it can capture the full advertising budget of the smaller firm. While the larger firm moves a larger fraction of its advertising budget to traditional awareness-generating advertising, this effect is small (as compared to the case of symmetric firms). Using keyword relevance scores, the search engine is therefore capturing the budget of the smaller firm, while at the same time effectively protecting the larger firm by charging the poaching firm higher prices. Hence, keyword relevance measures could be interpreted as a complex price discrimination mechanism: for the weak firm, poaching is a very desirable option; for this reason, the search engine can charge the weak firm a higher price than the strong firm which is creating the search volume. This result explains why search engines are in support of allowing bidding on trademarked keywords by competitors,[3] yet still employ keyword relevance measures to handicap poaching firms. We also present several extensions of our model to show the robustness of our results.

A growing theoretical and empirical literature on sponsored search advertising has enhanced our understanding of its different aspects; this includes Athey and Ellison (2009), Athey and Nikipelov (2010), Chan and Park (2010), Chen and He (2006), Desai et al. (2011), Ghose and Yang (2009), Jerath et al. (2011), Katona and Sarvary (2010), Liu et al. (2010), Park and Park (2010), Rutz and Bucklin (2007, 2011), Yang and Ghose (2010), Yao and Mela (2009) and Zhu and Wilbur (2011). Our work is distinctly different from the above work because they consider sponsored search advertising in isolation while we model it in a multichannel advertising setting. Joo et al. (2011) empirically show that television advertising increases Internet search volume; we use their finding as a building block in our model. Kim and Balachander (2010) model sponsored search in a multichannel setting. However, they do not consider poaching behavior of competing firms, and the resulting strategic response of the search engine (in terms of auction design). In our research,

[3] `http://searchengineland.com/bing-yahoo-align-with-googles-trademark-rules-64902`

the analysis of these two aspects leads to a rich set of results and insights which have anecdotal support. Finally, Bass et al. (2005) show the existence of free-riding in traditional advertising where one firm focuses on generic advertising to expand the market and its competitor focuses on brand advertising to steal market share. We also show the existence of free-riding effects, due to poaching, in our model. However, our study is very different from theirs because in sponsored search firms can target which customers to free ride on (for instance, by poaching only those who search a competitor's keyword), which is not possible in the scenario they consider. Moreover, we have search engine as a strategic agent in our model, an element which has no parallel in their model.[4]

The rest of the paper is structured as follows. In Section 2, we describe the model. In Section 3, we analyze the model with symmetric firms and discuss the firms' strategies as well as the equilibria of the game. In Section 4, we analyze the model with asymmetric firms. In Section 5, we analyze "keyword relevance scores" as a strategic device used by a search engine to control poaching, and show how it affects the search engine's revenue. In Section 6, we consider several extensions of the basic model and show that the key insights are unchanged under each extension, while we obtain additional interesting results. Finally, in Section 7, we conclude with a discussion and lay out some possible directions for future research.

2 Model

Our model consists of three entities: the firms, the users, and the search engine. Two firms, Firm 1 and Firm 2, produce identical products. Each firm has an exogenously specified total budget B

[4] We note that "downstream advertising," wherein the aim is to reach customers expected to have a high likelihood of purchase, is also possible in certain channels other than sponsored search. For instance, firms may advertise in yellow pages to reach customers when they are specifically looking for the provider of a product or service before making a purchase. However, targetability is weak in yellow pages which makes it difficult to poach a competitor's customers; for instance, among the customers who are consulting yellow pages, firms cannot distinguish between those who are interested in a competitor versus those who are already interested in the firm itself. Similarly, "checkout coupons" used in retail stores, powered by technology from providers such as Catalina Marketing, target customers based on their profiles (purchase history, gender, location, etc.). This allows targeting consumers who purchased a competitor's product in a category (Pancras and Sudhir 2007). However, in this case, the identification of the customer and subsequent targeting is after the current purchase is made (with the idea of making the customer switch at the next purchase occasion), which makes poaching less effective. Sponsored search, on the other hand, makes for a unique combination of features that make it an extremely effective channel for poaching—based on the keyword searched consumers self-identify whether they are interested in the competitor, the firm itself, or the category; the customers can be targeted after they are interested in the product but still before the purchase; based on the keyword searched, different customers can be targeted differently by showing them different ad copies and landing pages upon clicking.

allocated for advertising,[5] and has to decide how to allocate its advertising budget to traditional advertising and sponsored search advertising. We denote the money spent on traditional advertising by Firm i by T_i and the money spent on sponsored search by S_i, where $T_i + S_i = B$. We bundle all non-sponsored search channels of advertising together into traditional advertising.

As discussed earlier, we focus on the awareness-creating aspect of traditional advertising. When Firm i spends T_i on traditional advertising, it generates awareness for its product among $(1+\alpha)T_i$ customers, where $\alpha > 0$. These customers either buy the product directly or search for the product at a search engine. Each firm is associated with a specific keyword which consumers use to search for it on the search engine. For instance, if Apple sells the iPad and Samsung sells the Galaxy Tab, then the keywords associated with Apple and Samsung are "iPad" and "Galaxy Tab," respectively.[6] For simplicity, we assume that there is only one advertising slot available for each keyword, i.e., only one firm is shown in response to a keyword search. This simplification does not impact the insights from the model. When a customer searches for a keyword, the search engine uses a pay-per-click second-price auction with exogenous reserve price R to sell the advertising slot for that keyword to the firm that bids higher. However, when a consumer clicks on the sponsored link, the winner has to pay the loser's bid or the reserve price, which ever is higher.

The transaction of a customer who searches the product is either influenced by the sponsored links or not. It is not important in our model whether a customer purchases directly from the firm after being exposed to a traditional ad or searches but ignores the sponsored search results and then purchases from the firm. Therefore, without loss of generality, we assume that all the customers who search the product are influenced by sponsored search results. Specifically, we assume that out of the $(1+\alpha)T_i$ customers made aware by traditional advertising, αT_i customers buy the product independent of what they see in sponsored search,[7] and the remaining T_i customers carry out a search for Firm i's keyword at a search engine and purchase the product that they see in the sponsored search section of the results (which may or may not not be Firm i's product). The scaling assumptions above basically imply that the ratio of the number of transactions that were

[5] We make the budget endogenous in Section 6.4 and confirm that the results of our basic model are robust.

[6] We later consider the extension in which there is a third keyword which is the category keyword, such as "tablet" in the above example.

[7] Note that we are implicitly assuming that advertising response function is linear; however, our results apply to other response functions proposed in the literature like concave and S-shaped functions as well. Details of this analysis are available upon request.

not influenced by sponsored search to those that were influenced is α.

Note that out of the $(1+\alpha)T_i$ customers who are exposed to traditional advertising, only αT_i make a purchase directly. The remaining T_i customers overflow to the sponsored search channel and all of them purchase from the firm whose ad they see in response to their search. Therefore, the customers who are exposed to traditional advertising and are "upstream" in the AIDA framework have a smaller purchase-conversion rate (conversion rate equal to $\alpha/(1+\alpha)$) than the customers who are "downstream" in the AIDA framework and are exposed to sponsored search advertising (conversion rate equal to 1). Consequently, there is a trade-off between traditional advertising and sponsored search. On the one hand, the firm can decide to create awareness through traditional advertising and obtain some direct sales. On the other hand, the firm may choose to advertise on the competitor's keyword in sponsored search and rely on the awareness created by the competitor.

Key Intermediate Result

Due to each firm's traditional advertising, T_i customers search keyword i at the search engine. These customers arrive sequentially at the search engine and it runs a separate auction for each customer. In other words, the search engine sequentially runs T_i auctions for each keyword. In each auction, the firms submit their bids simultaneously. Each firm decides its bid in an auction based on the budget it has allocated for the keyword and how much of it is remaining when a specific customer arrives. Using subgame-perfect equilibrium, we show in Theorem A4 in the appendix that the unique outcome of this sequential second-price auction coincides with the outcome of a *market-clearing-price mechanism*.[8] We state this result in the lemma below.

Lemma 1 *Assume that Firm 1 spends L_1 and Firm 2 spends L_2 on keyword i. If $L_1 + L_2 \geq T_i R$ then $L_1/(L_1 + L_2) \cdot T_i$ customers purchase from Firm 1, and $L_2/(L_1 + L_2) \cdot T_i$ customers purchase from Firm 2. If $L_1 + L_2 < T_i R$ then L_1/R customers purchase from Firm 1 and L_2/R customers purchase from Firm 2.*

This result is interesting in itself and, to the best of our knowledge, is new to the auctions literature. This is also a very useful result as, for the analysis in the rest of the paper, it allows

[8]This result is robust to different variations. For instance, if all of the customers arrive at once, or if the firms cannot change the bids for each customer, or if the search engine uses a first-price auction instead of a second-price auction, we get the same outcome as in Lemma 1.

us to reduce bidding in a complicated sequential auction to a much simpler form that abstracts away from the auction and, in fact, represents a simple market-clearing allocation. In the following analysis, rather than modeling bidding between competitors in each and every scenario, we will simply use this lemma repeatedly.

Defining the Firms' Strategies

In general, a firm's strategy is any splitting of advertising budget between the traditional channel, its own keyword in sponsored search, and the competitor's keyword in sponsored search. For simplicity, we restrict the strategy space of the firms to three strategies, each focusing on one of the three channels. Specifically, we allow a firm to follow one of the following three pure strategies. A firm's strategy can also be a mixed strategy, meaning that each of the strategies below will be played with a certain probability.

1. Own Keyword Focus (Own): The firm focuses on its own keyword in sponsored search advertising, trying to target the consumers who are in late stages of purchases process.

2. Traditional Focus (Traditional): The firm focuses on traditional advertising, trying to create awareness and interest about the product.

3. Poaching Focus (Poaching): The firm focuses on the competitor's keyword in sponsored search, trying to steal potential customers of the competitor.

Let T^O, T^T and T^P be the amount of money that a firm spends on traditional advertising when using the Own, Traditional and Poaching strategies, respectively, the superscripts O, T and P stand for Own, Traditional and Poaching respectively.[9] We now consider these strategies one by one.

In the *Own strategy*, the firm focuses on its own keyword. A natural definition would be to assume that the firm spends all of its budget for advertising on its own keyword in sponsored search. However, this implies that the keyword will have zero search volume because nothing has

[9] Note that if $R > 1/\alpha$, then it is a strictly dominant strategy for the firms to spend all of their budget on traditional advertising. In other words, if the reserve price is so high that the cost of attracting a customer in sponsored search is higher than the cost in traditional advertising, the firms should spend all of their budgets on traditional advertising in any strategy. In reality, this situation is unlikely to happen because it means that the search engine has set the reserve price of sponsored search advertising so high that no advertiser wants to advertise on sponsored search.

been spent on awareness-generating traditional advertising, which implies that there will be no revenue. In other words, even when the firm wants to "maximally focus" on its own keyword, it should not spend all of its budget on its keyword in sponsored search, and should transfer some of its budget to traditional advertising to generate the necessary search volume for its keyword. In fact, we will define the budget allocation in the Own strategy in such a way that *any* strategy that allocates more budget to the firm's own keyword as compared to the Own strategy will always be *weakly dominated* by the allocation of the Own strategy. In this sense, the "Own Keyword Focus" strategy has "maximal focus" on the firm's own keyword. We now derive this allocation.

According to the second-price auction of the search engine, the firm has to pay at least R per customer in sponsored search advertising. And according to the model, if it spends T_i on traditional advertising, T_i customers would search the product. Therefore, the firm has to spend at least $T^O = \frac{B}{R+1}$ on traditional advertising even if it wants to focus on sponsored search advertising of its own keyword. Consequently, the firm spends $B - T^O = \frac{RB}{R+1}$ on its own keyword when using Own strategy. In general, it can be proved that spending more than $B - T^O$ for sponsored search advertising of own keyword is *weakly dominated* by spending $B - T^O$.

In the *Traditional strategy*, the firm focuses on traditional advertising. Similar to the Own strategy, we can show that spending all of budget on traditional advertising is a dominated strategy. As before, we define the Traditional strategy in a way that the firm has "maximal focus" on traditional advertising, i.e., under no conditions should the firm have an incentive to allocate more to traditional advertising than what it allocates in this strategy. Suppose that the firm has budget B and its competitor has budget B'. In the Traditional strategy, the amount of budget that the firm spends on traditional advertising is defined as $T^T = \min(B, \frac{B+B'}{R+1}, B+B'-\sqrt{\frac{B'(B+B')}{1+\alpha}})$. It can be proved that spending more than T^T on traditional advertising is weakly dominated by spending T^T on traditional advertising. In other words, no matter what strategy the competitor uses, the profit of the firm when spending T^T on traditional advertising is always greater than or equal to its profit when spending more than T^T on traditional advertising. Consequently, in the Traditional strategy, the firm spends $B - T^T$ for sponsored search advertising on its own keyword.

Finally, in the *Poaching strategy*, the firm spends all of its budget for sponsored search advertising on the competitor's keyword. Hence, we have $T^P = 0$.

It is critically important to note here that the above allocation of budget is *not* something that the players of the game are doing as part of the game. On the contrary, we as researchers are defining the strategies to set the strategy space such that the pure strategies are undominated, i.e., when a firm is following a strategy of focusing on one of the three types of advertising, the allocation is such that the firm spends maximum amount of budget on that form of advertising but still uses an undominated strategy. When the players play the game, they will pick one of these strategies to play, or play a mixed strategy.

Also note that, for simplicity, we are not considering any pure strategy other than the above three. In Section A5 in the appendix, we consider a more general model in which we allow firms to choose any allocation of advertising budget among traditional advertising, advertising on its own keyword in sponsored search, and poaching by advertising on the competitor's keyword in sponsored search. We show that the results and insights obtained in our basic model here are not affected. Specifically, the set of equilibria of this simpler model is the same as the set of equilibria of the more general model. This equivalency highlights the appropriateness of the strategy space in the simplified model.

The order of moves in the model is as follows. First, the search engine announces the rules of the auction (that it is a second-price, pay-per-click auction).[10] Second, the two firms simultaneously decide their budget allocation strategies. Third, consumers see traditional ads and a fraction $\alpha/(1+\alpha)$ of them buy directly from the firm whose ad they saw. Fourth, the remaining consumers go to the search engine sequentially and search the keyword of the firm whose traditional ad they saw, and the sequential second-price auction is played out. Fifth, each consumer who searches, purchases from the firm that is shown to her in the sponsored search results.

Finally, note that we have assumed the price of the product to be exogenous. We make this choice to focus solely on competition between firms in the sponsored search auction, and not confound it with price competition. In Section 6.6, we allow for price competition as well and confirm that the insights we obtain from our basic analysis hold.

[10] In the "keyword relevance" extension in Section 5, the search engine also decides and announces the relevance score multiplier for a bid by a firm on a competitor's keyword.

3 Analysis with Symmetric Firms

Examining the case of symmetric firms gives us some basic insights which make the subsequent analysis with asymmetric firms easier to understand.

3.1 Revenue Analysis

We use $\Pi^{i,j}$, where $i, j \in \{O, T, P\}$, to denote the revenue of a firm that is using strategy i while its competitor is using strategy j. For example, $\Pi^{P,O}$ denotes the revenue of a firm playing the Poaching strategy whose competitor is playing the Own strategy.

Own Strategy: According to definition of Own strategy, the revenue of a firm that is playing the Own strategy, as long as its competitor does not poach, will be $\Pi^{O,O} = \Pi^{O,T} = T^O(1+\alpha)$. However, if the competitor poaches, by Lemma 1, its revenue will be $\Pi^{O,P} = T^O(\alpha + \frac{B-T^O}{2B-T^O})$. The search engine's revenue from a firm that is playing the Own strategy will be $B - T^O$.

Traditional Strategy: According to definition of Traditional strategy, the revenue of a firm that is playing the Traditional strategy, as long as the other firm does not poach, will be $\Pi^{T,O} = \Pi^{T,T} = \alpha T^T + \min(\frac{B-T^T}{R}, T^T)$. However, if the competitor poaches, by Lemma 1, its revenue will be $\Pi^{T,P} = \alpha T^T + T^T(\frac{B-T^T}{2B-T^T})$. The search engine's revenue from a firm that is playing the Traditional strategy will be $B - T^T$.

Poaching Strategy: Consider a firm that is playing the Poaching strategy. If the competitor also poaches simultaneously, no money is spent on traditional advertising and hence no customer is gained. Therefore, the revenue of both firms will be zero, $\Pi^{P,P} = 0$. However, if the competitor plays the Own strategy, the revenue of the poaching firm will be $\Pi^{P,O} = T^O \frac{B}{2B-T^O}$. If the competitor plays the Traditional strategy, the revenue of the poaching firm will be $\Pi^{P,T} = T^T \frac{B}{2B-T^T}$.

Notice that a firm should not play Poaching if the competitor is playing Poaching (because $\Pi^{P,P} = 0$, which is less than both $\Pi^{O,P}$ and $\Pi^{T,P}$). Similarly, a firm should not play Poaching if the competitor is playing Own (because $\Pi^{P,O} < \Pi^{O,O}$). We find that the only way that a firm can benefit from playing Poaching is if the competitor plays Traditional, i.e., $\Pi^{P,O} < \Pi^{O,O}$. Furthermore, note that poaching can be beneficial only if $\Pi^{P,T} > \Pi^{O,T}$ (note that $\Pi^{O,T} > \Pi^{T,T}$

already holds), which gives the following condition.

$$R > \sqrt{\frac{2(1+\alpha)^3}{(1+2\alpha)^2}} - \frac{\alpha}{1+2\alpha}. \tag{1}$$

The above condition implies that Poaching is profitable only if R is large enough. Intuitively, if R is small, the firm finds it more profitable to conduct its own traditional advertising and capture the customers that overflow into search at the cheap reservation price, thus avoiding competition with the other firm. However, as R becomes larger, the customers from sponsored search do not come cheap any more. When R is large enough, the firm finds it more profitable to free ride on the awareness generation of the other firm (i.e., not spend anything from its own budget on awareness generation) and, in fact, use all of its advertising budget to compete with the other firm in the auction. Finally, the search engine's revenue from a firm playing Poaching will always be B unless the other firm is also playing Poaching, in which case the search engine's revenue will be 0.

3.2 Equilibrium Analysis

Each firm can pick one of the three strategies for allocating its advertising budget. This leads to the two-person normal-form game depicted in Table 1.

	Poach	Own	Traditional
Poach	$(\Pi^{P,P}, \Pi^{P,P})$	$(\Pi^{P,O}, \Pi^{O,P})$	$(\Pi^{P,T}, \Pi^{T,P})$
Own	$(\Pi^{O,P}, \Pi^{P,O})$	$(\Pi^{O,O}, \Pi^{O,O})$	$(\Pi^{O,T}, \Pi^{T,O})$
Traditional	$(\Pi^{T,P}, \Pi^{P,T})$	$(\Pi^{T,O}, \Pi^{O,T})$	$(\Pi^{T,T}, \Pi^{T,T})$

Table 1: Payoff matrix of the firms' strategies

Nash Equilibria: The game has both pure- and mixed-strategy Nash equilibria. Since $\Pi^{O,O} \geq \Pi^{P,O}$ and $\Pi^{O,O} \geq \Pi^{T,O}$, both firms using Own strategy is always a pure-strategy Nash equilibrium. One firm using Poaching and the other firm using Traditional may or may not be a pure-strategy Nash equilibrium. If the reserve price R is large enough such that the inequality in (1) holds, since $\Pi^{P,T} \geq \Pi^{O,T} \geq \Pi^{T,T}$ and $\Pi^{T,P} \geq \Pi^{O,P} \geq \Pi^{P,P}$, one firm using Poaching and the other firm using Traditional is also a pure-strategy Nash equilibrium; otherwise, it is not. Note that, even though the two firms are symmetric, the above is an asymmetric equilibrium in which one firm spends all of its budget on poaching while the other firm spends a larger portion of its budget on traditional advertising (as compared to the case when the competitor is not poaching).

The mixed-strategy equilibria of the game always conform to the following pattern. When R is small, only the (Own, Own) equilibrium is obtained in which no firm is poaching. As R increases and the inequality in (1) holds, two new types equilibria arise. One is the (Poach, Traditional) equilibrium discussed above (and its symmetric counterpart, the (Traditional, Poach) equilibrium). The third type of equilibrium is a mixed-strategy equilibrium in which one firm mixes between Poach and Own, and the other firm mixes between Traditional and Own. Note that the mixed-strategy equilibrium is also an asymmetric equilibrium. As R increases further and becomes larger than $1/\alpha$, both firms allocate all their budget to traditional advertising.

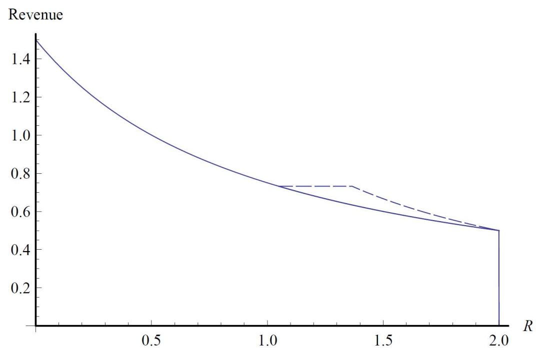
Figures 3(a) and 3(b) denote the revenues of firms. For clarity in the graphs for the cases with asymmetric equilibria, we designate one firm as the "poaching firm" (this firm always poaches in the (Poach, Traditional) equilibrium and mixes between Poach and Own in the mixed equilibrium) and the other firm as the "Traditional firm" (this firm always uses Traditional in the (Poach, Traditional) equilibrium and mixes between Traditional and Own in the mixed equilibrium). We can observe from Figure 3(a) that when poaching equilibria exist, the poaching firm's revenue is higher than in the non-poaching (Own, Own) equilibrium, while the non-poaching firm's revenue is lower. In other words, the non-poaching firm is accommodating the poaching firm's free-riding behavior.

Search Engine's Revenue: Different equilibria of the game have different revenue expressions for the search engine. Table 2 summarizes the search engine's revenue in each of the outcomes.

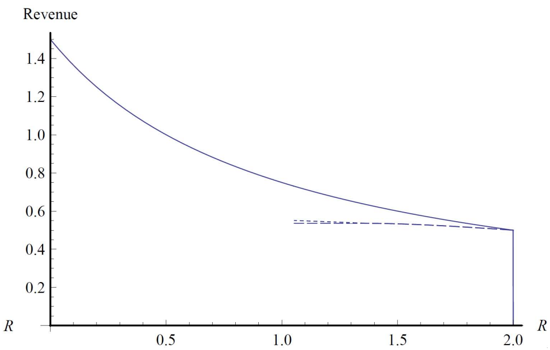
	Poach	Own	Traditional
Poach	0	$2B - T^O$	$2B - T^T$
Own	$2B - T^O$	$2B - 2T^O$	$2B - T^O - T^T$
Traditional	$2B - T^T$	$2B - T^O - T^T$	$2B - 2T^T$

Table 2: Search engine's payoff matrix

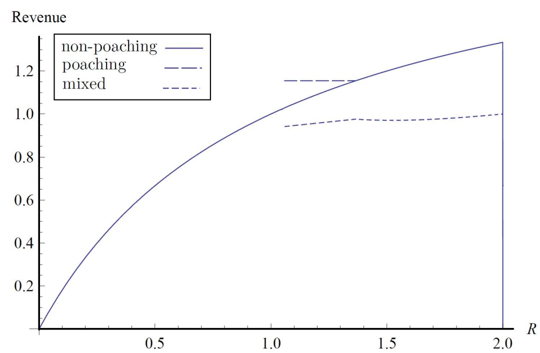
The search engine's revenue is depicted in Figure 3(c). First, note that this revenue increases in R until $R = 1/\alpha$ where it drops to zero. Second, poaching does not happen for low values of R. For high values of R, multiple equilibria exist and the search engine's revenue is the same from the (Own, Own) and the (Poach, Traditional) equilibria, which is larger than the revenue from the mixed equilibrium. Since there is a positive likelihood of the low-revenue mixed-strategy equilibrium existing, this implies that the existence of poaching may lower the search engine's



(a) Poaching Firm's Revenue



(b) Traditional Firm's Revenue



(c) Search Engine's Revenue

Figure 3: Revenue plots for $B = 1$ and $\alpha = 0.5$.

revenue for high values of R (even though poaching is increasing competition in the search engine's auction). This happens because the Traditional firm allocates more of its advertising budget to traditional advertising. The following proposition summarizes this important result.

Proposition 1 *Symmetric firms may follow asymmetric budget allocation strategies in which one firm allocates a larger fraction of its advertising budget to poaching on its competitor in sponsored search advertising, while the other firm allocates a larger fraction of its advertising budget to awareness-generating traditional advertising. Furthermore, the search engine's revenue may decrease in the presence of poaching.*

4 Asymmetric Firms with Different Advertising Budgets

In this section, we relax the symmetry assumption by assuming that the firms may have different advertising budgets. Without loss of generality, through scaling, we assume that the budget of one firm (the weak firm, denoted by subscript W) is 1, and the budget of the other firm (the strong firm, denoted by subscript S) is $B \geq 1$.

Definitions of Strategies

In this asymmetric firms case, we rederive the budget allocations for strong and weak firms based on the core idea that when a firm is following a strategy of focusing on one of the three types of advertising, the allocation is such that the firm spends maximum amount of budget on that form of advertising but still uses an undominated strategy.

Own Strategy: We have $T_W^O = \frac{1}{R+1}$ and $T_S^O = \frac{B}{R+1}$.

Traditional Strategy: We have $T_W^T = 1 + B - \sqrt{\frac{B(1+B)}{1+\alpha}}$, if $\frac{B+1}{R+1} \geq 1 + B - \sqrt{\frac{B(1+B)}{1+\alpha}}$, and $T_W^T = \frac{B+1}{R+1}$ otherwise. Similarly, we have $T_S^T = 1 + B - \sqrt{\frac{1+B}{1+\alpha}}$, if $\frac{B+1}{R+1} \geq 1 + B - \sqrt{\frac{1+B}{1+\alpha}}$, and $T_S^T = \frac{B+1}{R+1}$, otherwise.

Poaching Strategy: We have $T_W^A = T_S^A = 0$.

Revenue Analysis

Own Strategy: If no firm poaches on the keyword of the other, the situation is very similar to the symmetric case. Particularly, we have $\Pi_W^{O,O} = \Pi_W^{O,T} = T_W^O(1+\alpha)$ and $\Pi_S^{O,O} = \Pi_S^{O,T} = T_S^O(1+\alpha)$. If the strong firm poaches, $\Pi_W^{O,P} = T_W^O(\alpha + \frac{1-T_W^O}{B+1-T_W^O})$. Similarly, if the weak firm poaches $\Pi_S^{O,P} = T_S^O(\alpha + \frac{B-T_S^O}{1+B-T_S^O})$.

Traditional Strategy: The revenue of the weak firm, if the other firm poaches, is $\Pi_W^{T,P} = \alpha T_W^T + T_W^T \frac{1-T_W^T}{B+1-T_W^T}$. However, if the strong firm does not poach, the revenue of the weak firm is $\Pi_W^{T,O} = \Pi_W^{T,T} = \alpha T_W^T + \min(T_W^T, \frac{1-T_W^T}{R})$. The revenue of the strong firm is $\Pi_S^{T,P} = T_S^T \alpha + (\frac{B-T_S^T}{1+B-T_S^T}) T_S^T$, and $\Pi_S^{T,O} = \Pi_S^{T,T} = \alpha T_S^T + \min(T_S^T, \frac{B-T_S^T}{R})$.

Poaching Strategy: For the strong firm, we have $\Pi_S^{P,P} = 0, \Pi_S^{P,O} = T_W^O(\frac{B}{B+1-T_W^O})$ and $\Pi_S^{P,T} = T_W^T(\frac{B}{B+1-T_W^T})$. For the weak firm, we have $\Pi_W^{P,P} = 0, \Pi_W^{P,O} = T_S^O(\frac{1}{1+B-T_S^O})$, $\Pi_W^{P,T} = T_S^T(\frac{B}{1+B-T_S^T})$.

Equilibrium Analysis

The analysis in the previous section with symmetric firms provided basic insights. We now solve the game with asymmetric firms analytically in a complete manner. We provide the results and insights here; the derivations are provided in Section A1 in the appendix.

We use the following terminology for brevity. When describing equilibria, we assume that the first firm is the weak firm, and the second firm is the strong firm. For example, by (Poach, Traditional) equilibrium we mean an equilibrium in which the weak firm poaches and the strong firm uses Traditional. We will also see two types of mixed equilibria. In the first mixed equilibrium, which we call the Weak-Poach-mixed equilibrium, the weak firm mixes between Poach and Own, and the strong firm mixes between Traditional and Own. In the second mixed equilibrium, which we call the Strong-Poach-mixed equilibrium, the weak firm mixes between Traditional and Own, and the strong firm mixes between Poach and Own.

The different equilibria that arise for different values of the strong firm's budget B and the reserve price R can be understood by jointly examining Table 3 and Figure 4. (Recall that the weak firm's budget is normalized to 1, so a larger value of B denotes larger asymmetry between the firms. Also, if we set $B = 1$, which corresponds to the top edge of the plot, we obtain the results with symmetric firms where the budget of each firm is 1.) The seven regions in Figure 4 are labeled $\mathcal{A}, \mathcal{B}, \ldots, \mathcal{G}$. (See the appendix for analytical expressions for $R^W, R^S, R^*, R^{**}, R^*_m$ and $\overline{R}$, and analytical definitions of low, medium and high asymmetry.) Table 3 summarizes the set of possible equilibria in each region as well as whether the search engine benefits from poaching or not. Each row of the table represents one region indicated in the first column. The second column shows the list of possible equilibria in that region with letters O, T and P standing for Own, Traditional and Poaching, respectively. The third column of the table indicates the impact of poaching on the search engine's profit in that region. The letter Y means that poaching can increase the search engine's profit, the letter N means that poaching decreases the search engine's profit and Y/N means that poaching can increase or decrease the search engine's profit depending on equilibrium selection.[11] For region $\mathcal{H}$, (Poach, Traditional) is the unique equilibrium, so we can trivially say that poaching increases search engine's revenue. For region $\mathcal{A}$, we cannot make such a comparison since there is no poaching equilibrium, and we use the symbol "–" to denote this.

From the above results, we find that the weak firm poaches in all regions except region $\mathcal{A}$ while

[11] More precisely, the letter Y means that the set of equilibria with poaching equilibria weakly dominates (from the search engine's perspective) the set of equilibria without poaching equilibria. The letter N means that set of equilibria without poaching equilibria weakly dominates the set of equilibria with poaching equilibria. Y/N means that the sets of equilibria with and without poaching equilibria cannot be compared, i.e., depending on equilibrium selection, either option may have higher revenue for the search engine. See the appendix for the definition of "weak dominance" to compare sets of equilibria.

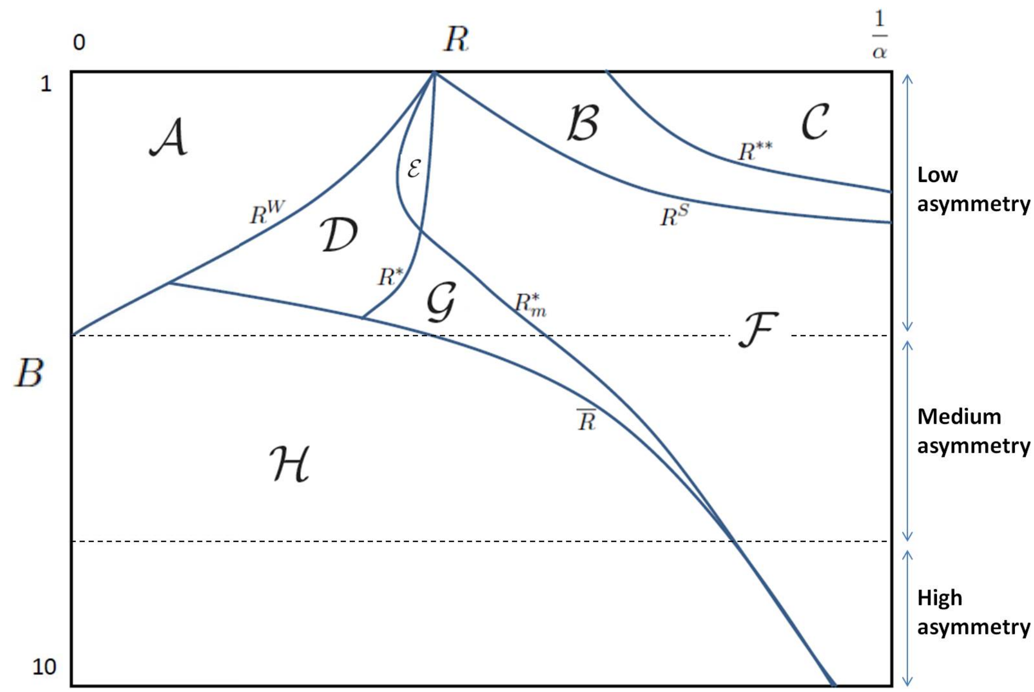
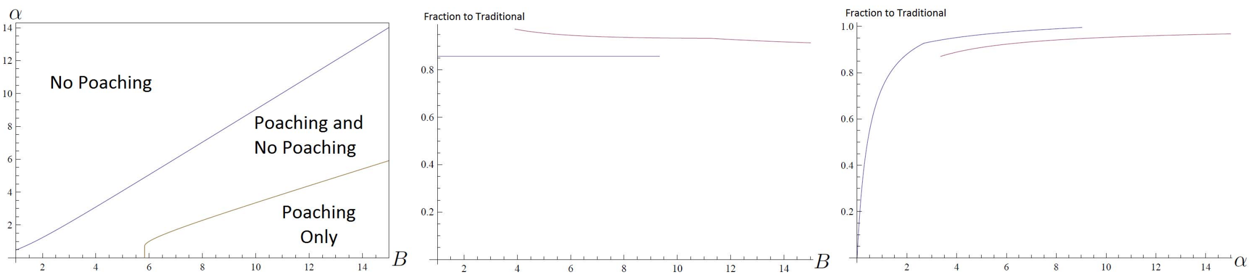


Figure 4: Poaching regions and search engine's revenue for different levels of budget asymmetry and different values of reserve price.

Region	Equilibria	Poaching Beneficial for Search Engine
$\mathcal{A}$	(O, O)	–
$\mathcal{B}$	(O, O), (P, T), (T, P), Weak-Poach-mixed, Strong-Poach-mixed	Y/N
$\mathcal{C}$	(O, O), (P, T), (T, P), Weak-Poach-mixed, Strong-Poach-mixed	N
$\mathcal{D}$	(O, O), (P, T), Weak-Poach-mixed	Y
$\mathcal{E}$	(O, O), (P, T), Weak-Poach-mixed	Y/N
$\mathcal{F}$	(O, O), (P, T), Weak-Poach-mixed	N
$\mathcal{G}$	(O, O), (P, T), Weak-Poach-mixed	Y
$\mathcal{H}$	(P, T)	Y

Table 3: Description of the regions in Figure 4

the strong firm poaches only in regions $\mathcal{B}$ and $\mathcal{C}$, i.e., the weak firm poaches in a larger parameter space as compared to the strong firm. We also show that the relative gain from poaching of the weak firm is larger than that of the strong firm (i.e., $\frac{\Pi_W^{P,T}}{\Pi_W^{O,O}} \geq \frac{\Pi_S^{P,T}}{\Pi_S^{O,O}}$). Moreover, the weak firm's incentive to poach increases with increasing budget asymmetry (i.e., $\frac{\Pi_W^{P,T}}{\Pi_W^{O,O}}$ is an increasing function of B). This is intuitively because the strong firm has a relatively large search volume; therefore,



(a) Poaching region

(b) Fraction of budget allocated to traditional advertising, by the firm being poached, as a function of B, for $\alpha = 3$

(c) Fraction of budget allocated to traditional advertising, by the firm being poached, as a function of α, for $B = 10$

Figure 5: Poaching region and the strategy of the firm being poached.

the poaching of the weak firm does not affect the sponsored search price significantly, and in turn, allows poaching at a relatively low price. In fact, if the firms are very asymmetric, the incentive to poach is so high that (Poach, Traditional) is the only equilibrium of the game (region $\mathcal{H}$). We state this in the proposition below.

Proposition 2 *When the advertising budget of one firm is larger than the advertising budget of the other firm, the lower-budget firm has a larger incentive to poach on the higher-budget firm. Moreover, if the asymmetry in budgets is large enough, poaching can increase the search engine's revenue.*

In the case of budget asymmetry, poaching may be beneficial for the search engine. This is intuitively because the weak firm can only steal a small fraction of the strong firm's customers. As a result, the strong firm's response to the weak firm's poaching is not as significant as in the case of symmetric firms, i.e., the strong firm does not shift a lot of its budget from sponsored search to traditional advertising in response to poaching. On the other hand, the weak firm spends all of its money on sponsored search. Hence, the search engine's revenue may increase in the presence of poaching. We state this in the proposition below.

Proposition 3 *If the asymmetry in the advertising budgets of firms is large enough, poaching can increase the search engine's revenue.*

We now examine the firms' strategies as functions of B (the budget asymmetry between firms) and α (the relative effectiveness of traditional advertising) for given exogenous R. The plot in Figure 5(a) is representative of the regions in which poaching occurs in the B-α space. Two interesting observations can be made from this figure. First, for a given level of α, poaching happens only if budget asymmetry is large enough. Intuitively, poaching becomes more attractive for a firm as its competitor's search volume becomes larger. Therefore, for a given α, poaching happens only when B is large enough.[12]

Second, for a fixed level of B, poaching does not happen if α is large enough, i.e., if the proportion of the consumers who are not influenced by sponsored search is large enough. Intuitively, if the proportion of the consumers who are influenced by sponsored search is small, trying to compete for and poach on those consumers is not a good strategy.[13] These two observations are summarized in the following proposition.

Proposition 4

(a) For a given level of budget asymmetry B between the firms, poaching happens only if the proportion of consumers who are influenced by sponsored search advertising is large enough.
(b) For a given proportion α of consumers who are not influenced by sponsored search advertising, poaching happens only if the budget of one firm is enough larger than the other firm.

Figures 5(b) and 5(c) explain the strong firm's strategy as a function of B and α. Due to the change in the weak firm's strategy (from poaching to not poaching or vice versa), the strong firm's strategy is not monotone in either graph. Moreover, because of the existence of multiple equilibria (poaching and non-poaching), there are two curves depicting the advertising strategy of the firm each corresponding to one equilibrium. In Figure 5(b), poaching happens only for $B > 3.90$. When $3.90 < B < 9.33$, both poaching and non-poaching equilibria exist. Finally, for $B > 9.33$, only poaching equilibrium exists. The jump in the percentage of budget allocated to traditional advertising in poaching equilibrium reflects the Traditional strategy of the strong firm. After the jump, we see that the percentage gradually decreases as B increases. This is because the strong firm is hurt less, and its response to poaching is moderated, as the level of asymmetry increases.[14]

In Figure 5(c), poaching happens only when $\alpha < 9.03$. Furthermore, if $\alpha < 3.35$, the only equilibrium is poaching equilibrium. Note that within each equilibrium (poaching or non-poaching),

[12]Mathematically, for poaching to happen we need $\Pi_W^{O,O} \leq \Pi_W^{P,T}$. In other words, poaching must be more profitable than the Own strategy for the weak firm to poach, given that the strong firm uses Traditional. It is easy to see that $\Pi_W^{P,T} - \Pi_W^{O,O}$ is increasing in B.

[13]Mathematically, this can be verified by observing that $\Pi_W^{P,T} - \Pi_W^{O,O}$ is decreasing in α.

[14]One can see that the percentage of traditional advertising in Traditional strategy converges to that of Own strategy as B grows.

percentage of budget allocated to traditional advertising increases with α. This is expected because as the proportion of consumers who are not influenced by sponsored search increases, the percentage of budget allocated to sponsored search advertising should decrease. We see that when switching from poaching to non-poaching equilibrium, the percentage of budget that the strong firm allocates to traditional advertising suddenly drops. This drop is because the strong firm changes its strategy from Traditional to Own. However, the percentage again increases as α increases.

5 Keyword Relevance Measures

All the major search engines transform an advertiser's submitted bid into an *effective bid* before determining the outcome of the sponsored search auction. A multiplier is typically used to compute the effective bid, and this multiplier depends on many parameters including the advertiser's past performance in terms of the click-through-rate (the probability that a customer who sees the advertiser's sponsored link clicks on the link), the quality and reputation of the advertiser's product or website, and the *relevance of the keyword* being bid on to the advertiser. Our focus here is on the last parameter and we explain it using the example below.

Consider the keyword "iPad" and the two firms Apple and Samsung. Apple is much more relevant to this keyword than Samsung, since Apple produces the iPad while Samsung only sells a competing product, namely Galaxy Tab, in the same category (tablets). Therefore, if the relevance of Apple to the keyword "iPad" is 1 on a scale from 0 and 1, the relevance of Samsung to this keyword is less than 1 and is, say, 0.5. For simplicity, assume that both firms have the same scores on other parameters used to calculate the multiplier for calculating the effective bid (click-through-rate, quality reputation, etc.). Suppose that Apple bids \$1 per click and Samsung bids \$1.5 per click to be displayed in response to the keyword "iPad." It seems natural that the search engine should prefer to display Samsung instead of Apple in this case (assuming only one is displayed) as Samsung should generate more revenue than Apple for it. However, surprisingly, in a situation like this, the search engine decides to display Apple because of higher relevance to the keyword. In fact, Samsung will have to bid and pay at least $\$1/0.5 = \$2$ to win this keyword. If Samsung bids \$1.5, Apple wins the keyword and has to pay only \$0.75 per click.

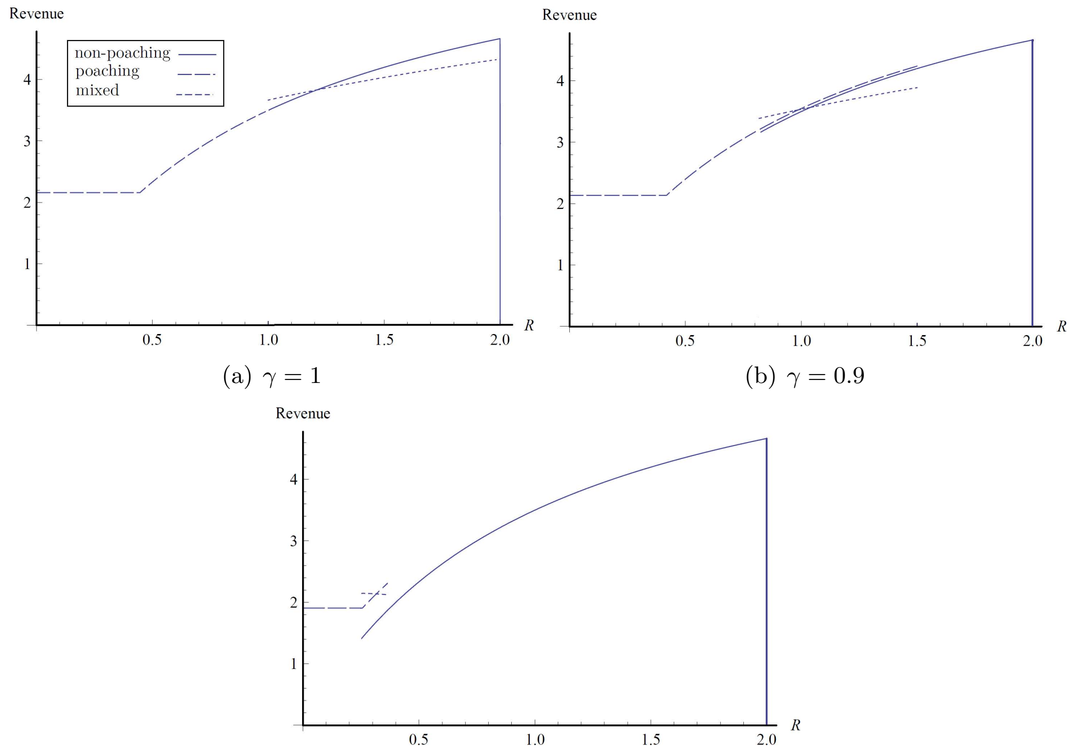
One explanation for the existence of "relevance measures" is that the search engine wants to improve user experience by showing ads most directly relevant to the keyword searched by the users. Although this is a reasonable explanation, we argue that it is probably not the only explanation.[15] We provide an alternative explanation—search engines may use keyword relevance measures to handicap poaching to the extent they want. In this section, we show that by employing the appropriate relevance factors, the search engine can increase its revenue. To simplify and only focus on the effect of relevance measures, we assume that both firms have the same click-through-rate, and the same quality and website reputation.

We assume that if a firm wants to bid on the keyword of the other firm, its bid (the bid of the poaching firm) will be multiplied by $0 \leq \gamma \leq 1$. If $\gamma = 1$, we are in the framework that we have been in so far: firms poach on each others' keywords without any handicap. On the other extreme, if $\gamma = 0$, firms can not bid on each others' keyword. This is similar to the situation where bidding on trademarked keywords is not allowed. We study the effect of intermediate values of γ on the search engine's revenue. To allow for asymmetric firms, we stay with the setting where one firm has budget $B \geq 1$ while the other firm has budget 1.

Definitions of Strategies

As before, we rederive the budget allocations for strong and weak firms based on the core idea that when a firm is following a strategy of focusing on one of the three types of advertising, the allocation is such that the firm spends maximum amount of budget on that form of advertising but still uses an undominated strategy. The Own and the Poaching strategies remain the same. However, the Traditional strategy changes slightly because the firm using Traditional strategy now knows that the bid of the Poaching firm is not as effective as it was when there was no poaching handicap. As a result, we have $T_W^T = 1 + \gamma B - \sqrt{\frac{\gamma B(1+\gamma B)}{1+\alpha}}$, if $\frac{\gamma B+1}{R+1} \geq 1 + \gamma B - \sqrt{\frac{\gamma B(1+\gamma B)}{1+\alpha}}$, and $T_W^T = \frac{\gamma B+1}{R+1}$, otherwise. Similarly, $T_S^T = \gamma + B - \sqrt{\frac{\gamma(\gamma+B)}{1+\alpha}}$, if $\frac{B+\gamma}{R+1} \geq \gamma + B - \sqrt{\frac{\gamma(\gamma+B)}{1+\alpha}}$, and $T_S^T = \frac{B+\gamma}{R+1}$, otherwise. The expressions are essentially the same as derived in Section 4, except that the when the weak firm uses Traditional against the strong firm's poaching it takes the strong firm's budget as γB instead of B and, similarly, when the strong firm uses Traditional against the

[15] Note that the argument of improving user experience has some weaknesses. First, organic and sponsored results are clearly demarcated from each other, and users will have higher expectations of directly relevant ads in the organic results, not in the sponsored results which are paid for. Second, if Samsung has a click-through-rate and quality reputation as high as Apple itself (as is the case in our example), then this indicates that users will appreciate Samsung's ad just as much as Apple's. From this point of view, reordering links will not improve user experience.



(a) $\gamma = 1$

(b) $\gamma = 0.9$

(c) $\gamma = 0.4$

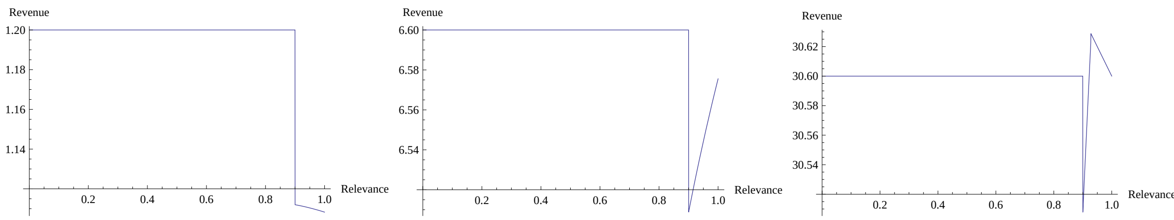
Figure 6: Revenue of the search engine as a function of the reserve price for different levels of relevance enforcement factor γ. The budget $B = 6$, $\alpha = 0.5$ and γ is set to three levels 1, 0.9 and 0.4.

weak firm's poaching it takes the weak firm's budget as $\gamma \cdot 1 = \gamma$ instead of 1.

Revenue and Equilibrium Analysis

The revenue and equilibrium computations are the same as in the previous cases. We omit the details here and focus on the results.

Figure 6 shows the search engine's revenues for three values of γ when $B = 6$, i.e., the advertising budget of the strong firm is six times the advertising budget of the weak firm. When $\gamma = 1$, for small values of R poaching is the only equilibrium; as R increases, non-poaching and mixed equilibria appear. Note that, in the poaching equilibria, the weak firm poaches and the strong firm uses Traditional. However, as we decrease γ to 0.9, non-poaching becomes the only equilibrium for high values of R. This is because decreasing γ handicaps poaching, and the effect of this handicap is more severe when R is large. Moreover, we see that for medium values of R, where both poaching and non-poaching equilibria exist, the revenue of the poaching equilibrium is greater than the revenue



(a) $B = 1$ (b) $B = 10$ (c) $B = 50$

Figure 7: Revenue of the search engine as a function of the relevance enforcement factor γ. The reserve price is $R = 1.5$, $\alpha = 0.5$ and B is set to three levels 1, 10 and 50.

of the non-poaching equilibrium. This difference becomes clearer as we decrease γ further down to 0.4.

Intuitively, in spite of being penalized, the weak firm poaches (for the same reasons as discussed in Section 4), in response to which the strong firm uses Traditional by moving more of its budget to traditional awareness-generating advertising. However, the "keyword relevance penalty" protects the strong firm to some extent by reducing the effective bid of the weak firm on its keyword, thus keeping bids from escalating too high. Shielded in this way, the strong firm does not need to shift as large a portion of its money to the traditional channel (as it would have done if poaching were not penalized). However, at the same time, the weak firm has to pay a higher price per click and the search engine extracts all the budget of the weak firm. In other words if the firms are asymmetric enough (the budget of one firm is enough larger than the other firm), the search engine benefits from penalizing (or handicapping) poaching. However, the penalty should only be up to the level where the weak firm still prefers to poach. In doing so, the strong firm will be protected and its response to poaching (i.e., moving more of its advertising budget to traditional advertising) will be moderated. Interestingly, keyword relevance measures could be interpreted as a complex *price discrimination mechanism.* For the weak firm, poaching is a very desirable option; for this reason, the search engine can charge the weak firm a higher price than the strong firm which is creating the search volume.

Figure 7 shows the search engine's revenue as a function of the relevance multiplier γ for different levels of asymmetry.[16] When the firms have similar budgets poaching hurts the search

[16]For the purpose of drawing Figure 7, wherever there are multiple equilibria, we take their simple average to calculate the search engine's revenue. We follow this somewhat non-standard approach for simplicity and clarity in the plots. The results are qualitatively robust to weighted average as well as to considering only one equilibrium at a time.

engine's revenue (as in Section 3). The search engine has the incentive to prevent poaching and its revenue is maximized at any small-enough γ. This is because any small-enough γ sets a high-enough penalty for poaching so that poaching does not happen. However, as the asymmetry increases, the search engine actually benefits from poaching at a medium level of penalty. This can be clearly observed in Figure 7 (c), where the peak at $\gamma = 0.9$ shows the best value of γ for the search engine. We summarize the above discussion in the following proposition.

Proposition 5 *If the advertising budget of the strong firm is large enough compared to that of the weak firm, the search engine handicaps poaching by competitors but does not prohibit it.*

The above proposition offers an explanation for why search engines are in support of allowing advertising by competitors on trademarked keywords (such as brand and company names), yet still employ keyword relevance measures to handicap poaching firms. Note also that in the above situation the weak firm practices poaching and benefits from it while the strong firm (with larger advertising budget) is hurt from poaching. The search engine also benefits from poaching by the weak firm. These results from the model support the observation that some leading firms in their respective industries (e.g., Rosetta Stone and Louis Vuitton) sued search engines in an effort to prevent them from following a policy of allowing bidding on trademarks by competitors.[17] The search engines won these lawsuits and have continued to allow poaching on trademarked keywords; at the same time, they continue to use keyword relevance scores to handicap poaching. In the above examples, the predictions from our model are in close agreement with the actual behavior of the string firms, the weak firms, and the search engines.

To summarize, our basic model shows that firms in an industry have the incentive to poach in sponsored search, especially firms with relatively smaller advertising budgets. The best response of competitors is to accomodate this poaching behavior. Surprisingly, even though poaching leads to more competition in its auctions, the search engine has the incentive to handicap poaching. We now proceed to extensions of the basic model.

[17] `http://paidcontent.org/article/419-trademark-lawsuit-against-googles-adwords-hits-the-appeals-court/`, `http://paidcontent.org/article/419-bing-will-run-ads-on-trademarked-searchesand-it-can-thank-google-for-th/`

6 Extensions

6.1 Category Keyword

In this extension, we assume that there is a category keyword which attracts customers from the traditional advertising of both firms. For example, in the context of tablets, "iPad" and "Galaxy Tab" are keywords specific to the companies Apple and Samsung, respectively, while "tablet" is a category keyword that describes both products. Some customers who see traditional awareness-generating ads of iPad or Galaxy Tab may search the keyword "tablet" instead of searching the product name. In accordance with this, we assume that some fraction of the customers who are exposed to traditional advertising of each firm search the category keyword instead of the firm-specific keyword. The insights obtained in Section 3 (asymmetric budget allocation strategies and reduction in search engine revenue due to poaching) also hold under this extension. More details are available in Section A2 in the appendix.

6.2 Reputation Effects

In the basic model, we assumed that a firm needs to advertise on the traditional channel to generate awareness and have non-zero search volume on the search engine. In this section, we drop this simplification and assume that a firm may have search volume even without recently-conducted awareness-generating advertising, say due to previous reputation. In other words, we assume that V customers search a firm's product even if it does not advertise on the traditional channel. We also let the firms to be asymmetric in this aspect by assuming that the reputation-based search volume of the "strong" firm is $V > 0$ while the "weak" firm has no reputation-based search volume (i.e., the weak firm's keyword will be searched only if it does awareness-generating advertising).

The detailed analysis of this extension is included in Section A3 in the appendix. We confirm that the insights obtained from the basic model hold. Furthermore, if V is large enough (the strong firm has much larger reputation-based search volume than the weak firm), only the weak firm wants to poach, and its incentive to poach increases with V. The strong firm, which already has high customer awareness, accommodates this poaching by spending more on traditional advertising. This gives us the following counter-intuitive result.

Proposition 6 *The firm that has larger reputation-based customer awareness spends even more* *on awareness-generating advertising as a best response to the poaching of its competitor.*

6.3 Display Advertising through the Search Engine

Internet display advertising is primarily an awareness-creating channel of advertising. Therefore, for the purpose of our modeling, we bundle Internet display advertising in the traditional channel (even though it has risen to prominence only in the last two decades). However, unlike other traditional advertising channels, Internet display advertising is largely controlled by the popular search engines that also control sponsored search advertising. Search engines usually match the website publishers (websites that attract Internet users) to the advertisers and collect a share of the advertising fees for this service. Examples of such services are "Ad Exchange" by Google, "AdECN" by Microsoft and "Right Media" by Yahoo!.

The above observation has an interesting implication—in response to poaching, a firm may move its money away from sponsored search, but spend some of this money on display advertising with the same search engine. In this case, the search engine still obtains the revenue, which might have different implications for its auction design strategy. We extend our model such that a fraction $0 < \delta < 1$ of the money spent on traditional channel goes to the search engine. We find that this extension produces the same insights as in the basic model in Section 3.

6.4 Endogenous Budget

In this section, we relax the assumption that the advertising budget of each firm is exogenously given, and allow each firm to decide how much to spend on advertising while trying to maximize its profit. Although there is no hard constraint on how much a firm can spend on advertising, we assume that spending more on advertising becomes harder as the firm spends more (Fernandez-Corugedo 2002), reflecting the fact that it is increasingly difficult to raise larger amounts of money. The profit of Firm i is

$$\Pi_i = \alpha T_i + \min(T_i \frac{S_i}{S_i + P_j}, \frac{S_i}{R}) + \min(T_j \frac{P_i}{S_j + P_i}, \frac{P_i}{R}) - \eta(T_i + S_i + P_i)^\rho$$

where $j = 3 - i$ represents the index of the other firm, T_i, S_i and P_i, respectively, represent the level of advertising on traditional channel, sponsored search of own keyword, and sponsored search of competitor's keyword. The parameter $\rho > 1$ captures the fact that increasing the advertising budget becomes harder as this budget becomes larger. Note that except for the budget term $\eta(T_i + S_i + P_i)^\rho$, the profit expression is the same as the profit expression presented in Section 2.

By numerically calculating the equilibria, we confirm that the results presented in Sections 3 and 4 are robust to budget endogeneity. Similar to the case of exogenous budget, we show that firms may use different advertising strategies in equilibrium. In particular, there exist asymmetric equilibria in which one firm focuses more on traditional advertising while its competitor poaches on its keyword. The slight difference from the exogenous budget setting is that symmetric firms with endogenous budget may poach on each others' keywords at the same time. However, the degree of poaching could be different for the two firms with one firm poaching more than the other one. Finally, similar to Section 5, we show that the search engine's revenue is maximized when poaching is slightly penalized. At a medium level of penalty, the firms accept the penalty and poach on each others' keywords. Since poaching is a desirable option for each firm, they spend more on poaching to compensate the penalty imposed by the search engine, which increases the search engine's revenue. However, if the penalty is too high, the firms' best responses are not to poach. This can decrease the search engine's revenue. Therefore, a medium level of penalty is where the search engine's revenue is maximized.

6.5 Firms' Decision Sequence

In the basic model in Section 2, we assumed that the firms decide how to allocate their budget to different channels of advertising simultaneously. However, one might argue that, in reality, the firms can observe each others' traditional advertising efforts when deciding about sponsored search advertising. The results presented in Sections 3 and 4 are robust under this alternative decision sequence. Intuitively, if one firm does not do traditional advertising and relies on poaching, the competitor is forced to do traditional advertising, otherwise there will be no search volume.

Consider an alternative model in which the firms first decide how much to spend on traditional advertising. Then, given the information on traditional advertising, the firms decide how much to spend on sponsored search advertising on their own keyword and on the competitor's keyword. Theorem A5 in the appendix shows that each firm will split its budget between the keywords in sponsored search advertising proportional to the search volume for that keyword. Therefore, if

Firm i spends T_i on the traditional channel, its profit will be

$$\Pi_i = \alpha T_i + \min\left((T_i + T_j)\frac{B - T_i}{2B - T_i - T_j}, \frac{B - T_i}{R}\right),$$

where $j = 3 - i$ represents the index of the other firm. The equilibrium profits of the firms with this new formulation remain the same as those in Section 3. Moreover, the poaching and non-poaching equilibria discussed in Section 3 are also equilibria of this game. Intuitively, if one firm spends zero on traditional advertising, its competitor's best response is to use Traditional strategy. Given that the competitor uses Traditional strategy, spending zero on traditional channel and poaching on the competitor's keyword is the best response. Also, similar to the analysis of Section 3, the best response to the Own strategy is to use the Own strategy.

6.6 Consumers' Purchase Model and Price Competition

In our basic model, we assumed that product prices are determined exogenously, and consumers purchase passively at the price offered to them by the firm whose traditional or sponsored advertisement they see most recently. In this extension, we model price competition between firms using a model in which consumers are horizontally differentiated. We assume that consumers get aware of a firm only if they see an ad of the firm, which can either be a traditional ad or an ad in sponsored search. Therefore, consumers that are poached become aware of both firms and compare prices across firms while making their purchase decisions, which leads to price competition. The consumers who see ads from only one firm do not compare prices as they are not aware of the second firm. More details of the model are available in Section A4 in the appendix.

We solve the model numerically and confirm that our original results are robust under price competition—equilibria exist in which one firm focuses on traditional advertising and the other focuses on poaching, and the search engine's revenue is maximized with a medium level of penalty on poaching. A new interesting result from this model is that the poaching firm sets a lower price than the other firm. In this way, the poaching firm can maximize the effect of poaching on its competitor's keyword and win more of the comparison shoppers. The firm that is being poached does not decrease the price as much because it is benefiting from the customers who are not aware of the product of the poaching firm (i.e., customers not influenced by sponsored search).

7 Conclusions and Discussion

In this research, we study the poaching behavior of firms in sponsored search advertising. A firm can spend on traditional channels of advertising such as television, print and radio to create awareness, attract the customers, and increase the search volume of its keyword at a search engine. Alternatively, a firm may limit its awareness-creating activities and spend its budget on stealing the potential customers of its competitor by advertising on the competitor's keyword in sponsored search, which we call poaching.

We find that even when the firms are identical, they may follow different advertising strategies—one firm focuses on the traditional channel and spends most of its budget for creating awareness, while the other firm spends most of its budget on poaching. Although poaching seems to be beneficial for the search engine as it increases the competition on the keywords, we find that it actually may decrease the search engine's revenue. Since poaching increases prices (bids) in sponsored search, thus increasing per-customer acquisition costs in this channel, the firms may spend less on this channel. Therefore, the search engine may increase its revenue by making poaching harder for the firms and keeping bids in check.

When the firms are asymmetric, and the advertising budget of one firm is significantly larger than the advertising budget of the other firm, there is an interesting twist in the above results. First, the stronger firm does not want to poach while the weaker firm has much more incentive to poach (as compared to the symmetric case). Furthermore, unlike the case of symmetric firms, poaching may increase the search engine's revenue. Since the stronger firm has a large search volume, the effect of the weaker firm's poaching is small. In other words, poaching of the weaker firm does not make sponsored search much less efficient for the stronger firm. Thus, the stronger firm keeps almost the same portion of its budget in sponsored search. On the other hand, the weaker firm does not need to create awareness and can spend its entire budget in sponsored search, which increases the search engine's revenue.

We find that in asymmetric case, the best strategy for the search engine is to handicap poaching but not too much so that the weak firm still prefers to poach. This handicap can be implemented through charging the poaching firm a higher price than the non-poaching firm for the same keyword. Interestingly, we see that well-known search engines, e.g., Google, Yahoo! and Bing, have already implemented such penalties through "keyword relevance" multipliers. A firm has to pay higher price than its competitor for appearing in response to the competitor's keyword, even if it has the same click-through-rate and quality measures as its competitor. By including keyword relevance measures in our model, we confirmed that a medium level of penalty maximizes the search engine revenue. Our results agree with the industry observations that the search engines, when sued by firms for allowing poaching, defended their practice of allowing bids on trademarked keywords, but at the same time are penalizing poaching through keyword relevance multipliers.

We also consider various extensions of the model which confirm the robustness of our results and also provide additional insights. Specifically, we consider an extension in which one firm has some search volume for its keyword even without recent awareness-generating advertising (say, because of previous reputation). We find that, surprisingly, the firm that has higher exogenous search volume due to reputation-based customer awareness has greater incentive to invest in traditional advertising to drive even more search volume to its keyword. In another extension, where the firms compete on price, we find that the poaching firm has incentive to set a lower price than its competitor.

Our work sheds light on the poaching behavior of firms in a multi-channel advertising setting. There are many other related problems that may be studied in future work. In particular, the firms are not vertically differentiated in our model. Perhaps, a joint model of our work and Desai et al. (2011), that allows differentiation in a multi-channel advertising model, would be an interesting direction for future work. Another interesting direction to look at is the consequences of poaching among *partners*. For example, online travel agencies such as Orbitz bid on keywords such as "Sheraton Hotel in San Francisco" trying to *steal and resell* the potential customers of Sheraton back to Sheraton. This poaching not only decreases Sheraton's profit from its own customers (because it has to share a part of the revenue with Orbitz for delivering this customer), but also increases the price of sponsored search advertising for Sheraton. It would be interesting to know how partners should react to such poaching behavior.

Appendix

A1 Derivations for the Asymmetric Firms Case

We use the following terminology for brevity. When describing equilibria, we assume that the first firm is the weak firm, and the second firm is the strong firm. For example, by (Poach, Traditional) equilibrium we mean an equilibrium in which the weak firm poaches and the strong firm uses Traditional. We will also see two types of mixed equilibria. In the first mixed equilibrium, which we call the Weak-Poach-mixed equilibrium, the weak firm mixes between Poach and Own, and the strong firm mixes between Traditional and Own. In the second mixed equilibrium, which we call the Strong-Poach-mixed equilibrium, the weak firm mixes between Traditional and Own, and the strong firm mixes between Poach and Own.

Because of the existence of multiple equilibria in our setting, we define the *weak dominance* concept to compare sets of equilibria. We say that equilibrium set S_1 weakly dominates equilibrium set S_2, from Player P's perspective, if for every equilibrium $e_1 \in S_1$ and $e_2 \in S_2$, Player P's profit in e_1 is greater than or equal to her profit in e_2, with the inequality being strict for at least one pair (e_1, e_2). This definition is particularly useful when comparing the revenue of the search engine with and without the presence of poaching.

We start the analysis by assuming a low level of asymmetry between the firms' advertising budgets. Then, we show how the results are generalized for higher levels of asymmetry.

Low Level of Asymmetry

To aid the exposition of the derivation, we use Figure A1, which plots the firms' and the search engine's revenues for $\alpha = 0.5$ and $B = 1.5$ (which is a low asymmetry case). In Figure A1(a) we see the existence of multiple equilibria for $R > 0.60$. For $R \geq 0.6$, since $\Pi_W^{P,T} > \Pi_W^{O,O}$, the weak firm may poach on the strong firm's keyword. Similarly, for $R > 1.58$ since $\Pi_S^{P,T} > \Pi_S^{O,O}$, the strong firm may poach on weak firm's keyword. In general, let R^W be the threshold value of R for which $\Pi_W^{P,T} > \Pi_W^{O,O}$ if $R > R^W$.[A1] Similarly, let R^S be the threshold value of R for which $\Pi_S^{P,T} > \Pi_S^{O,O}$ if

[A1] R^W is the value of R at which $\Pi_W^{P,T} = \Pi_W^{O,O}$. The solution is $R^W = \sqrt{\frac{(1+\alpha)^3(1+B)}{(\alpha+B+\alpha B)^2}} + \frac{1-(1+\alpha)B}{\alpha+B+\alpha B}$.

$R > R^S$.[A2] Using elementary calculus, it can be proved that $R^W < R^S$. In other words, the weak firm starts poaching for lower values of R. In the example of Figure A1, $R^W = 0.6$ and $R^S = 1.58$.

When $R < R^W$, the unique equilibrium is the (Own, Own). When R is between R^W and R^S, there are three equilibria: (Own, Own), (Poach, Traditional) and mixed. Note that in (Poach, Traditional) equilibrium, weak firm poaches and strong firm uses Traditional. Similarly, in mixed equilibrium, weak firm mixes between Poach and Own, and strong firm mixes between Traditional and Own. When R is larger than R^S, there are five equilibria: (Own, Own), (Poach, Traditional), (Traditional, Poach) and two mixed equilibria. In the first mixed equilibrium, the weak firm mixes between Poach and Own, and the strong firm mixes between Traditional and Own. However, in the second mixed equilibrium, the weak firm mixes between Traditional and Own, and the strong firm mixes between Poach and Own. For brevity, throughout the rest of this section, we refer to the mixed equilibrium in which the weak firm mixes between Poach and Own, and the strong firm mixes between Traditional and Own as the Weak-Poach-mixed equilibrium, and to the other one, as the Strong-Poach-mixed equilibrium.

Figure A1(c) shows the revenue of the search engine for different equilibria as functions of reserve price R. In (Own, Own) equilibrium the search engine's revenue is $1 + B - T_S^O - T_W^O$. In (Poach, Traditional) equilibrium (for $R > R^W$), the search engine's revenue is $1 + B - T_S^T$. And, in (Traditional, Poach) equilibrium (for $R > R^S$), the search engine's revenue is $1 + B - T_W^T$. In the Weak-Poach-mixed equilibrium, the search engine's revenue is $1 + B - (1 - p_W^*)T_W^O - p_S^* T_S^T - (1 - p_S^*)T_S^O$, where p_W^* and p_S^* represent the probability of poaching of weak firm and Traditional of strong firm, respectively. Similarly, in the Strong-Poach-mixed equilibrium, the search engine's revenue is $1 + B - (1 - p_S^{**})T_S^O - p_W^{**} T_W^T - (1 - p_W^{**})T_W^O$, where p_W^{**} and p_S^{**} represent the probability of Traditional of weak firm and poaching of strong firm, respectively. Note that the probabilities p_W^*, p_S^*, p_W^{**} and p_S^{**} can be calculated analytically, using the equilibrium conditions, as follows.

$$p_S^* \Pi_W^{P,T} + (1 - p_S^*)\Pi_W^{P,O} = p_S^* \Pi_W^{O,T} + (1 - p_S^*)\Pi_W^{O,O} \Rightarrow p_S^* = \frac{\Pi_W^{P,O} - \Pi_W^{O,O}}{\Pi_W^{P,O} + \Pi_W^{O,T} - \Pi_W^{P,T} - \Pi_W^{O,O}}$$

$$p_S^{**} \Pi_W^{T,P} + (1 - p_S^{**})\Pi_W^{T,O} = p_S^{**} \Pi_W^{O,P} + (1 - p_S^{**})\Pi_W^{O,O} \Rightarrow p_S^{**} = \frac{\Pi_W^{T,O} - \Pi_W^{O,O}}{\Pi_W^{T,O} + \Pi_W^{O,P} - \Pi_W^{T,P} - \Pi_W^{O,O}}$$

[A2] $R^S = \sqrt{\frac{(1+\alpha)^3 B(1+B)}{(1+\alpha+\alpha B)^2}} + \frac{B-\alpha-1}{1+\alpha+\alpha B}$.

$$p_W^* \Pi_S^{T,P} + (1 - p_W^*)\Pi_S^{T,O} = p_W^* \Pi_S^{O,P} + (1 - p_W^*)\Pi_S^{O,O} \Rightarrow p_W^* = \frac{\Pi_S^{T,O} - \Pi_S^{O,O}}{\Pi_S^{T,O} + \Pi_S^{O,P} - \Pi_S^{T,P} - \Pi_S^{O,O}}$$

$$p_W^{**} \Pi_S^{P,T} + (1 - p_W^{**})\Pi_S^{P,O} = p_W^{**} \Pi_S^{O,T} + (1 - p_W^{**})\Pi_S^{O,O} \Rightarrow p_W^{**} = \frac{\Pi_S^{P,O} - \Pi_S^{O,O}}{\Pi_S^{P,O} + \Pi_S^{O,T} - \Pi_S^{P,T} - \Pi_S^{O,O}}$$

Let $R^* = \frac{\sqrt{\frac{1+B}{1+\alpha}}}{1+B-\sqrt{\frac{1+B}{1+\alpha}}}$. Using the expressions derived for search engine's revenue, we have that the revenue of (Poach, Traditional) equilibrium is the same as the revenue of (Own, Own) equilibrium for any value of R larger than R^*. In other words, for $R \geq R^*$, we have $1 + B - T_S^T = 1 + B - T_W^O - T_S^O$. In Figure A1(c), we have $R^* = 1.07$, showing the point where the curve representing the poaching equilibrium joins the curve representing the non-poaching equilibrium. We can similarly define $R^{**} = \frac{\sqrt{\frac{B(1+B)}{1+\alpha}}}{1+B-\sqrt{\frac{B(1+B)}{1+\alpha}}}$ to be the threshold value of R beyond which the search engine's revenue of (Traditional, Poach) equilibrium is equal to the revenue of (Own, Own) equilibrium. In other words, for $R \geq R^{**}$, we have $1+B-T_W^T = 1+B-T_W^O - T_S^O$. In Figure A1(c), we have $R^{**} = 1.72$, indicating the point where the curve representing the (Traditional, Poach) equilibrium joins the curve representing (Own, Own) equilibrium. When $R > R^*$ and $R < R^S$, not allowing poaching weakly dominates allowing poaching from search engine's perspective. In this region, the revenues of (Own, Own) equilibrium and (Poach, Traditional) equilibrium are the same, and larger than the revenue of the mixed equilibrium. Similarly, when $R > R^{**}$, not allowing poaching weakly dominates allowing poaching. In this region, (Poach, Traditional), (Traditional, Poach) and (Own, Own) equilibria have the same revenue for the search engine; but they are higher than the revenues of the two mixed equilibria.

Let R_m^* be the value of R at which $1+B-T_S^O - T_W^O = 1+B-(1-p_W^*)T_W^O - p_S^* T_S^T - (1-p_S^*)T_S^O$. In other words, R_m^* is the value of R at which the revenue of the search engine in the Weak-Poach-mixed equilibrium is equal to the revenue of the search engine in (Own, Own) equilibrium. In Figure A1(c), we have $R_m^* = 0.98$. For $R > R^W$ and $R < R_m^*$, revenues of the search engine from the Weak-Poach-mixed equilibrium and from (Poach, Traditional) equilibrium are larger than the revenue from (Own, Own) equilibrium. In other words, for $R^W \leq R < R_m^*$, the set of equilibria in presence of poaching (when poaching is allowed) weakly dominates the set of equilibria without poaching (when poaching is not allowed), from search engine's perspective.

To summarize, R can be in one of the following intervals:

1. $[0, R^W)$: The unique equilibrium is (Own, Own).

2. $[R^W, R^*_m)$: There are three equilibria: (Own, Own), (Poach, Traditional) and the Weak-Poach-mixed. Allowing poaching weakly dominates not allowing poaching from search engine's perspective.

3. $[R^*_m, R^*)$: There are three equilibria: (Own, Own), (Poach, Traditional) and the Weak-Poach-mixed. Search engine's revenue of the mixed equilibrium is lower than (Own, Own), and revenue of (Poach, Traditional) equilibrium is higher than (Own, Own) equilibrium.

4. $[R^*, R^S)$: There are three equilibria: (Own, Own), (Poach, Traditional) and the Weak-Poach-mixed. Not allowing poaching weakly dominates allowing poaching from search engine's perspective.

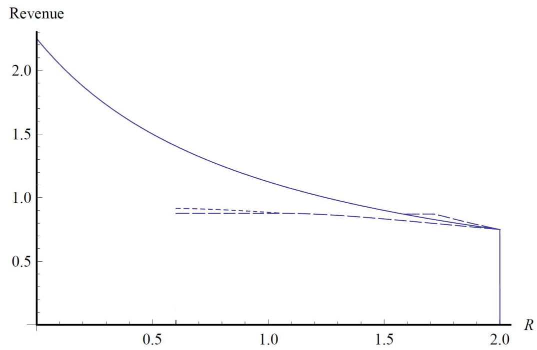
5. $[R^S, R^{**})$: There are five equilibria. Search engine's revenue may be lower or higher in presence of poaching, depending on equilibrium selection.

6. $[R^{**}, \frac{1}{\alpha}]$: There are five equilibria. Not allowing poaching weakly dominates allowing poaching from search engine's perspective.

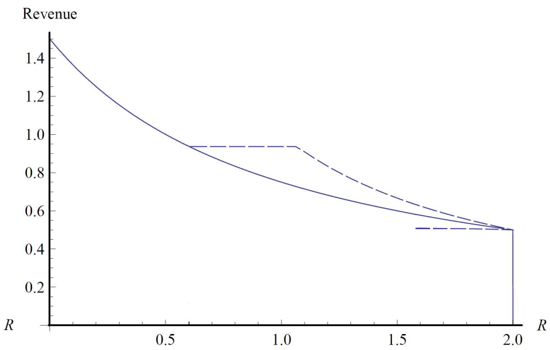
7. $(\frac{1}{\alpha}, \infty)$: Both firms spend all of their budget on traditional channel. Search engine's revenue is 0.

In case of symmetric firms, $R^W = R^S$. In other words, intervals 2, 3 and 4 do not exist.

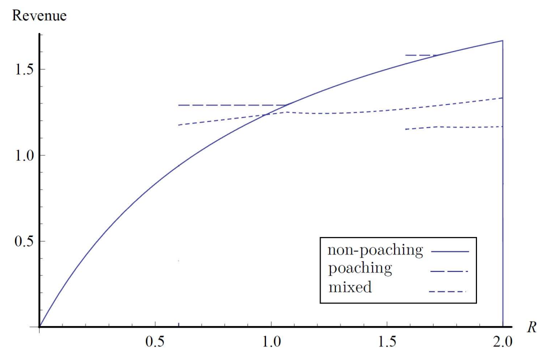
We also find that the weak firm's relative gain from poaching is larger than that of the strong firm. In other words, $\frac{\Pi_W^{P,T}}{\Pi_W^{O,O}} \geq \frac{\Pi_S^{P,T}}{\Pi_S^{O,O}}$. Moreover, the weak firm's incentive to poach increases with increasing budget asymmetry. In other words, $\frac{\Pi_W^{P,T}}{\Pi_W^{O,O}}$ is an increasing function of B. This is intuitively because the strong firm has a relatively large search volume; therefore, the poaching of the weak firm does not affect the sponsored search price significantly, and in turn, allows poaching at a relatively low price. In fact, if the firms are very asymmetric, the incentive to poach is so high that (Poach, Traditional) is the only equilibrium of the game. Next, we will discuss the equilibria of the game at medium and high levels of asymmetry.



(a) Strong Firm Revenue



(b) Weak Firm Revenue



(c) Search Engine's Revenue

Figure A1: Revenue plots when the Strong firm has budget $B \geq 1$ and the weak firm has budget 1, for parameters $B = 1.5$ and $\alpha = 0.5$.

Medium and High Level of Asymmetry

In this section, we study the effect of *degree* of budget asymmetry, on the results presented in the previous section. As we will show, the results are qualitatively similar. However, the degree of budget asymmetry has interesting effects on the size and location of the intervals.

The first interesting observation is that R^S is increasing in B. In other words, as budget asymmetry increases the intervals in which the strong firm poaches on weak firm's keyword (intervals 5 and 6) shrink. If B is large enough, R^S becomes larger than $1/\alpha$. In other words, if one firm is enough larger than the other firm, the strong firm does not poach on the weak firm's keyword under any condition.

Reverse of this effect exists for R^W. As B increases, R^W decreases. If B is large enough, R^W becomes 0. In other words, if one firm is enough larger than the other firm, weak firm poaching on strong firm's keyword is always an equilibrium. These changes in interval thresholds are consistent with Proposition 2 that says weak firm's incentive to poach increase and strong firm's incentive to poach decrease as budget asymmetry increases.

Mathematically speaking, if $B \geq \frac{1}{\alpha}$ then $R^S > \frac{1}{\alpha}$. Under this condition, strong firm does not poach on weak firm's keyword. Furthermore, $B \geq \frac{3+3\alpha+\alpha^2}{1+\alpha}$ implies $R^W = 0$. Under this condition, weak firm poaching on strong firm's keyword is always an equilibrium. We define the values of B where $B < \max(\frac{1}{\alpha}, \frac{3+3\alpha+\alpha^2}{1+\alpha})$ as *low* level of budget asymmetry. For such values of B, the results are what we discussed in the previous section. However, when $B \geq \max(\frac{1}{\alpha}, \frac{3+3\alpha+\alpha^2}{1+\alpha})$ we have *medium* or *high* level of asymmetry.

For medium level of asymmetry, R can be in one of the following intervals.

1. $[0, R^*_m)$: There are three equilibria: (Own, Own), (Poach, Traditional) and Weak-Poach-mixed. Allowing poaching weakly dominates not allowing poaching from search engine's perspective.

2. $[R^*_m, R^*)$: There are three equilibria: (Own, Own), (Poach, Traditional) and Weak-Poach-mixed. Search engine's revenue of the mixed equilibrium is lower than (Own, Own), and revenue of (Poach, Traditional) equilibrium is higher than (Own, Own) equilibrium.

3. $[R^*, \frac{1}{\alpha}]$: There are three equilibria: (Own, Own), (Poach, Traditional) and Weak-Poach-mixed. Not allowing poaching weakly dominates allowing poaching from search engine's perspective.

4. $(\frac{1}{\alpha}, \infty)$: Both firms spend all of their budget on traditional channel. Search engine's revenue is 0.

Note that $R^* = \frac{\sqrt{\frac{1+B}{1+\alpha}}}{1+B-\sqrt{\frac{1+B}{1+\alpha}}}$, is also a decreasing function of B. In other words, the first two intervals shrink and the third interval grows as B increases.

A condition that could not exist for low level of asymmetry and could occur for high level of asymmetry is $\Pi_W^{P,O} > \Pi_W^{O,O}$. In other words, if the firms are asymmetric enough, even if the strong firm uses Own strategy, the weak firm prefers to poach. In this situation, (Own, Own) cannot be an equilibrium anymore. Using simple calculus, we see that this condition is satisfied if $B \geq 1 + \alpha$ and $R < \frac{-1-\alpha+B}{1+\alpha+\alpha B}$. Define $\overline{R} = \frac{-1-\alpha+B}{1+\alpha+\alpha B}$. Note that $\overline{R}$ converges to $\frac{1}{\alpha}$ as B increases. This means that for large enough values of B, the only equilibrium is when weak firm poaches on strong firm's keyword for *almost* all values of R (except, of course for $R > \frac{1}{\alpha}$ where no firm uses sponsored search advertising at all). In summary, for high level of asymmetry, R can be in one of the following intervals.

1. $[0, \overline{R})$: There is one equilibrium: (Poach, Traditional). Allowing poaching has the same revenue as not allowing poaching for the search engine.

2. $[\overline{R}, \frac{1}{\alpha}]$: There are three equilibria: (Own, Own), (Poach, Traditional) and Weak-Poach-mixed. Not allowing poaching weakly dominates allowing poaching from search engine's perspective.

3. $(\frac{1}{\alpha}, \infty)$: Both firms spend all of their budget on traditional channel. Search engine's revenue is 0.

As mentioned, the second interval shrinks and the first interval grows as B increases. Finally, we should mention that the transition from the interval structure in medium level of asymmetry to the interval structure in high level of asymmetry is through the growth of $\overline{R}$. Depending on how large $\overline{R}$ is, the $[0, \overline{R})$ interval of high asymmetry case can override the fist interval, or the first and the second intervals of the medium asymmetry case.

A2 Category Keyword

We extend our model and assume that there exists a category keyword which attracts customers from the traditional ads of both firms. We categorize the customers into three categories as follows: (1) The customer buys the product directly after seeing the traditional ad, or searches for the product keyword after seeing the traditional ad, but ignores the sponsored search results and eventually converts to the advertised product. (2) The customer searches for the product keyword after seeing the traditional ad, and converts to the product advertised in the sponsored search result; in the case that no product is advertised in sponsored search, the customer will not convert. (3) The customer searches for the category keyword after seeing the traditional ad, and converts to the product advertised in the sponsored search result; in the case that no product is advertised in sponsored search, the customer will not convert. We assume that the "scaled probability" that a customer is in Category 1 is α, in Category 2 is 1, and in Category 3 is β. Therefore, if a firm spends x in traditional advertising, there will be αx customers in Category 1, x customers in Category 2, and βx customers in Category 3. We now derive the expressions for the revenues of the firms under different strategies.

Definitions of Strategies

We rederive the budget allocations for strong and weak firms based on the core idea that when a firm is following a strategy of focusing on one of the three types of advertising, the allocation is such that the firm spends maximum amount of budget on that form of advertising but still uses an undominated strategy.

Let C^J be the amount spent on the category keyword in sponsored search in strategy J, where $J \in \{N, P, D\}$; all other notation is carried over from the basic model.

Own Strategy: First, assume that there is only one firm in the market. If the firm spends x in traditional advertising, to win the customers of the product keyword she has to spend at least xR in sponsored search of the product keyword, and βxR in sponsored search of the category keyword, where R is the reserve price of sponsored search auction set by the search engine. The optimal amount of money to be spent on traditional advertising in this case is $T^O = \frac{B}{1+R(1+\beta)}$. Consequently, since in Own strategy, the firm does not advertise on the keyword of the other firm (i.e., $S_2^O = 0$), the amount of money that he spends on sponsored search is $S_1^O + C^O = B - T^O = \frac{BR(1+\beta)}{1+R(1+\beta)}$. Since the the number of queries to the product keyword and the category keyword are proportional to 1 and β, by Theorem A5, we have $S_1^O\beta = C^O$, which gives $S_1^O = \frac{BR}{1+R(1+\beta)}$ and $C^O = \frac{BR\beta}{1+R(1+\beta)}$.

Poaching Strategy: The poaching firm's spending on its own keyword and on traditional advertising is zero, i.e., $T^P = S_1^P = 0$. Using Theorem A5, the poaching firm's spending on the competitor's keyword is $S_2^P = B/(1+\beta)$, and on the category keyword is $C^P = \beta B/(1+\beta)$.

Traditional Strategy: In the Traditional strategy, the firm assumes that the other firm poaches; given this assumption, the firm's revenue is $\alpha T + (B-T)/R$ if $2B - T < T(1+\beta)R$, and is $\alpha T + T(1+\beta)\frac{B-T}{2B-T}$ otherwise. Assuming $\alpha < (1+\beta)/R$, the optimal solution to this problem is $T^T = B(2 - \sqrt{2(1+\beta)/(1+\alpha+\beta)})$ if $\frac{2}{R(1+\beta)+1} \geq 2 - \sqrt{2(1+\beta)/(1+\alpha+\beta)}$, and $T^T = \frac{2B}{(1+\beta)R+1}$ otherwise. Since $S_1^T + C^T = B - T^T$, by Theorem A5, $S_1^T = \frac{1}{1+\beta}(B - T^T)$ and $C^T = \frac{\beta}{1+\beta}(B - T^T)$.

Revenue Analysis

Both Firms Own: If both firms choose Own strategy, the revenue of each firm is $\Pi^{O,O} = T^O(1+\alpha+\beta)$.

Both Firms Traditional: If both firms choose Traditional strategy, the revenue of each firm is $\Pi^{T,T} = \alpha T^T + \min(T^T, \frac{S_1^T}{R}) + \min(\beta T^T, \frac{C^T}{R})$.

Both Firms Poaching: The revenue of both firms in this case is of course zero, i.e., $\Pi^{P,P} = 0$.

One Firm Poaching, One Firm Own: In this case, the number of queries on the product keyword is T^O and on the category keyword is βT^O. Therefore, the Own firm's revenue is $\Pi^{O,P} = \alpha T^O + T^O \frac{S_1^O}{S_1^O + S_2^P} + \beta T^O \frac{C^O}{C^P + C^O}$, and the Poaching firm's revenue is $\Pi^{P,O} = T^O \frac{S_2^P}{S_1^O+S_2^P} + \beta T^O \frac{C^P}{C^P+C^O}$.

One Firm Traditional, One Firm Own: In this case, the number of queries on the Traditional firm's product is T^T and on the Own firm's product is T^O. Also, the number of queries on the category keyword is $\beta(T^O + T^T)$. Hence, the Own firm's revenue is $\Pi^{O,T} = \alpha T^O + T^O + \min(\frac{C^O}{R}, (\frac{C^O}{C^O+C^T})\beta(T^O+T^T))$, and the Traditional firm's revenue is $\Pi^{T,O} = \alpha T^T + \min(T^T, \frac{S_1^T}{R}) + \min(\frac{C^T}{R}, (\frac{C^T}{C^O+C^T})\beta(T^O+T^T))$.

One Firm Poaching, One Firm Traditional: In this case, the price will be greater than or equal to R for category keyword and the product keyword; therefore, $\Pi^{T,P} = \alpha T^T + T^T \frac{S_1^T}{S_1^T+S_2^P} + \beta T^T \frac{C^T}{C^T+C^P}$ and $\Pi^{P,T} = T^T \frac{S_2^P}{S_1^T+S_2^P} + \beta T^T \frac{C^P}{C^T+C^P}$.

We use the above expressions to analyze the equilibrium of the two-person normal-form game as before. We find that the results and insights from the basic model in Section 3 (without category keyword) continue to hold.

A3 Reputation Effects

Suppose that Firm i has some exogenous search volume V_i for its keyword, which is independent of how much it has recently spent on creating awareness for its product. This may be, for instance, because of the previous reputation that the firm holds. For simplicity, we assume that $V_1 = V$ and $V_2 = 0$, i.e., V customers search the keyword of the "strong" firm (denoted by subscript S) without traditional advertising, while no customers search the keyword of the "weak" firm (denoted by subscript W) without traditional advertising. As before, we assume that spending x on awareness advertising creates search volume x; hence, if the strong firm spends x on awareness advertising, the search volume for its keyword will be $V + x$.

Definitions of Strategies

We rederive the budget allocations for strong and weak firms based on the core idea that when a firm is following a strategy of focusing on one of the three types of advertising, the allocation is such that the firm spends maximum amount of budget on that form of advertising but still uses an undominated strategy.

Own Strategy: For the weak firm, the Own strategy has not changed and is as before: $T_W^O = \frac{B}{R+1}$. However, for the strong firm, if $B \le VR$, we have $T_S^O = 0$; otherwise, $T_S^O = \frac{B-VR}{R+1}$.

Traditional Strategy: For the weak firm, the Traditional strategy does not change. However, notice that when the strong firm poaches, Traditional is not necessarily the best response from the weak firm as it may want to poach too. If $\frac{2}{R+1} \ge 2 - \sqrt{2/(1+\alpha)}$, $T_W^T = B(2 - \sqrt{2/(1+\alpha)})$; otherwise, $T_W^T = 2B/(R+1)$. For the strong firm, recall that in Traditional strategy the firm assumes that the other firm poaches. Given this assumption, the firm's revenue is $\alpha T + (B-T)/R$ if $2B - T < (T+V)R$, and is $\alpha T + (T+V)\frac{B-T}{2B-T}$ if $2B - T \ge (T+V)R$. Therefore, if $\frac{2B-VR}{R+1} \ge B(2 - \sqrt{\frac{2B+V}{B(1+\alpha)}})$, then $T_S^T = B(2 - \sqrt{\frac{2B+V}{B(1+\alpha)}})$; otherwise, if $2B \ge VR$, $T_S^T = \frac{2B-VR}{R+1}$; otherwise, $T_S^T = 0$.

Poaching Strategy: By definition, $T_W^P = T_S^P = 0$.

Revenue Analysis

Own Strategy: For the weak firm, as long as it is not poaching, V does not have an impact. Therefore, $\Pi_W^{O,O} = \Pi_W^{O,T} = T_W^O(1+\alpha)$ and $\Pi_W^{O,P} = T_W^O(\alpha + \frac{B-T_W^O}{2B-T_W^O})$. For the strong firm, if $B \ge VR$, $\Pi_S^{O,O} = \Pi_S^{O,T} = T_S^O(1+\alpha) + V$; otherwise, $\Pi_S^{O,O} = \Pi_S^{O,T} = B/R$. Similarly, if $B \ge VR$, $\Pi_S^{O,P} = T_S^O\alpha + \frac{B-T_S^O}{2B-T_S^O}(V + T_S^O)$; otherwise, $T_S^O = 0$ and hence, if $2B \ge VR$, $\Pi_S^{O,P} = \frac{B}{2B}V = V/2$; otherwise $\Pi_S^{O,P} = B/R$.

Traditional Strategy: Nothing changes for the weaker firm, which implies $\Pi_W^{T,P} = \alpha T_W^T + T_W^T\frac{B-T_W^T}{2B-T_W^T}$ and $\Pi_W^{T,O} = \Pi_W^{T,T} = \alpha T_W^T + \min(T_W^T, \frac{B-T_W^T}{R})$. For the strong firm, if $2B - T_S^T \ge (V+T_S^T)R$, $\Pi_S^{T,P} = T_S^T\alpha + \frac{B-T_S^T}{2B-T_S^T}(T_S^T+V)$; otherwise, as in the previous case, $\Pi_S^{T,P} = T_S^T\alpha + \frac{B-T_S^T}{R}$. Similarly, if $B - T_S^T \ge (V + T_S^T)R$, $\Pi_S^{T,O} = \Pi_S^{T,T} = T_S^T(1+\alpha) + V$; otherwise, $\Pi_S^{T,O} = \Pi_S^{T,T} = T_S^T\alpha + \frac{B-T_S^T}{R}$.

Poaching Strategy: If the stronger firm poaches, value of V does not affect its utility. Therefore, $\Pi_S^{P,P} = 0$, $\Pi_S^{P,O} = T_W^O(\frac{B}{2B-T_W^O})$ and $\Pi_S^{P,T} = T_W^T(\frac{B}{2B-T_W^T})$. If the weaker firm poaches, if $B \geq VR$, $\Pi_W^{P,P} = V$; otherwise, $\Pi_W^{P,P} = B/R$. Similarly, if $2B - T_S^O \geq (V + T_S^O)R$, $\Pi_W^{P,O} = \frac{B}{2B-T_S^O}(V + T_S^O)$; otherwise, $\Pi_W^{P,O} = B/R$. Finally, if $2B - T_S^T \geq (V + T_S^T)R$, $\Pi_W^{P,T} = \frac{B}{2B-T_S^T}(V + T_S^T)$; otherwise, $\Pi_W^{P,T} = B/R$.

We use the above expressions to analyze the equilibrium of the two-person normal-form game as before.

A4 Consumers' Purchase Model and Price Competition

We consider a Hotelling line of length 1, with consumers distributed uniformly on it and each firm located at one end of the line. We assume that the valuation of each consumer for either firm's product is $V = 1$, and travel cost (misfit cost) along the line is $t > 0$ per unit distance traveled by a consumer. The consumers do not know about the existence of the firms initially. A firm can make consumers aware of its product through traditional advertising. More specifically, if Firm i spends T_i on traditional advertising, $(1 + \alpha)T_i$ consumers become aware of Firm i's product, and we assume that these consumers are uniformly distributed on the Hotelling line.[A3] After being exposed to traditional advertising, some consumers search the firm's keyword on a search engine, in response to which they may see this firm's ad or the competing firm's ad. Some of the consumers who become aware of both firms (through one firm's traditional ad and the other firm's sponsored ad) compare prices before purchasing, which leads to price competition.

The consumers who eventually purchase the product from Firm i could be in one of the following categories ($j = 3 - i$ is the index of Firm i's competitor):

1. Exposed to traditional advertising of Firm i, not influenced by sponsored search advertising and purchase from Firm i;

2. Exposed to traditional advertising of Firm i, influenced by sponsored search advertising, see sponsored search advertising of Firm i and purchase from Firm i;

[A3]We assume that the total consumer population is large enough that it is unlikely that a consumer is exposed to traditional advertising of both firms. Therefore, after traditional advertising, each consumer knows about at most one product.

3. Exposed to traditional advertising of Firm j, influenced by sponsored search advertising, see sponsored search advertising of Firm i, without comparing prices purchase from Firm i;

4. Exposed to traditional advertising of Firm i, influenced by sponsored search advertising, see sponsored search advertising of Firm j, compare prices and purchase from Firm i;

5. Exposed to traditional advertising of Firm j, influenced by sponsored search advertising, see sponsored search advertising of Firm i, compare prices and purchase from Firm i.

Consumers in Category 1 are not influenced by sponsored search. Consumers in Categories 2, 3, 4 and 5 are influenced by sponsored search. Price competition between firms is only due to Categories 4 and 5. This feature of the model implies that consumers who are poached, and therefore become aware of both firms, also compare prices across firms, which leads to price competition.

Let $C_{a,b}$ (where $a, b \in \{1, 2\}$) be the number of customers who are exposed to traditional advertising of Firm i and sponsored search advertising of Firm j, where each $C_{a,b}$ is a function of the firms' advertising budget allocations. There is a total of $C_{1,2} + C_{2,1}$ customers who are exposed to advertising (traditional or sponsored search) of both firms. We assume that χ fraction of them compare the prices of the two firms while $1 - \chi$ fraction purchase from the firm that is shown in sponsored search.[A4] From Firm i's point of view, Categories 1, 2, 3, 4 and 5 have $\alpha T_i, C_{i,i}, (1 - \chi)C_{j,i}, \chi C_{i,j}$ and $\chi C_{j,i}$ consumers, respectively. The number of consumers in each category depend on the firms' advertising budget allocations. Using the formulation of Section 2, we have $C_{i,j} = \min(T_i \frac{P_j}{S_i+P_i}, \frac{P_j}{R})$, where T_i, S_i and P_i represent how much Firm i spends on traditional advertising, sponsored search advertising of its own keyword, and poaching on competitor's keyword, respectively.

For Categories 1, 2 and 3, in which consumers do not compare prices across firms, $(1-p_i)/t$ of the consumers purchase. For Categories 4 and 5, in which consumers compare prices, $1/2+(p_j-p_i)/(2t)$ of the consumers purchase from Firm i (and the rest from Firm j).[A5] Therefore, assuming that

[A4]Note that those customers who are exposed to advertising of both firms but without comparing prices purchase from the firm that did traditional advertising are already counted in αT_i and are categorized in the Category 1.

[A5]Let x be a consumer's distance from Firm i, and p_i and p_j be the prices of Firms i and j, respectively. If this consumer considers only Firm i, she purchases if $1 - tx - p_i \geq 0$. If she considers both firms, she purchases from Firm i if $1 - tx - p_i \geq 1 - t(1 - x) - p_j$. While solving, we consider boundary effects as needed.

the marginal cost of production is zero, the profit of Firm i is:

$$\Pi_i = p_i \left((\alpha T_i + C_{i,i} + (1-\chi)C_{j,i}) \frac{1-p_i}{t} + \chi(C_{i,j} + C_{j,i})(\frac{1}{2} + \frac{p_j - p_i}{2t}) \right).$$

The parameter χ captures the price competition between the firms due to poaching. Note that if $\chi = 0$, the model collapses to the model in Section 2 (and the optimal price is 1/2).

We solve the above model numerically and confirm that the results presented in Sections 3 and 4 are robust under price competition. We see that symmetric firms may use different strategies in equilibrium with one firm focusing on traditional advertising and the other firm focusing on poaching. Moreover, as in Section 5, the search engine's revenue is maximized with a medium level of penalty on poaching. A new interesting result from this model is that the poaching firm sets a lower price than the other firm. In this way, the poaching firm can maximize the effect of poaching on its competitor's keyword and win more of the comparison shoppers. The firm that is being poached does not decrease the price as much because it is benefiting from the customers who are not aware of the product of the poaching firm (i.e., customers not influenced by sponsored search).

A5 Strategy Space Discretization

In the model in Section 2, we discretize the game by restricting the strategy space of each firm to three strategies, namely, Own, Poaching and Traditional. Although we allow the firms to use mixed strategies, we do not allow them to split their budget among the channels as a pure strategy. The purpose of the discretization is to make the model easier to solve and understand. In contrast, we could leave the strategy space *continuous*, allowing each firm to allocate arbitrary portion of its budget to each of the channels. In this section, we show that our results are robust under continuous strategy space. In particular, we show that under certain conditions, the set of equilibria when the strategy space of the firms is continuous coincides with the set of equilibria when their strategy space is discrete.

Let game G be the discrete game between the two firms as defined in Section 2. Let game H be the continuous version of game G. In other words, in game H, each firm decides how to allocate its budget to different channels of advertising, and does not have to necessarily follow exactly one of the three Poaching, Own or Traditional strategies. We show that when the reserve price R is large enough, the poaching equilibrium of game G is also an equilibrium of game H. Furthermore, if the relevance score γ is small enough, the non-poaching equilibrium of G is also an equilibrium of H. Finally, we prove that if R is large enough, game H has no equilibrium other than those in G. Therefore, the results obtained for discrete game G throughout the paper also apply to continuous game H.

Theorem A1 *For sufficiently large reserve price R (when $R \geq \frac{2B-T^T}{T^T}$), the poaching equilibrium of the discrete game G is also an equilibrium of the continuous game H.*

Proof: Consider a poaching equilibrium of game G where Firm 1 poaches on Firm 2's keyword. Firm 2's response, Traditional strategy, is calculated over the continuous strategy space and hence is a best response to poaching of Firm 1 in the continuous game H as well, by definition. Firm 1's utility from poaching all of its budget is $\frac{B}{R}$ because $R \geq \frac{2B-T^T}{T^T}$. If Firm 1 deviates and spends x on poaching and $B-x$ on traditional channel and its own keyword, assuming that it splits $B-x$ optimally between traditional channel and its own keyword, its utility will be $\frac{x}{R} + (1+\alpha)\frac{B-x}{R+1}$. From Section 3 we know that $\alpha < \frac{1}{R}$. This proves that the deviation is dominated for any $x < B$. Therefore, poaching with all of the budget is best response to Traditional. Consequently, poaching equilibrium of discrete game G is also an equilibrium of continuous game H. $\square$

The theorem below states that in equilibrium, one firm spends all of its budget poaching on the other firm's keyword.

Theorem A2 *For sufficiently large reserve price R (when $R \geq \frac{2B-T^T}{T^T}$), the poaching equilibrium of the continuous game H is the only equilibrium of game H.*

Proof: Suppose that in equilibrium Firm i spends t_i, s_i and p_i on traditional channel, its own keyword and competitor's keyword, respectively. Let $x_i = t_i + s_i$, and $y_i = p_i$. In other words, x_i corresponds to the amount of money that a firm spends on its own channel, and y_i corresponds to the amount of money that a firm spends on its competitor's channel. We know that, not considering the opponent's response, increasing x_i does not affect the marginal utility of y_i and vice versa. Therefore, by first order conditions, in equilibrium, the marginal utility of x_i and y_i must be equal unless $y_i = 0$ or $y_i = B$.

First, suppose that $y_i \neq 0$ and $y_i \neq B$ for both firms. Let d_{x_i} and d_{y_i} denote the marginal utility of x_i and y_i. We have $d_{x_1} = d_{y_1}$ and $d_{x_2} = d_{y_2}$. Lemma A1 shows that $d_{y_1} > d_{x_2}$ and $d_{y_2} > d_{x_1}$.

This proves that $d_{x_1} = d_{y_1} > d_{x_2} = d_{y_2} > d_{x_1}$ which is a contradiction. Therefore, we conclude that at least one of y_is must be B or 0.

Now, we prove that at least one of y_is must be B. Suppose that $y_1 < B$ and $y_2 < B$. This implies that $d_{y_1} \leq d_{x_1}$ and $d_{y_2} \leq d_{x_2}$. Using the same argument as before, we get $d_{x_1} \geq d_{y_1} > d_{x_2} \geq d_{y_2} > d_{x_1}$ which is a contradiction. $\square$

Lemma A1 *For sufficiently large reserve price R (when $R \geq \frac{x_j+y_i-T^T}{T^T}$), assuming equilibrium conditions, we have $d_{y_i} > d_{x_j}$ for $i \neq j$.*

Proof: When R is sufficiently large, using basic calculus, we get $d_{y_i} = \frac{1}{R}$ and $d_{x_j} = \frac{\alpha+1}{R+1}$. Since $R < \frac{1}{\alpha}$, we have $d_{y_i} > d_{x_j}$. $\square$

Theorem A2 shows that the continuous game H is slightly different from game G as there is no non-poaching equilibrium in game H. However, Theorem A3 shows that if poaching is penalized, the non-poaching equilibrium of discrete game G is also an equilibrium of game H.

Theorem A3 *If the poaching is penalized by a multiplier γ, for sufficiently small γ (where $\gamma \leq \frac{(1+\alpha)R}{1+R}$) non-poaching equilibrium of game G is also an equilibrium of game H.*

Proof: Consider the non-poaching equilibrium of G in which each firm uses Own strategy. We know, by definition, that as long as the firms do not want to poach on each others' keywords, Own strategy is the optimum way of splitting budget between traditional channel and own keyword on sponsored search. Consider a deviation where Firm 1 spends $x > 0$ poaching on Firm 2's keyword while Firm 2 is playing Own strategy. Also, assume that Firm 1 splits the remaining $B - x$ optimally between between traditional channel and its own keyword. Firm 1's utility after deviation is $\gamma T^O \frac{x}{B-T^O+x} + (1+\alpha)\frac{B-x}{R+1}$ while before deviation it is $(1+\alpha)\frac{B}{R+1}$. Using elementary calculus we see that the deviation is beneficial if and only if $\gamma \geq \frac{(1+\alpha)(BR+x+Rx)}{B(1+R)}$. Therefore, if $\gamma \leq \frac{(1+\alpha)R}{1+R}$ then non-poaching equilibrium of game G is also an equilibrium of game H. $\square$

A6 Proofs of Theorems Used As Intermediate Results

Suppose that a seller want to sell n units of an item. The seller can sell the units one by one, each in a second price auction. We call this mechanism a *sequential second price auction.* This mechanism roughly describes how search engines sell their advertising slots. Whenever a consumer searches a keyword, the search engine runs a (generalized) second price auction to sell the advertising slot. The seller can instead sell the n units using *Market Clearing Price Mechanism.* In market clearing price mechanism, the seller sets the highest price p at which the market clears, i.e., demand meets supply. The following theorem proves that the two mechanisms essentially lead to the same outcome. The theorem helps us to analyze the outcome of a sequential second price auction, a result which we use frequently throughout the paper.

Theorem A4 *Suppose that n identical items are sold in a sequential second price auction with reserve price R. Two bidders 1 and 2 with budgets B_1 and B_2 are participating in the auctions; each bidder wants to maximize the number of items that she wins. The outcome of any subgame perfect equilibrium of the game is equivalent to the outcome of market clearing price mechanism with reserve price R.*

Proof: First suppose $\lfloor B_1/R \rfloor + \lfloor B_2/R \rfloor \geq n$, i.e., the market clearing price is at least R. Let p be the market clearing price; i.e., $\lfloor B_1/p \rfloor + \lfloor B_2/p \rfloor = n$. Note that if the first player bids p in all rounds, he can make sure that he wins at least $n - \lfloor B_2/p \rfloor = \lfloor B_1/p \rfloor$ items because his opponent has to pay p for every item that he wins. Similarly, if the second player bids p in all rounds, he can make sure that he wins at least $n - \lfloor B_1/p \rfloor = \lfloor B_2/p \rfloor$ items. Since, $\lfloor B_1/p \rfloor + \lfloor B_2/p \rfloor = n$, we see that player i cannot win more than $\lfloor B_i/p \rfloor$ items, which means that he wins exactly $\lfloor B_i/p \rfloor$ items.

Now, consider the case where $\lfloor B_1/R \rfloor + \lfloor B_2/R \rfloor < n$. In this case, we know that if the largest bid in the auction is smaller than R, the item in that round will be left unallocated. Also, if the larger bid is at least R, but the smaller bid is less than R, the item will be allocated, but at price R (instead of the second highest bid). Given this information, bidding anything below R, in any round, is weakly dominated. Also, by bidding R, bidder i can make sure that he wins at least $\lfloor B_i/R \rfloor$ items. Since bidder i can never win more than $\lfloor B_i/R \rfloor$ items, in any subgame perfect equilibrium, he wins exactly $\lfloor B_i/R \rfloor$ items. $\square$

It is interesting to know that the subgame perfect equilibrium of a sequential second price auction is not unique.[A6] In particular, there are many different *optimal* actions that the players

[A6] In our initial proof for this theorem, we characterize the set of equilibria of a sequential second price auction. Later, we came up with the presented proof which is shorter and easier to understand. Our initial proof and the characterization of subgame perfect equilibria are available upon request.

may take in each period, but they all eventually lead to the same outcome described in Theorem A4.

Lemma A2 *The function* $\frac{x}{C+x}$ *is monotonically increasing and concave in* x*, for any* $x \geq 0$ *and fixed* $C \geq 0$.

Proof: The first derivative in x is $\frac{C}{(C+x)^2}$ and the second derivative is $-\frac{2C}{(C+x)^3}$. $\square$

Theorem A5 *Suppose that there are two investment options. The revenue of the first one has the functional form* $\alpha Q \frac{x}{\alpha C+x}$ *if* x *is invested, while the revenue of the second one is* $\beta Q \frac{x}{\beta C+x}$*. Then, the optimal way to split* x *between the two options is to invest* $\frac{\alpha x}{\alpha+\beta}$ *in the first one and* $\frac{\beta x}{\alpha+\beta}$ *in the second one.*

Proof: The proof directly follows from Lemma A2 and first-order conditions. $\square$

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