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Integrating the Dirichlet-Multinomial and Multinomial Logit Models of Brand Choice

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Abstract

This paper discusses the interpretative benefits that arise from merging the Dirichlet-multinomial (DM) model as a loyalty variable in the multinomial logit (MNL) model of brand choice. The estimated parameters of this hybrid model compare favorably to those of a "pure" DM model (with no marketing mix variables) as well as those of a standard MNL model with an exponentially smoothed loyalty measure. The hybrid model offers an index of consumer heterogeneity and estimates of each brand's "preference share," adjusted for the effects of price and promotional activities. We illustrate the unique interpretations offered by these parameters across four different product categories, showing how changes in heterogeneity (across categories or model specifications) are closely related to changes in the overall impact of marketing mix variables.

In choosing among different methods to capture the effects of cross-sectional heterogeneity in the multinomial logit choice model, the selection criteria are most often related to model fit and predictive validity. Although fit is an important criterion, researchers tend to overlook other factors, such as the interpretability of the heterogeneity measure and its parameters:

- Does the measure indicate the overall degree of heterogeneity across the sample households?
- Can the researcher infer anything about the importance of consumer heterogeneity, vis-à-vis marketing mix effects, in the dataset(s) under analysis?
- Does the measure offer an aggregate indication of each brand's relative popularity, adjusted for marketing mix effects?

A fourth consideration is of critical importance if the heterogeneity measure is to have any practical value to analysts and managers:

- Can the measure and its required parameters be calculated in a parsimonious, computationally efficient manner?

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This paper discusses a heterogeneity measure that achieves all of these goals. Although this measure is fairly new in the MNL context, it relates directly to the Dirichlet-multinomial (DM) model, a choice model well known for its ability to capture heterogeneity across consumers. The DM model is quite adept at accounting for heterogeneity and offers interpretable parameters, but does not easily allow for explanatory (marketing mix) variables, which typically play a major role in MNL models. We construct a hybrid DM-MNL model that features the "best of both worlds." This model is a standard MNL model (accounting for the effects of marketing mix variables on choice behavior) with a simultaneously estimated DM component to capture heterogeneity across households.

The model discussed here is a prominent special case of a more general model employed by Fader and Lattin (1993), i.e., the Dirichlet-multinomial-geometric, which was developed to address the issue of nonstationarity in consumer choice. In contrast to that paper, we concentrate less on model specification and performance, and more on parameter interpretability. These interpretations become even more illuminating when compared across several different product categories.

The paper proceeds as follows. After briefly introducing the MNL model, we discuss the DM model and the interpretations offered by its parameter estimates. We compare three different models (the "pure" DM model with no marketing mix effects included, the DM-MNL hybrid, and an MNL model with an exponentially smoothed loyalty variable) within one product category, and then investigate possible cross-category patterns using summary statistics from three additional categories. We close by discussing some extensions of the modeling techniques presented here.

1. The multinomial logit model and measures of consumer heterogeneity

In many marketing applications, the multinomial logit model takes on the following structure:

$$MNL_{hjt} = \frac{e^{v_{hjt}}}{\sum_k e^{v_{hkt}}}, \text{ where} \tag{1}$$

MNL_{hjt} = the MNL probability that household h chooses brand j on purchase occasion t,
$v_{hjt} = \beta_{j0} + \beta_l LOY_{hjt} + \Sigma_m \beta_m x_{hjtm}$
= the deterministic component of utility of brand j for household h at purchase occasion t,
x_{hjtm} = explanatory variable for brand j and household h on purchase occasion t,
β_m = estimated logit coefficient for explanatory variable m,
β_{j0} = intercept term for brand j,
LOY_{hjt} = measure of consumer heterogeneity or "brand loyalty."

The x_{hjtm} generally include marketing mix variables such as price and different types of promotions. In most MNL models, however, the so-called loyalty variable often proves to be the most significant variable of all. The most popular loyalty measure, from Guadagni and Little (1983), uses an exponential smoothing approach to weight the past purchase history for each household:

$$LOY_{hj}(t+1) = \lambda LOY_{hj}(t) + (1-\lambda)y_{hjt}, \text{ where} \tag{2}$$

$LOY_{hj}(t)$ = "loyalty" of household h to brand j on purchase t,
y_{hjt} = 1 if household h buys brand j on purchase occasion t, 0 otherwise,
λ = smoothing parameter, $0 \leq \lambda \leq 1$

Many MNL applications (e.g., Gupta 1988, Kalwani et al. 1990, Kannan and Wright 1991) have used this measure (hereafter called SMOOTH), although researchers have developed other purchase-based approaches to convert observed past purchase patterns into heterogeneity measures.[1] If judged only by their ability to produce good-fitting models, most of these measures work fairly well. But they do suffer from several drawbacks. First, none of these measures meets all of the criteria mentioned at the start of the paper. None provide summary statistics that convey either the relative loyalty for each brand or the overall level of cross-sectional heterogeneity in the market. In fact, it is difficult to make *any* valid inferences from these measures. One can not, in general, draw any meaningful conclusions by comparing the estimated coefficient for the heterogeneity measure (β_l) with the brand-specific constants (β_{j0}'s).

A second problem lies in the updating mechanisms undering these measures. All of them initially treat the brand-specific intercepts (β_{j0}'s) as homogeneous "priors" for each household's loyalty towards each brand. As each purchase history grows, the LOY_{hjt} term moves away from these "priors" towards a heterogeneous (presumably more accurate) set of "posterior" values. But note that the relative weight placed on the brand-specific constants versus the time-varying measure never changes. This is strongly counterintuitive: as the purchase history grows, the importance of the aggregate-level "priors" should diminish. Thus, an additional criterion we can place on a "good" heterogeneity measure is that it should shift its weight from its "priors" to the updated household-level information in a monotonically increasing fashion over time. This does not occur in any of the simple additive approaches discussed so far.[2]

Finally, none of these methods was designed to act as a nested probabilistic choice model that can stand on its own outside of the MNL framework. A standalone heterogeneity model can be treated as a very appropriate benchmark for the full MNL model (Kalwani, Meyer, and Morrison 1992) and can help the analyst sort out some of the effects of heterogeneity from the impact of marketing mix variables.

The Dirichlet-multinomial model can address all of these concerns. First, its parameters yield meaningful interpretations, providing convenient measures of relative brand loyalty in addition to a well-accepted measure of overall heterogeneity. Second, it features a logical updating process that allows its aggregate-level "priors" to diminish over time. Third, it has been used as a stand-alone probabilistic choice model numerous times. And finally, the DM model can be derived as a special case of the MNL model, and therefore can be estimated using standard, commercially available MNL software.

2. The Dirichlet-multinomial model

The Dirichlet-multinomial model was popularized in marketing principally by Jeuland, Bass, and Wright (1980) and Goodhardt, Ehrenberg, and Chatfield (1984). In recent years, several researchers have demonstrated the suitability of the DM model for analyzing UPC scanner data (e.g., Elrod and Winer 1991; Jain, Bass, and Chen 1990; Kalwani, Meyer, and Morrison 1992). We refrain from discussing the DM model in much detail; we choose instead to focus more closely on its household-level updating scheme, which we adopt as our heterogeneity measure. The DM model integrates aggregate-level prior information with observed household-level purchasing behavior as follows:

$$DM_{hjt} = \frac{\alpha_j + n_{hjt}}{S + n_{ht}}, \text{ where} \tag{3}$$

DM_{hjt} = the DM probability that household h picks brand j on purchase occasion t,
α_j = brand-specific Dirichlet parameter, and $S = \Sigma_j \alpha_j$,
n_{hjt} = total purchases of brand j by household h up to (but not including) time t, and $n_{ht} = \Sigma_j n_{hjt}$.

The brand-specific Dirichlet parameters (α_j's) indicate the relative market-level preference for each brand in the absence of any other information. Assuming all brands are available at the start of each household's recorded purchase history, the probability of choosing brand j will be α_j/S. As the household's purchase history grows, past purchase information is used to update each household's set of DM_{hjt} estimates away from these priors and towards the heterogeneous (but unobservable) values that best conform to that household. Because the values of n_{hjt} cannot decrease, this household-level information will grow continuously, outweighing the impact of the priors over time, as desired.[3]

Besides offering an indication of relative preference towards each brand, the α_j's also indicate the degree of cross-sectional heterogeneity present in the market. If the parameters all take on high values, then purchase-history updating tends to play a small role; i.e., each household's choice probabilities stay close to α_j/S even as its purchase history grows. Conversely, low α_j's indicate relatively high heterogeneity, since observed purchase information will quickly dominate the α_j's in

equation (3). A summary statistic commonly used to convey the overall level of heterogeneity is the *polarization index* (Jeuland, Bass, and Wright 1980; Sabavala and Morrison 1977):

$$\phi = \frac{1}{1 + S} \tag{4}$$

This statistic is bounded between 0 and 1, with values of ϕ near 0 implying very low heterogeneity.

3. The DM-MNL marriage

With its long, successful history as a stand-alone probabilistic choice model, and its ability to capture and explain many different types of heterogeneity, the basic DM model shown in (3) is a worthy candidate to serve as the LOY_{hjt} variable, as an alternative to the other operationalizations of loyalty mentioned earlier. This combination offers several novel modeling benefits.

First, it allows the researcher to estimate the ordinary DM model as a special case of the MNL model. If the DM probabilities (i.e., the probability that household h chooses brand j at time t) are included as the sole variable in the MNL specification, then the MNL model can be collapsed into the DM model. In this case, the only parameter estimates would be the α_j's, which would retain all of the properties discussed earlier. Kalwani, Meyer, and Morrison (1992) recommend using this simple model as an appropriate benchmark for full MNL models.

Second, in estimating a full MNL model (with marketing mix variables) using the DM heterogeneity measure, the α_j's are estimated simultaneously with the MNL coefficients (β_m's), and therefore take into account the role of marketing mix effects. Thus, instead of interpreting the α_j's as the "best guess at relative brand preferences *in the absence of any information*," they can be interpreted as "the best guess at relative brand preferences *given the observed store environment information*." Differences will show how prices, promotions, etc., can distort the parameter estimates from a "pure" DM model (i.e., no marketing mix effects). For example, in the pure DM model, a low-priced high-share brand will have a relatively high value for α_j, regardless of its price. However, when prices are accounted for in the DM-MNL model, some of this brand's *apparent* popularity will be properly attributed to its low price; its estimated value of α_j would likely become smaller.

Changes in the α_j's from the pure DM model to the DM-MNL model also reflect the overall importance of the marketing mix in the choice process. Consider an extreme case where marketing mix effects are virtually irrelevant in the choice decision. In such a case, the α_j's (and the polarization index, ϕ) would barely change from the pure DM model to the DM-MNL model. Larger changes across these models indicate that one or more marketing variables are having some impact. Thus, in comparing the pure and hybrid DM models, the overall magnitude of changes in the α_j's (or ϕ) is an indication of the importance of marketing mix variables in general.

Pursuing this logic a bit further, the polarization index also has the potential to offer insights *across* datasets. For example, following the reasoning in the previous paragraph, changes in this summary statistic can give an indication of the impact of marketing mix effects across different product categories, geographical regions, or time periods. In an empirical section to follow, we examine the relationship between the polarization index and the impact of marketing mix effects across four different datasets.

3.1. Modeling approach

To estimate a pure DM model, we set the deterministic component of the logit utility function equal to the logarithm of the updated DM probabilities:

$$v_{hjt} = \ln(DM_{hjt}) \tag{5}$$

The logarithm cancels out the exponentiation that takes place in the logit model (equation 1), thereby making the MNL probabilities exactly equal to the household-level DM probabilities (equation 3). Note that there are no β_{j0} or β_m coefficients in (5); the brand-specific constants are now imbedded in the DM loyalty expression and the MNL coefficient for the DM term is constrained to 1. Thus, the only parameters in this logit model are the J α_j's that comprise the DM model.

In practice, model estimation can be simplified slightly. Because the denominator of DM_{hjt} is the same for all brands, it cancels out when introduced into the top and bottom of the MNL choice probability expression (1). Equation 5 can therefore be rewritten as:

$$v_{hjt} = \ln(\alpha_j + n_{hjt}) \tag{6}$$

Marketing mix covariates can be added to the model in a straightforward manner:

$$v_{hjt} = \ln(\alpha_j + n_{hjt}) + \Sigma_m \beta_m x_{hjtm} \tag{7}$$

Given a set of α_j's, it is no more difficult to calculate this heterogeneity measure than the exponentially smoothed one in (2). Whether or not this simple formula is truly suitable as a heterogeneity measure is an empirical question, which we will address at length shortly.

To obtain parameter estimates of the nonlinear α_j parameters along with the linear MNL coefficients (β_m), we use an iterative procedure (Fader, Lattin, and Little 1992) that provides maximum likelihood estimates for nonlinear parameters using ordinary (linear) MNL software. Our experience with this procedure has been very successful. Across several different applications, we have observed

reasonably rapid convergence, usually requiring no more than 5 to 7 iterations of the MNL model.

One technical caveat should be noted: when marketing mix variables are used in the model, the DM component no longer constitutes a full choice model as it does in (3); rather, it acts solely as an MNL loyalty variable. Strictly speaking, it is incorrect to refer to this variable as "Dirichlet-multinomial" when other explanatory variables are present. However, this does not prohibit using equation 3 as a loyalty variable, nor does it in any way affect the interpretations of ϕ and the α_j parameters as indications of cross-sectional heterogeneity and relative preference, etc. For convenience, we will continue to refer to this variable as "DM loyalty," although we recognize that only the "pure" DM model with no covariates is fully consistent with the underlying Dirichlet-multinomial choice theory.[4]

4. Illustration: Refrigerated orange juice

Our database consists of 200 randomly chosen households in Marion, IN, that made at least one purchase of refrigerated orange juice in 1984. We analyze the six leading brands in the market (all are 64-ounce packages): four national brands (Citrus Hill, Minute Maid, Tropicana Regular, and Tropicana Premium), a regional brand, and a store brand. These six brand-sizes account for 80% of all category purchases by our sample panelists. The model calibration period covers 1984, including 1589 purchases. We also use 1490 purchases from 1983 to initialize the loyalty variables, and reserve 666 purchases from the first half of 1985 as a forecast period. Explanatory variables include regular (depromoted) price, promotional price cuts, and newspaper feature activity. We estimate two DM-based logit models (the pure model and one with marketing mix variables) and compare them to an MNL model that uses the more traditional exponentially smoothed loyalty variable shown in equation 2.

Table 1 shows the maximum likelihood parameter estimates and some relevant summary statistics for the three models mentioned earlier. It should come as no surprise that the pure DM model fits notably worse than the two models with marketing mix variables. On the other hand, it is interesting that the incremental improvements in moving away from the pure DM model are rather modest ($\rho^2 = 0.14$ for the standard MNL model and $\rho^2 = 0.10$ for DM-MNL). These relatively small gains are consistent with the results of Kalwani, Meyer, and Morrison (1992), who found the pure DM model to be a tough benchmark for the full MNL model.

Both marketing mix models fit the data reasonably well. Over the calibration period, the standard MNL model fits conspicuously better than the DM-MNL hybrid, but the relative performances in the forecast period are the opposite. This modest reversal in forecasting ability is at least partially attributable to the different updating mechanisms embodied by the SMOOTH and DM measures, as discussed earlier.

The logit coefficients for the marketing mix variables shown in table 1 are sig-

Table 1. Estimation results: refrigerated orange juice

	Standard MNL model		Pure DM model		Hybrid DM-MNL model	
	coef.	(t-stat)	coef.	(t-stat)	coef.	(t-stat)
DM parameters (α's)						
Citrus Hill	—		1.043	(6.95)	1.315	(6.85)
Minute Maid	—		0.701	(5.99)	1.161	(5.81)
Regional Brand	—		0.542	(5.76)	0.409	(5.31)
Tropicana Regular	—		0.594	(5.45)	0.459	(4.83)
Store Brand	—		1.162	(4.88)	0.262	(3.49)
Tropicana Premium	—		0.075	(2.49)	0.218	(2.66)
Loyalty coefficient	3.927	(23.55)	—		—	
Marketing mix coefficients (β's):						
Feature	0.606	(6.01)	—		0.444	(4.71)
Regular price	−2.482	(−9.59)	—		−1.923	(−11.78)
Price cut	2.343	(9.29)	—		2.065	(8.78)
Brand-specific constants:						
Citrus Hill	1.161	(5.82)	—		—	
Minute Maid	1.133	(4.66)	—		—	
Regional Brand	0.219	(1.18)	—		—	
Tropicana Regular	0.404	(2.05)	—		—	
Store Brand	0.000[a]	—	—		—	
Tropicana Premium	0.884	(2.95)	—		—	
Smoothing constant	0.831	(59.36)	—		—	
Polarization index (φ)	—		0.195		0.207	
Calibration period (N = 1589):						
Log likelihood	−1438		−1667		−1503	
Parameters estimated	10		6		9	
Fit statistic ($\bar{\rho}^2$)	0.417		0.326		0.391	
Forecast period (N = 666):						
Log likelihood	−621		−723		−614	
Fit statistic (ρ^2)	0.370		0.267		0.377	

[a]Brand-specific constant for store brand constrained to zero.

nificant, intuitively sound, and similar in magnitude and direction. However, the standard MNL model coefficients are slightly larger than those of the DM-MNL model. This is a natural consequence that arises when comparing two different MNL models with such a marked difference in fit (see, e.g., Guadagni and Little 1983, page 207). The brand-specific parameters are also fairly similar across the different models. However, the DM parameters offer several insights that cannot be obtained from the MNL brand-specific intercept terms. For example, one may examine the "preference share" (i.e., α_j/S) as a indicator of the apparent propensity to purchase each brand, assuming all else is held constant.

Looking first at the preference shares for the pure DM model (shown in table 2), the most startling observation is how the preference share for the store brand compares to its overall market share. This difference is clearly due to the store brand's limited distribution. As shown in the final column, this brand is available at only 40% of the purchase occasions in the database. At the other extreme, the opposite effect is apparent, though not as strong, for the two most widely distributed (Citrus Hill and Minute Maid); in this case, high availability suggests that market share overstates apparent popularity.

Perhaps the most interesting comparison is between the α_j's for the two DM models. The two sets of parameters are positively correlated (r = 0.407), but the DM-MNL α_j's tell a different story about relative brand preferences than the "pure" α_j's. Notice that the preference shares for the three "flagship" national brands (Citrus Hill, Minute Maid, and Tropicana Premium) go up when marketing mix effects are accounted for, while the three lower priced brands all become less favored.[5] The store brand, which appeared to be the most preferred brand under the pure DM model, is apparently much less popular when its low price is taken into account. The DM-MNL preference shares give an indication of how market shares might look if all brands were equally available and had identical prices and promotions.

Finally, the value of the polarization index (φ) increases slightly in moving from the pure DM model to DM-MNL, indicating a higher level of heterogeneity in the market. After stripping away the effects of marketing mix variables, consumers begin to look less alike. In the next section we examine this phenomenon more closely, looking at comparable results across three additional product categories.

Table 2. Comparison of preference shares from Dirichlet models

	Market share	α/S (Pure DM)	α/S (DM-MNL)	Percent availability[a]
Citrus Hill	0.288	0.253	0.344	0.992
Minute Maid	0.236	0.170	0.304	0.997
Regional Brand	0.151	0.132	0.107	0.821
Tropicana Regular	0.146	0.144	0.120	0.826
Store Brand	0.137	0.282	0.069	0.404
Tropicana Premium	0.042	0.018	0.057	0.814

[a]Fraction of all purchase occasions in calibration period at which each brand was available.

5. Cross-category comparison

Three of the principal DM results for refrigerated orange juice can be summarized as follows:

1. The polarization index, or implied level of heterogeneity, in the pure DM model is $\phi = 0.195$.
2. The apparent level of heterogeneity is slightly higher for the DM-MNL model ($\phi = 0.207$).
3. Going from the pure DM model to the DM-MNL model (i.e., adding marketing mix variables) leads to an incremental improvement in model fit of $\rho^2 = 0.10$.

In this section we examine these three summary measures in a more systematic manner, using similar statistics from four product categories (including orange juice) to explore different relationships between the polarization index and the incremental value of marketing mix variables.

Earlier we discussed how changes in the α_j's and ϕ indicate *something* about the importance of marketing mix effects. We now formalize this link: as marketing mix variables have a larger impact on the consumer choice process, we expect to see a larger increase in the polarization index (ϕ) in moving from the pure DM model to the DM-MNL model.

Consider an extreme case in which price (or any other marketing variable) is far and away the most important influence on brand choice. If all consumers face the same shopping environment and have similar degrees of price responsiveness (as assumed by the class of logit models used here), then the pure DM α_j's will merely capture differences in price. Consumers will *appear* to be extremely homogeneous, and the polarization index, ϕ, will be very low. However, the DM-MNL model is able to dissentangle this price effect from cross-sectional heterogeneity. When the price effect is captured, the α_j's are free to pick up remaining sources of heterogeneity; we would expect the α_j's to drop (and ϕ to increase) rather dramatically from the pure DM model to the DM-MNL model.[6]

5.1. The Data

We have chosen three datasets that have been used in previously published studies, thereby eliminating almost any control over the choice of sample panelists, brands, and time periods. As we will demonstrate, these categories have substantial differences in the impact of marketing mix variables, so there is sufficient variability to discern whether or not the aforementioned relationship holds true.[7]

The new categories include crackers (used by Bucklin and Lattin 1991), coffee (Guadagni and Little 1983), and aseptic (shelf-stable, boxed) fruit drinks (Abe 1991). Each category is described briefly in an appendix; we refer the reader to the cited studies for further details about each dataset.

Table 3. Cross-category estimation results

	Log likelihood (calibration period)			Incremental ρ^2
	Standard MNL	Pure DM	DM-MNL	(Pure DM ⇒ DM-MNL)[a]
Aseptic Drinks	−1423	−1535	−1446	0.058
Orange Juice	−1438	−1667	−1503	0.098
Crackers	−464	−595	−484	0.187
Coffee	−1060	−1466	−1019	0.305

[a]Defined as the incremental gain towards perfect model (LL = 0): $(LL_{DM} - LL_{DM\text{-}MNL})/LL_{DM}$.

Table 4. Cross-category polarization indices

	Polarization index (ϕ)		
	Pure DM	DM-MNL	Δ%[a]
Aseptic Drinks	0.440	0.446	1.1%
Orange Juice	0.195	0.207	1.5
Crackers	0.252	0.475	29.8
Coffee	0.231	0.390	20.7

[a]Defined as the incremental percent gain towards pure heterogeneity ($\phi = 1$): $(\phi_{DM\text{-}MNL} - \phi_{DM})/(1 - \phi_{DM})$.

5.2. Estimation Results

For each of the three new categories, we estimate the same types of models as shown earlier for refrigerated orange juice. We include all marketing mix variables in their simplest forms, with no additional variables (e.g., past purchase dummy variables) that might have been included in the published models. The key estimation results are presented in table 3.

The pure DM model fits worse than the two marketing mix models in all cases, and standard MNL beats DM-MNL in two of the three new categories. However, the sizeable gap between standard MNL and DM-MNL observed for orange juice is considerably smaller for the other three categories. Although DM-MNL does not outperform the standard model, its advantages in parameter interpretability compensate to a large extent, particularly from a managerial perspective.

The final column in table 3 reflects striking differences in marketing mix effects across the four datasets. Hints of these differences can be seen in the papers that originally used these datasets. For example, the t-statistics for promotion variables reported in Abe (1991) for aseptic drinks are 2.3 and 3.3 (for feature and display, respectively); in contrast, Guadagni and Little (1983) report a promotional t-statistic of 14.1 for coffee. The results for orange juice (shown in table 1) and crackers lie between these two extremes.

The cross-category polarization indices, shown in table 4, offer evidence reasonably consistent with the suggested hypothesis. The two categories with slight marketing mix effects (aseptic drinks and orange juice) show very small increases in ϕ; on the other hand, crackers and coffee, with relatively large effects of marketing variables, show large increases in heterogeneity across the two DM models. Across the four categories, the correlation between these pairs of changes is 0.743.

Of course, these four product categories do not offer definitive proof of a link between marketing mix effects and the estimates of heterogeneity, but the strength of this apparent trend merits closer attention from choice modelers.

6. Conclusions

This paper has brought together two well-established modeling paradigms in a way that makes both models better off. The Dirichlet-multinomial and multinomial logit models have been applied extensively in managerial contexts, but no one has previously considered the benefits of combining them together. One useful implication of this work is the improved accessibility for the DM model. It might be fair to say that some researchers and practitioners, including frequent users of MNL, have avoided the DM model because of difficulties in parameter estimation. We have overcome this problem by demonstrating how the pure DM model can be estimated as a special case of an ordinary logit model. Fader and Schmittlein (1992) employ this estimation procedure for the DM model and compare it to more traditional, aggregate DM estimation methods (e.g., Uncles 1989). They show that the latter approach does not account for differences in brand availability over time and across stores, and therefore leads to biased estimates of consumer heterogeneity.

We have also shown that the basic DM structure is a promising heterogeneity variable for use in the MNL framework. The advantages shown here – primarily interesting and relevant parameter estimates – are benefits that might encourage modelers to employ this type of loyalty variable on a more regular basis. Fader and Lattin (1993) go a step further in also capturing nonstationarity in brand preferences using a multivariate (Dirichlet) extension of the beta-binomial-geometric model (Sabavala and Morrison 1981) instead of the stationary DM model used here. They discuss shortcomings in the ability of SMOOTH to sort out heterogeneity from nonstationarity, and demonstrate further improvements in fit and predictive ability beyond the simple models discussed here.

We have also offered insights about the relationship between cross-sectional heterogeneity and the influence of marketing mix variables. We observed that household brand preferences are generally more heterogeneous than indicated by the pure DM model. The size of this change appears to be directly related to the impact of prices, promotions, etc., in the brand choice process. Both of these important characteristics can be conveyed by a single summary statistic, the polarization index (ϕ). We strongly endorse the widespread use of this simple measure, and encourage other logit modelers to report it on a regular basis.

Notes

1. These include using dummy variables to indicate the brand last chosen by each household (e.g., Mayhew and Winer 1992), and each household's share of past purchases (e.g., Krishnamurthi and Raj 1988).
2. The semiparametric random effects model (Chintagunta, Jain, and Vilcassim 1991) is a powerful alternative approach that meets several of the criteria mentioned earlier. This measure enables the estimation of the relative preference and degree of heterogeneity for each brand. But this model does not offer a simple summary statistic for the overall extent of heterogeneity in the market, and estimation can be unwieldy: Fader and Lattin (1993) find that this model is roughly 1000 times more computationally intensive than the ordinary MNL estimation procedures used here.
3. Because the past purchases contained in n_{hjt} reflect the influence of marketing activities, the DM measure is contaminated by marketing mix effects. Bhattacharya, Fader, and Lodish (1992) examine this general issue, which affects all purchase-based heterogeneity measures.
4. It is theoretically possible to create a DM-MNL model that incorporates "true" Dirichlet heterogeneity, even in the presence of time-varying marketing mix effects. Such a model would have a different form than equation 7, but would likely require computationally intensive numerical integration procedures to obtain parameter estimates.
5. The average shelf prices for the six brands over the calibration period are Tropicana Premium, $2.26; Minute Maid, $1.98; Citrus Hill, $1.83; regional brand, $1.76; Tropicana Regular, $1.75; and store brand, $1.33.
6. Other factors might mitigate this pattern to some extent. For example, all consumers might not face the same shopping environment (i.e., they might choose among a set of stores with different pricing policies), and there will probably be some heterogeneity in price responsiveness. Nevertheless, these assumptions are likely to be fairly reasonable; moderate violations might not affect the proposed hypothesis to any great extent.
7. It is also worth noting that these categories also differ in that they fall into three different promotional clusters according to the Fader and Lodish (1990) cluster configuration.

Appendix: Description of data sets

Aseptic Drinks This IRI dataset contains three brands of aseptic fruit drink (packs of three single-serving paper cartons). It covers the period from 12/29/86 through 2/6/89, comprising a total of 3221 purchases by 143 panelists. We use the first 26 weeks (744 purchases) for initialization and the remaining 2477 purchases for calibration. Abe (1991) used a subset of these data, including only 33 of the heaviest purchasing households.

Coffee This is the classic SAMI database from Guadagni and Little (1983). The data cover 78 weeks from 9/14/78 through 3/12/80, including a 25 week initialization period. The calibration period consists of 1021 purchases of eight different brands made by 100 households in Kansas City.

Crackers Bucklin and Lattin (1991) use two years of IRI saltine cracker data from Williamsport, PA. There are 152 panelists and six brands. The first year (954 purchases) is used for initialization, while the calibration period consists of the 943 second-year purchases.

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