A Latent Change Point Model for Intertemporal Discounting with Reference Durations[1]

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Abstract

Products and services are increasingly offered with contracts of different lengths. Consumers' choice of a specific contract involves an intertemporal decision, as they have to discount future utility. Given the long duration of some contracts, consumers' instantaneous utility for a service may be time dependent arising from potential changes in their future needs. We study individuals' discounting of future benefits while allowing for changes in instantaneous utility. We gather experimental data from price matching tasks for a subscription-based service and identify discounting patterns using a latent change-point model. Our results show that models with change-points fit the individual's discounting pattern significantly better than models without. There is also conceptual superiority of including change-points as these are highly correlated with durations that consumers may consider in their decisions (e.g., planning horizon or time till graduation). Interestingly, the individual's discounting pattern is consistent with exponential discounting in the absence of change-points but follows a hyperbolic discounting pattern when allowing consumers' instantaneous utility to change over time.

Keywords: Intertemporal Discounting, Time dependent Preferences, Change-Point Model.

1 Introduction

Consumers frequently contend with intertemporal choices between an outcome in the present and in the future. Such decisions may involve choosing an immediate consumption (e.g., buying a car) or saving money for future consumption (e.g., for retirement) or the choice between a short-term contract with more flexibility but a high price per time unit and a longer-term contract with less flexibility but a smaller price per time unit.

Subscriptions or flat rate plans are popular pricing mechanism for a variety of products and services such as health clubs, online information services, newspapers, Internet access, software updates or pay TV. In many cases, subscriptions are offered with quite long contract periods. For example, in the U.S., the Economist offers a 2-year subscription of its magazine for \$335 and in Europe a 3-year subscription at a price of €715. Similarly, McAfee offers a 3-year subscription of its Internet security software for \$149 and Microsoft offers students a 4-year subscription of its product Office 365 University for \$79. Still other services such as 24 Hour Fitness (24hourfitness.com) offers a 2-year membership to its over 400 gyms nationwide at a price of \$1,230 or a 3-year membership at a price of \$1,497. From a firm's perspective, there are several advantages of selling longer contracts to customers. First, the marginal cost, especially for largely fixed cost-based products or services, are low or even zero. Second, customer lock-in effects can be achieved. Third, as customers usually pay the full subscription price at the beginning, a firm can leverage the increased assets based on interest rates in the marketplace.

From consumers' perspective, the primary incentive for choosing a longer subscription comes from price discounts. Beyond that, consumers' choice of a specific plan or tariff involves an intertemporal decision, as they have to discount their expected future utility. Past literature on intertemporal discounting shows that uncertainty and potential changes in future needs plays an important role in how consumers make decisions in the present (DellaVigna and Malmendier 2004; O'Donoghue and Rabin 2000, 1999). In the case of long contracts, consumers' instantaneous utility for a service may be time dependent arising from potential changes in their future needs (e.g., Kahneman and Snell 1992; Simonson 1990; Walsh 1995; Loewenstein et al. 2003). For example, Loewenstein, O'Donoghue, and Rabin (2003) show that people anticipate a change in their preferences over time. Consumers anticipation of changing tastes and utility may affect their intertemporal decisions when subscribing to contract-based products or services and in particular when such contracts last over such long durations as two to four years.

A variety of underlying drivers can be responsible for why individuals may expect a change in their instantaneous utility for a service. First, they may expect changes in technology or in their personal life in the near future, which can render a service less attractive.[1] Second, past studies show that "habit formation" (Pollak 1970; Ryder and Heal 1973; Wathieu 1997) and reference points have a strong impact on the individuals' expectations of future utility and discounting behavior (Loewenstein 1988; Loewenstein and Prelec 1992; Shelley 1993). It is likely that consumers' preference for a service changes when its contract length is beyond a reference duration (Loewenstein and Angner 2003; Meier and Sprenger 2010; Baucells and Sarin 2010; Cohen and Axelrod 1984; O'Donoghue and Rabin 2002). The reference time points (reference durations) that individuals use in how they discount future benefits may differ from one context to another. In some situations, it may be obvious what specific reference time point consumers will use (e.g., retirement age for retirement benefits). In other contexts, however, there could be multiple reference time points. For instance, when choosing a contract for health club membership, consumers may evaluate available contracts by comparing them to the typical contract length they subscribe for or to their individual personal planning horizon (which they consider when making a decision about future consumption). Thus, while identifying individuals' discounting patterns, it would be important to accommodate changes in their expected instantaneous utility. In terms of a modeling framework, a latent change point model (Barry and Hartigan 1993; Khodadadi and Asgharian 2008; Kapur et al. 2011; Fong and DeSarbo 2007), which has been used in other marketing contexts to accommodate changes in model parameters before and after an event, is eminently suitable to capture changes in expected instantaneous utility when estimating the individuals' discounting patterns. To the best of our knowledge, our paper is the first that employs a latent change-point framework while determining consumers' discounting patterns.

[1] According to the OECD Employment Outlook 2013, employees in the U.S. as well as in Europe are more often changing their jobs and job tenure is decreasing in all OECD countries (20% of all employees have to change their job in less than 12 months). Also the geographic mobility of consumers is increasing. According to the U. S. Census Bureau, more than 20% of all U.S. citizens move every year resulting in 17 million long-distance moves annually. With such movement, long service contracts become less attractive. Furthermore in many product categories, the technology underlying the product is changing rapidly (e.g. the product lifecycle of cellphones is in many cases less than one year).

In this paper, we study and analyze the individuals' discounting behavior allowing for temporal changes in their expected instantaneous utility (or per-period flow utility) of a product or service. To demonstrate the importance of latent change-points in the identification of the discount function, we collect data following an experimental paradigm and elicit consumers' discounting behavior for future benefits in the context of subscriptions. We employ experimental studies that involve matching tasks in which participants state a price that would make them indifferent between a baseline contract of an Internet access service (e.g., a one-month service contract with a price $30) and one with longer duration (e.g., a six-month contract). To identify discounting patterns from the matching tasks, we propose a model of consumer's intertemporal discounting that accounts for changes in their expected instantaneous utility from consuming the service via latent change-points. Our model thus accommodates shifts in the consumers' discounting behavior due to the interplay of contract length, and the consumers' change in expected instantaneous utility.

We emerge from the work with three key findings. First, we show that models with latent change-points in the expected instantaneous utility fit the data on individuals' valuation of a service significantly better than models with no change-point. Second, there is also conceptual superiority of including latent change-points in the modeling framework as these are highly correlated with typical reference durations that consumers may consider in their decisions (e.g., time to graduation). This further emphasizes the need to account for reference durations in consumers' discounting behavior. Third, while consumers' discount function is an exponential (constant) function in the absence of change-points, an inclusion of change-points allows us to identify that individuals discount future benefits following a hyperbolic pattern. Our results also have important implications for firms offering their products or services with subscription contracts as determining the correct discount rate matters for such managerial decisions as optimal pricing (Yao et al. 2012; Dubé et al. 2014) or targeting a service subscription to a specific customer segment.

The remainder of the paper is organized as follows. In Section 2, we describe an online experiment using matching tasks that provides initial evidence for the presence of reference durations and the need to accommodate latent change points. In Section 3, we propose a model incorporating latent change-points that allows us to infer discounting patterns from the matching tasks. Section 4 formally analyzes the data from the first experiment. In Section 5, we describe a second study that allow us to provide a more direct evidence of a causal link between reference durations and change points as well as to show the robustness of the results in Study 1. Section 6 concludes with a summary of results, discussion of underlying theoretical constructs and directions for future research.

2 Study 1 - Online Experiment

In our first study, we use an adaptive online survey to determine the pattern of the consumers' discounting of future benefits that they receive from access to a service. We choose Internet access as the focal service based on a pretest that indicated a majority of consumers use the service and are comfortable with answering questions related to its pricing. Participants are asked to state a price that would make them indifferent between a baseline contract (e.g., a one-month contract with a price $30) and one with longer duration (e.g., a six-month contract). We use pairwise matching tasks as our primary interest is in inferring consumers' discounting pattern for different contract durations and not in their absolute willingness-to-pay (WTP) for the focal service (e.g., Thaler 1981).

Procedure

In the first study, 212 graduate and undergraduate students from a Swiss university completed the online survey. At the beginning of the study, we asked respondents three questions related to the price they are willing to pay for Internet access - (Q1) the price which makes them indifferent between having Internet access for one month and no service, (Q2) the number of months of Internet access they expect if they pay CHF 150[2] and (Q3) the price which makes them indifferent between having Internet access for three months and no service. Using the responses to the three questions, an online program calculated the average price which a participant is willing to pay for one month of Internet access (p_1).[3]

Past research on intertemporal preferences has used matching tasks to determine how con-

[2] CHF 150 correspond to $ 150 at the time when the experiment was conducted

[3] To determine the average price for a respondent, the online program divided CHF 150 by their answer for question (Q2) and their answer for question (Q3) by three. Using these numbers, the online program then averaged the answers of the three questions (Q1) to (Q3). The responses to the three questions were highly correlated with the correlations being 0.79 (between Q1 and Q2), 0.91 (between Q1 and Q3) and 0.78 (between Q2 and Q3).

sumers discount monetary outcomes at future time points (e.g., receive $100 in the present versus $150 after one year) and calculate their discount rate from the ratio of current and future outcomes (Thaler 1981). In line with past research, we also used a matching task and asked participants to answer questions of the following nature in which they stated the monthly fee (in CHF) that makes them indifferent between a one-month baseline contract and a longer contract duration, i.e.,

> *"For which monthly price p_j, would you choose a subscription contract of duration T_j months as compared to a one-month contract of price p_1? $CHF\ p_j.-$"*

with p_j being the monthly payment and T_j the contract duration (e.g., 12 months). As noted earlier, the price p_1 for the one-month baseline contract duration for each respondent was customized based on their responses to the three initial questions. We term the price p_j as the monthly indifference price in the following discussion.

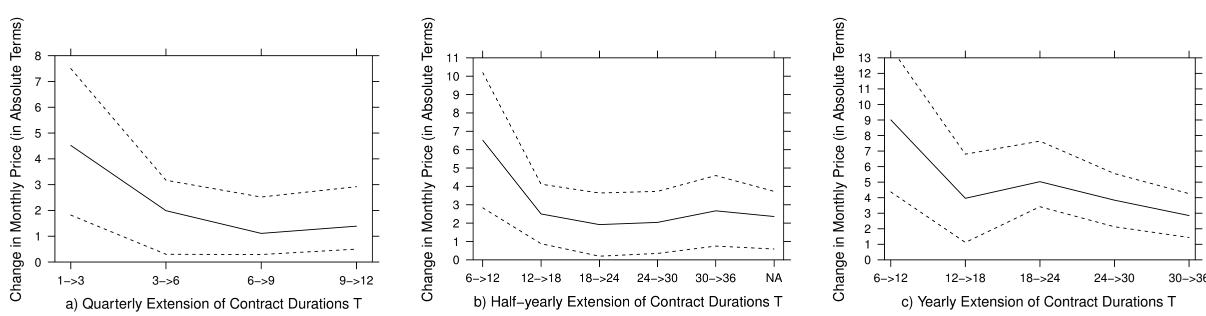
Each participant answered ten questions about their monthly indifference price for subscriptions with various contract durations (i.e., 3, 6, 9, 12, 18, 24, 30, 36, 48, and 60 months). The order of questions was counterbalanced across participants. All participants were instructed that they would be paying the access fee for the entire contract at the beginning.

After completing the matching tasks, we collected data from participants on possible durations they may use as a reference while considering subscription plans for Internet access - (1.) maximum contract duration they would consider to subscribe, (2.) maximum contract duration they have ever subscribed to, (3.) the time until their (expected) graduation, and (4.) their actual personal planning horizon (in months).

Model-Free Analysis

To provide model-free evidence, Table 1 shows the changes in respondents' average monthly indifference price for Internet service with different durations. To illustrate how we determine the changes in the monthly indifference price, consider the value in the second quarter (- 1.99). We calculate this value as follows: for each respondent, we take the difference between their monthly indifference price for a six-month contract and three-month contract and then average it across respondents. The changes in the average monthly indifference price for other durations (e.g., quarter, half-yearly and yearly) are calculated in a similar manner. The results show that the change in the average monthly indifference price decreases (in absolute terms) as a duration is more distant from the present. However, when the contract duration gets extended by a fourth half year (when considering half-year durations) or a third year (when considering yearly durations), there is an increase (in absolute terms) in the monthly indifference price for Internet access. We perform a paired comparison t-test to determine whether each successive change in the monthly indifference price is significantly different from the previous value. For the yearly extensions in contract duration, all changes are significant ($p < 0.05$).[4] Figure 1 graphically illustrates the changes in the monthly indifference price (in absolute terms) when the contract duration extends by a quarter, half-year and a year, respectively. The dashed lines show the 95% confidence interval.

Figure 1: Study 1 (Online Survey) - Change in Monthly Indifference Price with Contract Duration (in Absolute Terms)



In summary, from the model-free analysis, it is unclear whether individuals are discounting future benefits in a hyperbolic or an exponential manner. In fact, the non-monotonic pattern as shown in Figure 1 cannot be explained by either hyperbolic or exponential discounting if each individual keeps a constant instantaneous benefit over time. One possible explanation for the observed pattern in the monthly indifference prices is that the discounting pattern may still be described by a hyperbolic (or even an exponential) function albeit consumers' valuation of the service are changing with contract duration. For instance, consumers may value a subscription

[4]We show the monthly indifference price for Internet service averaged across respondents and it has a non-monotonic pattern with duration. While there may be heterogeneity across respondents in their valuation of the service, under either exponential or hyperbolic discounting, the individual plots should also be monotonically decreasing. And, the sum (or average) of a set of monotonically decreasing functions would be monotonically decreasing as well. Thus, heterogeneity across respondents alone cannot explain the non-monotonic pattern that we see in our data. With that being said, in the formal model presented later, we account for customer heterogeneity on multiple dimensions.

Table 1: Study 1 (Online Survey) - Change in Monthly Indifference Price with Contract Duration

Extension of Contract Duration T_j	Average Change in Monthly Price p_j
$1 \rightarrow 3$ (1. quarter)	-4.52
$3 \rightarrow 6$ (2. quarter)	-1.99
$6 \rightarrow 9$ (3. quarter)	-1.11
$9 \rightarrow 12$ (4. quarter)	-1.39
$1 \rightarrow 6$ (1. half year)	-6.51
$6 \rightarrow 12$ (2. half year)	-2.50
$12 \rightarrow 18$ (3. half year)	-1.92
$18 \rightarrow 24$ (4. half year)	-2.04
$24 \rightarrow 30$ (5. half year)	-2.67^x
$30 \rightarrow 36$ (6. half year)	-2.36
$1 \rightarrow 12$ (1. year)	-9.01
$12 \rightarrow 24$ (2. year)	-3.96*
$24 \rightarrow 36$ (3. year)	-5.03*
$36 \rightarrow 48$ (4. year)	-3.84*
$48 \rightarrow 60$ (5. year)	-2.85*

* $p < 0.05$ $\quad$ x -2.67 is significantly lower than -1.92

less so when its duration is beyond what they typically subscribe for. Such decrease in the valuation of a service will translate into a large price discount which consumers require to subscribe to a longer contract duration. To analyze the discounting pattern in a more formal manner, we propose a model of consumers' intertemporal preferences in the next section.

3 A Model of Consumers' Intertemporal Preferences

In this section we derive a model of consumers' intertemporal preferences. We begin with a discounted utility model that incorporates consumer discounting of future benefits over continuous durations. Next, we discuss the specifics of the discount function and propose a change-point model to capture temporal changes in consumers' valuation for service. We conclude with a discussion of model estimation.

3.1 Utility Model

Consider consumer i at time $t = 0$ facing J plans, each of which provide access to a service (e.g., a health club or Internet access service). Each plan j ($j = 1, \ldots, J$) is described in terms of length of time the consumer can access the service (starting at time $t = 0$) and a one-time fee for the entire contract duration to be paid at the beginning of the contract (e.g., a health club membership for three months for a fee of $300). While the time unit for the length of contract can be general (e.g., a day, a week or a month), we use months to be consistent with the data collection in our experiments. For plan j, let the contract duration be T_j months, where $T_j \geq 1$. We assume that the expected utility consumer i associates with plan j, $v_{ij}(T_j)$, starts from zero, $v_{ij}(0) = 0$, and increases with duration. We formulate a discounted utility specification (Samuelson 1937) in a way that the expected utility from plan j depends on the duration that a consumer has access to the service. Thus,

$$v_{ij}(T_j) = \int_{t=0}^{T_j} \nu_i \delta_i(t) dt, \tag{1}$$

where $\nu_i \geq 0$ is the expected instantaneous utility (or per-period flow utility) for consuming the service.[5] The individual-specific function $\delta_i(t)$ is for discounting future utility from ongoing access to the service.

Note that the impact of the two components of the discounted utility specification (the per-period flow utility and the discount function) on the overall valuation of the service cannot be identified non-parametrically without imposing structure. To this end, we leverage past research

[5]Note ν_i is assumed to be time invariant. Later, we extend the model and allow ν_i to be time dependent.

on intertemporal discounting that has shown that individuals have present-biased preferences, which results in a hyperbolic pattern in their discount rates (Strotz 1955; Thaler 1981). It is, therefore, reasonable to expect such a discount pattern in the present context as well. We postulate a functional form that allows for hyperbolic discounting, and nests constant (exponential) discounting, as discussed and proposed in Loewenstein and Prelec (1992). Thus, we assume the following generalized hyperbola for the discount function:

$$\delta_i(t) = (1+\alpha_i t)^{-\frac{\beta_i}{\alpha_i}}, \quad \alpha_i, \beta_i > 0. \tag{2}$$

The parameter α_i captures the divergence from constant discounting. As the parameter α_i becomes close to 0, the discount function $\delta_i(t)$ becomes an exponential function with the discount rate β_i, i.e. $\delta_i(t) = exp(-\beta_i t)$.

To develop the empirical specification of the model, let Y_i be the income of consumer i and $EU_i(Y_i)$ denote his expected utility of income Y_i evaluated at $t = 0$. For consumer i, let $p_i^o(1)$ denote the *observed* price by a researcher for a one-month contract (i.e., the baseline plan with the shortest duration). Let $p_i^*(T_j)$ denote the price for plan j with duration T_j months ($T_j > 1$) paid at $t = 0$ that makes a consumer *indifferent* at time $t = 0$ between the one-month plan at price $p_i^o(1)$ and switching to plan j. For a consumer, the expected utility for a one-month plan at price $p_i^o(1)$ and for a plan with a contract duration of T_j months, with a total price of $p_i^*(T_j)$, can be equated as follows:

$$\int_{t=0}^{1} \nu_i \delta_i(t)dt \; + \; EU_i(Y_i - p_i^o(1)) \; = \; \int_{t=0}^{T_j} \nu_i \delta_i(t)dt \; + \; EU_i(Y_i - p_i^*(T_j)) \tag{3}$$

A rearrangement of terms in Equation (3) leads to

$$EU_i(Y_i - p_i^o(1)) \; - \; EU_i(Y_i - p_i^*(T_j)) \; = \; \int_{t=0}^{T_j} \nu_i \delta_i(t)dt \; - \; \int_{t=0}^{1} \nu_i \delta_i(t)dt. \tag{4}$$

Let $Y_i \gg p_i^*(T_j)$. In this case, the marginal utility of income, k_i, can be treated as a constant in the range of $[Y_i - p_i^*(T_j), Y_i]$. Following a Taylor Expansion to the first order, Equation (4) leads to the following:

$$p_i^*(T_j) - p_i^o(1) \;=\; \int_{t=0}^{T_j} \frac{\nu_i}{k_i}\,\delta_i(t)dt - \int_{t=0}^{1} \frac{\nu_i}{k_i}\,\delta_i(t)dt$$

$$p_i^*(T_j) \;=\; \int_{t=0}^{T_j} \frac{\nu_i}{k_i}\,\delta_i(t)dt \;+\; p_i^o(1) \;-\; \int_{t=0}^{1} \frac{\nu_i}{k_i}\,\delta_i(t)dt \tag{5}$$

We divide Equation (5) by duration T_j to obtain the *monthly* price that makes consumers indifferent between a one-month contract and a contract with duration T_j. Thus,

$$\frac{p_i^*(T_j)}{T_j} \;=\; \int_{t=0}^{T_j} \frac{\nu_i}{T_j k_i}\,\delta_i(t)dt \;+\; \frac{p_i^o(1)}{T_j} \;-\; \int_{t=0}^{1} \frac{\nu_i}{T_j k_i}\,\delta_i(t)dt \tag{6}$$

Let $p_i^o(T_j)$ denote the *observed* total indifference price of consumer i for a contract of duration T_j. We assume that:

$$\frac{p_i^o(T_j)}{T_j} \;=\; \frac{p_i^*(T_j)}{T_j} + \varepsilon_{iT_j}, \quad \varepsilon_{iT_j} \sim N(0, \sigma_\epsilon) \tag{7}$$

where ε_{iT_j} captures the measurement error from the survey that is assumed to be *i.i.d.* across all plans and respondents. In sum, the relationship between indifference prices for a one-month contract duration and longer contract durations can be described as follows:

$$\frac{p_i^o(T_j)}{T_j} \;=\; \int_{t=0}^{T_j} \frac{\nu_i}{T_j k_i}\,\delta_i(t)dt \;+\; \frac{p_i^o(1)}{T_j} \;-\; \int_{t=0}^{1} \frac{\nu_i}{T_j k_i}\,\delta_i(t)dt + \;\varepsilon_{iT_j}, \tag{8}$$

3.2 Change-Point Model

Consumers' valuation for a service can be time dependent. For instance, the valuation may change when the contract length for a service is beyond a threshold duration (Loewenstein and Angner 2003; Meier and Sprenger 2010; Baucells and Sarin 2010; Cohen and Axelrod 1984; O'Donoghue and Rabin 2002). Such a change in valuation can have multiple underpinnings e.g., potential changes regarding either the future need for the service (would a subscription to a health club be useful beyond a year) or the ability to use it (any relocation will make a long contract for the local health club unattractively). In this paper, we capture the temporal changes in service valuation (as described below) but do not focus on disentangling among the different reasons.

There are two approaches for capturing changes in consumers' valuation of a service - allow for temporal discontinuity in either the expected instantaneous (or per-period flow) utility, ν_i, or in the parameters of the discount function, α_i, β_i. We estimate and test both approaches, but adopt the former approach for the following reasons. First, past work in the area of intertemporal discounting shows that a discount function plays a fundamental role in how consumers value future rewards. Several different psychological mechanisms may account for a hyperbolic pattern, including visceral factors and impulsivity (e.g., Ainslie 1975; Loewenstein 1996) and differences in cognitive representations between near and future events (e.g., Malkoc and Zauberman 2006; Zauberman and Lynch 2005). These findings collectively are indicative of the robust nature of discount function. Thus, an approach that allows for temporal changes in the parameters of a discount function may be inconsistent with prior findings. Second, in our empirical setting, our proposed approach fits the data significantly better than one that allows for temporal changes in the parameters of the discount function.

We accommodate changes in individuals' valuation for contracts by allowing the expected instantaneous (per-period flow) utility for consuming the service, ν_i, to vary over time. Therefore, we employ a change-point framework to capture structural breaks in the consumers' expected instantaneous utility (Khodadadi and Asgharian 2008).

For consumer i, suppose there is a single change-point in their expected instantaneous utility, let τ_i denote the latent change-point. We specify that $\nu_{it} = \nu_{i1}$ for $t < \tau_i$ and $\nu_{it} = \nu_{i2}$ for $t > \tau_i$. Assuming a single change-point, the expected utility for the service from plan j with duration T_j can be specified as follows:

$$v_{ij}(T_j) = \begin{cases} \int_{t=0}^{T_j} \nu_{i1}\ \delta_i(t)dt & \text{if } T_j < \tau_i \\ \\ \int_{t=0}^{\tau_i} \nu_{i1}\ \delta_i(t)dt + \int_{\tau_i}^{T_j} \nu_{i2}\ \delta_i(t)dt & \text{if } T_j \geq \tau_i \end{cases} \tag{9}$$

The expected utility function (9) together with the assumed functional form of the discount function (2) and a latent change-point (τ_{1i}), leads to the following estimable model:

$$\frac{p_i^o(T_j)}{T_j} = \begin{cases} \frac{\nu_{i1}}{T_j\ k_i\ (\alpha_i-\beta_i)} \left[\lambda(T_j)-1\right] + \eta_i\ +\ \varepsilon_{iT_j}, & \text{if } T_j < \tau_i \\ \\ \frac{\nu_{i1}}{T_j\ k_i\ (\alpha_i-\beta_i)} \left[\lambda\left(\tau_i\right)-1\right] + \frac{\nu_{i2}}{T_j\ k_i\ (\alpha_i-\beta_i)} \left[\lambda(T_j)-\lambda(\tau_i)\right] + \eta_i\ +\ \varepsilon_{iT_j}, & \text{if } T_j \geq \tau_i \end{cases} \tag{10}$$

with

$$\lambda\left(t\right) = \left(1+\alpha_i t\right)^{1-\frac{\beta_i}{\alpha_i}} \qquad \text{and} \qquad \eta_i = \frac{p_i^o(1)}{T_j}\ -\ \frac{\nu_{i1}\ \left[\lambda(1)-1\right)]}{T_j\ k_i\ (\alpha_i-\beta_i)}.$$

The model can easily accommodate multiple change-points in consumers' expected instantaneous utility. For instance, assuming two latent change-points, the expected utility for the service from plan j with duration T_j is specified as follows:

$$v_{ij}(T_j) = \begin{cases} \int_{t=0}^{T_j} \nu_{i1}\ \delta_i(t)dt & \text{if } T_j < \tau_{1i} \\ \\ \int_{t=0}^{\tau_{1i}} \nu_{i1}\ \delta_i(t)dt + \int_{\tau_{1i}}^{T_j} \nu_{i2}\ \delta_i(t)dt & \text{if } \tau_{1i} \leq T_j \leq \tau_{2i} \\ \\ \int_{t=0}^{\tau_{1i}} \nu_{i1}\ \delta_i(t)dt + \int_{\tau_{1i}}^{\tau_{2i}} \nu_{i2}\ \delta_i(t)dt + \int_{\tau_{2i}}^{T_j} \nu_{i3}\ \delta_i(t)dt & \text{if } \tau_{2i} \leq T_j \end{cases} \tag{11}$$

Capturing two latent change-points (τ_{1i}, τ_{2i}) in consumers' preferences for a service, the expected utility function (11), together with the discount function (2) results in the following estimable model (for $j > 1$):

$$\frac{p_i^o(T_j)}{T_j} = \begin{cases} \frac{\nu_{i1}}{T_j\ k_i\ (\alpha_i-\beta_i)} \left[\lambda(T_j)-1\right] + \eta_i\ +\ \varepsilon_{iT_j}, & \text{if } T_j < \tau_{1i} \\ \\ \frac{\nu_{i1}\ \left[\lambda(\tau_{1i})-1\right]}{T_j\ k_i\ (\alpha_i-\beta_i)} + \frac{\nu_{i2}\ \left[\lambda(T_j)-\lambda(\tau_{1i})\right]}{T_j\ k_i\ (\alpha_i-\beta_i)} + \eta_i\ +\ \varepsilon_{iT_j}, & \text{if } \tau_{1i} \leq T_j < \tau_{2i} \\ \\ \frac{\nu_{i1}\ \left[\lambda(\tau_{1i})-1\right]}{T_j\ k_i\ (\alpha_i-\beta_i)} + \frac{\nu_{i2}\ \left[\lambda(\tau_{2i})-\lambda(\tau_{1i})\right]}{T_j\ k_i\ (\alpha_i-\beta_i)} + \frac{\nu_{i3}\ \left[\lambda(T_j)-\lambda(\tau_{2i})\right]}{T_j\ k_i\ (\alpha_i-\beta_i)} + \eta_i + \varepsilon_{iT_j}, & \text{if } \tau_{2i} \leq T_j \end{cases} \tag{12}$$

Note that we can capture more continuous changes in consumers' expected instantaneous utility by allowing for more change points. Model estimations on our data (discussed in the following section of the paper) suggest that a single change point is sufficient.

We adopt a Bayesian framework for simulation-based inference. The details of the Bayesian estimation are given in the Appendix, which also describes an extensive simulation study that we conducted to assess parameter recovery and to test the identification of the correct number of latent change points in the flow utility. Next we discuss the estimation results from our model using the data from Study 1.

4 Study 1 - Results of Model Estimation

We estimate our proposed model with the generalized hyperbola as well as a nested model with exponential discounting. We refer to these models as the ”model considering hyperbolic discounting” and the ”model considering exponential discounting”. To assess structural breaks in the consumers’ expected instantaneous utility, we estimate both models without a change-point as well as with one and two latent change-points. A comparison of the fit of these models will show the benefit of accounting for latent change-points and reveal the magnitude of hyperbolic discounting. We use Markov chain Monte Carlo (MCMC) methods to estimate the models. For each model, we ran sampling chains for 400,000 iterations and assessed convergence by monitoring every 10th value of the time series of the draws. We report the results based on 30,000 draws retained after discarding the initial 10,000 draws as burn-in iterations.

We compare models using the Deviance Information Criterion (DIC), which accounts for the hierarchical model structure, penalizes model complexity and can be used to compare non-nested models(Ando 2007; Spiegelhalter et al. 2003a). Table 2 and Table 3 summarizes the results of our model estimations. Comparing the DIC of models without change-points, exponential discounting is superior to hyperbolic discounting. However, an opposite pattern emerges while comparing the models with latent change-points (see Table 2). Models considering hyperbolic discounting are significantly better than those considering exponential discounting. Furthermore, both models with hyperbolic discounting and change-points are superior to the one with exponential discounting and no change-point. Finally, the model that fits our data best is one with a single latent change-point and hyperbolic discounting. That a single change-point is sufficient has face validity based on the model-free evidence presented earlier.[6]

Parameter Estimates. Table 2 and Table 3 report posterior means as well as posterior variances for all model parameters and their respective posterior 95% intervals. The posterior mean of the latent change-point is $\tau_1 = 21.02$ months in the best fitting model with one latent change-point and the posterior mean of the expected instantaneous utility (or per-period flow utility) is $\nu_{i1} =$

[6] We also compared the model specification that fits our data best (i.e., a single latent change-point in the expected instantaneous, flow utility, ν_i, with hyperbolic discounting) with a model that accommodates a change point in the parameters of the discount function (α_i, β_i) (while assuming that flow utility, ν_i, is constant). The DIC of the latter model is 4892.03, which is significantly worse than our proposed model (4776.28).

53.25 and $\nu_{i2} = 26.21$, respectively. The distribution of the latent change point is captured using a beta distribution (please see details in the Appendix) and the estimates are $Beta(2.78, 5.18)$. The average mean squared error of the model with a single latent change-point and hyperbolic discounting is 0.68 and the average error variance is 2.99.[7]

The change-point models also provide information regarding when a change-point is likely to occur for each individual (recall that the change-point is individual specific). Figure 2 shows the probability density function (PDF) of the estimated change-point τ for all participants for the best fitting model (model considering hyperbolic discounting with one latent change-point). The figure also plots the empirical distribution of participants' personal planning horizon, their time until graduation, the maximum contract duration they would consider to subscribe to (Longest Subscription) as well as the maximum contract duration they have ever subscribed to (Maximum Contract Duration).[8] The distributions of the participants' personal planning horizon, the maximum contract duration they would consider to subscribe to as well as the maximum contract duration they have ever subscribed to have multiple modes while the distribution of students time until graduation is predominantly unimodal. The figure shows that the probability of a change-point is highest at a duration of around 27 months, which is quite close to the modes of participants' planning horizon and their time for graduation. In addition, we find that there is a strong positive correlation between participants' estimated latent change-points and their time to graduation (0.6057) as well as their planning horizon (0.5275). The correlation between participants' estimated latent change-points and the maximum contract duration they would consider to subscribe (0.1142) as well as the maximum contract duration they have ever subscribed to (0.1980) is significantly lower.

[7]We calculated the individual discounted price predictions based on the single latent change point model and constructed the 95% confidence interval. Across all observations, none of the 95% confidence interval around the price predictions contained 0.

[8]We applied a smoothing density estimator to the empirical distribution using the MATHEMATICA function smoothhistogram.

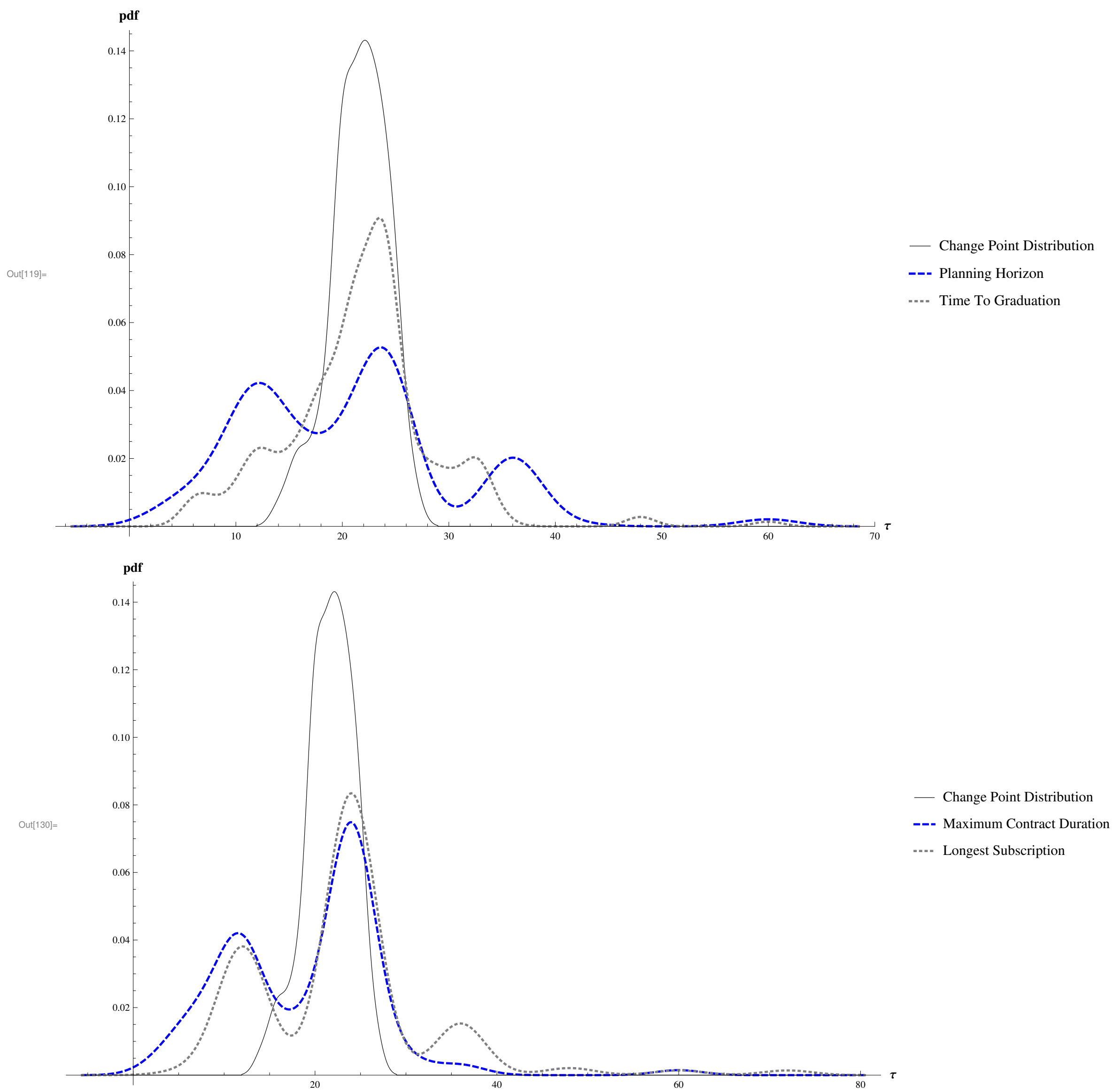
Table 2: Study 1 - Parameter Estimates and Model Fit (considering Hyperbolic Discounting)

Model without latent change-point			Model with 1 latent change-point			Model with 2 latent change-points		
Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*	Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*	Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*
α	0.01 *(0.00, 0.01)*	0.006 *(0.002, 0.009)*	α	0.62 *(0.38, 0.92)*	0.11 *(0.02, 0.22)*	α	0.75 *(0.45, 1.10)*	0.13 *(0.02, 0.32)*
β	0.03 *(0.03, 0.03)*	0.003 *(0.001, 0.006)*	β	0.05 *(0.04, 0.08)*	0.01 *(0.00, 0.04)*	β	0.07 *(0.04, 0.10)*	0.02 *(0.00, 0.09)*
ν	44.1 *(42.19, 46.18))*	21.35 *(16.72, 27.30)*	$\nu_{(1)}$	53.25 *(50.24, 56.48)*	27.99 *(21.72, 36.21)*	$\nu_{(1)}$	54.26 *(51.15, 57.51)*	28.68 *(22.17, 37.05)*
			$\nu_{(2)}$	26.21 *(24.59, 27.92)*	8.64 *(6.25, 11.77)*	$\nu_{(2)}$	32.63 *(18.45, 55.54)*	13.89 *(0.8343, 14.11)*
			τ_1	21.61 *(21.1, 22.11)*	9.99 *(7.65, 12.86))*	$\nu_{(3)}$	26.57 *(24.89, 28.35)*	8.97 *(6.46, 12.29)*
						$\tau_{(1)}$	21.64 *(21.13, 22.16)*	9.96 *(7.61, 12.91)*
						$\tau_{(2)}$	58.68 *(57.17, 59.69)*	1.52 *((0.07, 6.42))*
DIC	7352.48		DIC	4776.28		DIC	4846.69	

Table 3: Study 1 - Parameter Estimates and Model Fit (considering Exponential Discounting)

Model without latent change-point			Model with 1 latent change-point			Model with 2 latent change-points		
Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*	Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*	Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*
β	0.03 *(0.03, 0.03)*	0.001 *(0.000, 0.001)*	β	0.02 *(0.02, 0.02)*	0.001 *(0.000, 0.001)*	β	0.02 *(0.02, 0.02)*	0.001 *(0.000, 0.001)*
ν	43.99 *(42.04, 46.07)*	21.25 *(16.66, 27.22)*	$\nu_{(1)}$	41.56 *(39.79, 43.45)*	17.63 *(13.91, 22.43)*	$\nu_{(1)}$	42.14 *(40.36, 44.05)*	18.21 *(14.34, 23.13)*
			$\nu_{(2)}$	26.66 *(25.16, 28.25)*	7.79 *(5.51, 10.86)*	$\nu_{(2)}$	33.28 *(31.11, 35.72)*	17.57 *(11.85, 25.37)*
			τ_1	21.02 *(20.23, 21.83)*	2.42 *(2.05, 4.38))*	$\nu_{(3)}$	7.44 *(2.53, 12.4)*	2.79 *(0.02, 16.67)*
						$\tau_{(1)}$	25.16 *(24.52, 25.77)*	4.48 *(3.07, 6.31)*
						$\tau_{(2)}$	58.73 *(58.01, 59.64)*	1.29 *(0.43, 3.16)*
DIC	7337.09		DIC	6219.74		DIC	5932.62	

Figure 2: Study 1 (Online Survey) - Distribution of Change-Points and Participants Personal Planning Horizon, Time to Graduation, Longest Subscription as well as Maximum Considered Contract Duration



5 Study 2 - Fixed-Baseline Valuation

Study 1 shows that participants' change point is correlated with their reference durations (e.g., planning horizon and time to graduation). The objective of Study 2 is to provide a more direct evidence of a causal link between reference durations and change points. To address this objective, we manipulate the reference duration that participants consider while valuing a service. Study 2 also allows us to test the robustness of the results in Study 1. To do so, the matching task in Study 2 is similar to that used in Study 1 with the sole difference that the baseline reference price of a monthly contract is fixed across participants.

Procedure

Study 2 had a total of 307 participants who were assigned to two conditions termed as *"No Priming"* and *"12-Month Priming"*. All participants were masters-level students at a German university and trained in business economics. We chose Internet access as the focal service with baseline contract duration of one-month and price of € 30[9].

The *"No Priming"* condition had 137 participants. Each participant stated their maximum monthly price (in €) that would make them indifferent between the baseline contract of one-month at € 30 and a contract with a longer duration (i.e., 3, 6, 9, 12, 18, 24, 30, 36, 48, and 60 months). The order of questions was counterbalanced across participants. After the matching tasks, each participant also stated their critical durations when subscribing to an Internet access service - (1.) maximum contract duration they would consider to subscribe, (2.) maximum contract duration they have ever subscribed to, (3.) the time until their (expected) graduation, and (4.) their actual personal planning horizon.

The *"12-Month Priming"* condition had 170 participants. Each participant was primed that *after 12 months* an event may happen that will prevent them from using Internet service thereafter (the instructor primed participants to think about a lucrative job offer abroad or similar life-changing events). After the priming, the participants answered 10 matching tasks, same as those in priming condition.

[9]€ 30 correspond to $ 30 at the time when the experiment was conducted

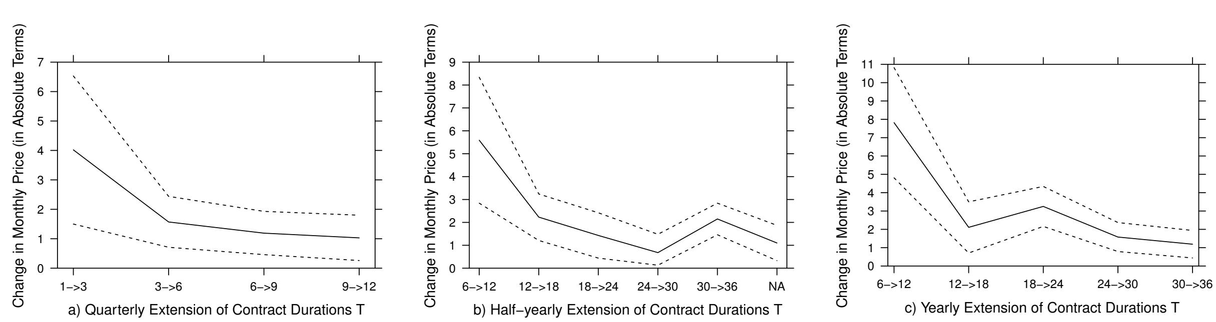
Following, we present first the results of a model-free analysis and after results from different model estimations without a change-point as well as with one or two latent change-points.

5.1 "No Priming" Condition

Model-Free Analysis

Table 4 and Figure 3 shows the change in respondents' average monthly indifference price for Internet service with different durations (calculated in a similar manner as described in Study 1). The pattern of changes in the monthly indifference price is similar to the one presented in Study 1. The change in the average monthly indifference price is decreasing (in absolute terms) as a duration is more distant from the present. When the contract duration gets extended by a fourth half year (when considering half-year durations) or a third year (when considering yearly durations), we find again an increase (in absolute terms) in the monthly price for an additional contract period. We performed paired comparison t-test to determine whether each successive change in the monthly indifference price is significantly different from the previous value. For yearly extensions, the change from the second to the third year is significant ($p < 0.05$).

Figure 3: Experiment 2a (No Priming) - Change in Monthly Indifference Price with Contract Duration (in Absolute Terms)



Model-based Results

Table 4: Experiment 2a (No Priming) - Change in Monthly Indifference Price with Contract Duration

Extension of Contract Duration T_j	Average Change in Monthly Price p_j
$1 \rightarrow 3$ (1. quarter)	-4.02*
$3 \rightarrow 6$ (2. quarter)	-1.57
$6 \rightarrow 9$ (3. quarter)	-1.19
$9 \rightarrow 12$ (4. quarter)	-1.03
$1 \rightarrow 6$ (1. half year)	-5.59*
$6 \rightarrow 12$ (2. half year)	-2.23
$12 \rightarrow 18$ (3. half year)	-1.43
$18 \rightarrow 24$ (4. half year)	-0.68
$24 \rightarrow 30$ (5. half year)	-2.15
$30 \rightarrow 36$ (6. half year)	-1.10
$1 \rightarrow 12$ (1. year)	-7.82*
$12 \rightarrow 24$ (2. year)	-2.11
$24 \rightarrow 36$ (3. year)	-3.25*
$36 \rightarrow 48$ (4. year)	-1.58
$48 \rightarrow 60$ (5. year)	-1.19

* $p < 0.05$

We estimate our proposed model[10] with the generalized hyperbola as well as a nested model with constant (exponential) discounting. As in Study 1, to assess structural breaks in consumers' expected instantaneous utility we estimate both models without a change-point and with one and two latent change-points. For each model, we ran sampling chains for 400,000 iterations and assessed convergence by monitoring every 10th value of the time series of the draws. We report the results based on 30,000 draws retained after discarding the initial 10,000 draws as burn-in iterations. Table 5 shows the estimates of our proposed model with the generalized hyperbola. A comparison of the models on Deviance Information Criterions (DICs) shows the benefit of accounting for latent change-points and reveals the magnitude of hyperbolic discounting. Consistent with the model-free evidence, a model with one latent change-point and hyperbolic discounting fits our data best.

[10]Our model in Section 3 still applies for Study 2 (Experiment 2a). The only change is to set all $p_i^o = €\ 30$

The parameter estimates in Table 5 show that participants' discounting of future benefits is heavily biased towards the present. Recall that α equal to 0 leads to exponential discounting. The parameter estimates of the latent-change-point distribution are $Beta(1.21, 2.16)$. The average mean squared error of the model with a single latent change-point and hyperbolic discounting is 0.264 and the mean error variance is estimated to be 0.34.[11]

Table 6 shows the estimates of our proposed model with exponential discounting. Similar to Study 1, it is noteworthy that with no latent change-point, a model with exponential discounting is superior to one with hyperbolic discounting. However, the inclusion of latent change-points in our modeling framework allows us to uncover that individuals are discounting future benefits in a hyperbolic fashion. Finally, the analysis also provides information on participants' individual latent change-points in their instantaneous (per-period flow) utility. We find that there is a strong, significant positive correlation between participants' individual latent change-points and their individual time to graduation (0.583) as well as their individual planning horizon (0.478). The correlation between participants' latent change-points and the maximum contract duration they would consider to subscribe (0.228) as well as the maximum contract duration they have ever subscribed to (0.143) is significantly lower.

[11]We calculated the individual discounted price predictions based on the single latent change point model and constructed the 95% confidence interval. Across all observations, none of the 95% confidence interval around the price predictions contained 0.

Table 5: Experiment 2a (Experiment Without Priming) - Parameter Estimates and Model Fit (considering Hyperbolic Discounting)

Model without latent change-point			Model with 1 latent change-point			Model with 2 latent change-points		
Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*	Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*	Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*
α	0.01 *(0.01, 0.02)*	0.001 *(0.000, 0.001)*	α	0.91 *(0.55, 1.50)*	0.41 *(0.11, 0.94)*	α	0.53 *(0.29, 0.77)*	0.25 *(0.04, 0.61)*
β	0.03 *(0.03, 0.03)*	0.00 *(0.00, 0.00)*	β	0.11 *(0.08, 0.16)*	0.00 *(0.00, 0.00)*	β	0.08 *(0.06, 0.10)*	0.00 *(0.00, 0.00)*
ν	25.6 *(24.91, 26.33)*	15.65 *(11.97, 20.33)*	$\nu_{(1)}$	27.53 *(26.53, 28.68)*	15.25 *(11.55, 20.14)*	$\nu_{(1)}$	26.94 *(26.1, 27.82)*	15.46 *(11.72, 20.25)*
			$\nu_{(2)}$	17.02 *(16.28, 17.8)*	9.834 *(6.32, 14.74)*	$\nu_{(2)}$	18.45 *(17.52, 19.46)*	9.21 *(1.87, 19.84)*
			τ_1	22.14 *(21.03, 23.22)*	3.99 *(1.37, 4.38))*	$\nu_{(3)}$	7.09 *(2.10, 11.59)*	6.32 *(0.02, 9.65)*
						$\tau_{(1)}$	25.53 *(24.79, 26.25)*	3.45 *(2.10, 5.45)*
						$\tau_{(2)}$	58.6 *(57.59, 60.04)*	2.20 *(0.57, 5.64)*
DIC	3273.21		DIC	2403.61		DIC	2443.38	

Table 6: Experiment 2a (Experiment Without Priming) - Parameter Estimates and Model Fit (considering Exponential Discounting)

Model without latent change-point			Model with 1 latent change-point			Model with 2 latent change-points		
Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*	Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*	Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*
β	0.02 *(0.02, 0.03)*	0.00 *(0.00, 0.00)*	β	0.02 *(0.02, 0.02)*	0.00 *(0.00, 0.00)*	β	0.02 *(0.02, 0.02)*	0.00 *(0.00, 0.00)*
ν	25.19 *(24.54, 25.87)*	15.17 *(11.65, 19.77)*	$\nu_{(1)}$	24.48 *(23.87, 25.11)*	12.44 *(9.53, 16.19)*	$\nu_{(1)}$	24.71 *(24.10, 25.35)*	12.46 *(9.57, 16.28)*
			$\nu_{(2)}$	18.80 *(17.89, 19.81)*	9.73 *(5.75, 15.41)*	$\nu_{(2)}$	24.56 *(22.25, 27.29)*	17.58 *(2.66, 38.69)*
			τ_1	19.11 *(17.20, 21.17)*	6.74 *(4.72, 9.49)*	$\nu_{(3)}$	17.67 *(15.96, 19.13)*	7.73 *(1.69, 18.07)*
						$\tau_{(1)}$	23.01 *(20.65, 25.5)*	6.92 *(4.56, 10.14)*
						$\tau_{(2)}$	58.85 *(53.23, 59.97)*	3.49 *(0.93, 5.97)*
DIC	3294.58		DIC	2783.52		DIC	2815.52	

5.2 "12- Month Priming" Condition

Model-Free Analysis

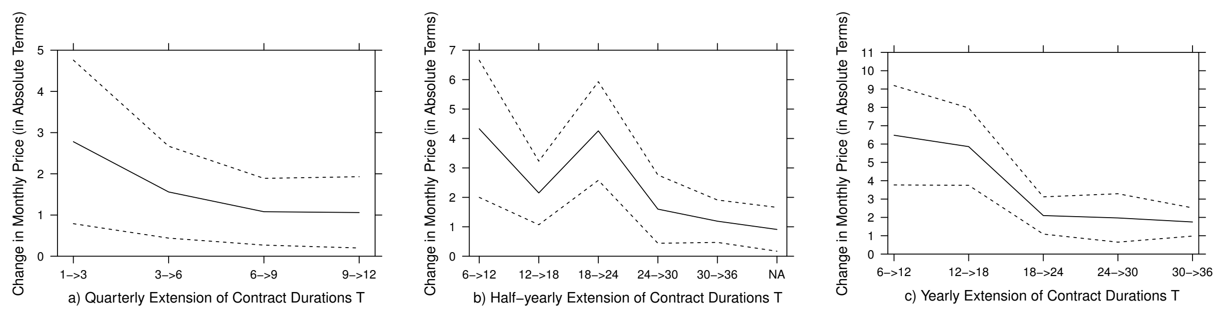
Table 7 and Figure 4 shows the changes in respondents' average monthly indifference price with duration (e.g., quarter, half-yearly and yearly). There is a noticeable shift after a contract duration of 12-months suggesting that our priming influenced the pattern of monthly indifference prices. For instance, a closer look at the half-yearly duration plot suggests that when the contract duration is beyond 12-months, we find an increase (in absolute terms) in the monthly indifference price for an additional contract period. Additionally, we find that there is a small, non-significant, increase around 30 months. This pattern suggests that there may be two latent change-points in the instantaneous utility of consuming the service - one change-point, due to our priming and a second change-point, due to any intrinsic duration that participants consider.

Table 7: Experiment 2b (Experiment with 12-Month Priming) - Change in Monthly Indifference Price with Contract Duration

Extension of Contract Duration T_j	Average Change in Monthly Price p_j
$1 \rightarrow 3$ (1. quarter)	-2.78
$3 \rightarrow 6$ (2. quarter)	-1.56
$6 \rightarrow 9$ (3. quarter)	-1.08
$9 \rightarrow 12$ (4. quarter)	-1.06
$1 \rightarrow 6$ (1. half year)	-4.33
$6 \rightarrow 12$ (2. half year)	-2.15*
$12 \rightarrow 18$ (3. half year)	-4.26*
$18 \rightarrow 24$ (4. half year)	-1.60*
$24 \rightarrow 30$ (5. half year)	-1.19
$30 \rightarrow 36$ (6. half year)	-0.91
$1 \rightarrow 12$ (1. year)	-6.48
$12 \rightarrow 24$ (2. year)	-5.86*
$24 \rightarrow 36$ (3. year)	-2.10*
$36 \rightarrow 48$ (4. year)	-1.97
$48 \rightarrow 60$ (5. year)	-1.75

* $p < 0.05$

Figure 4: Experiment 2b (Experiment with 12-Month Priming) - Change in Monthly Indifference Price with Contract Duration (in Absolute Terms)



Model-based Results

Table 8 shows the estimates of our proposed model with the generalized hyperbola. Consistent with model-free evidence, a two change-point model with hyperbolic discounting fits our data best. The Deviance Information Criterion (DIC) of the model with two latent change-points is lower than the DIC of all other estimated models (see Table 8 and Table 9). The posterior mean of the latent change-point provides further evidence of the impact of the priming on the instantaneous utility from consuming the service. The first latent change-point occurs around 11.36 months (close to the primed duration of 12 months) while the second change-point is around 22.06 months. That the first latent change-point is estimated to be close to the primed duration provides evidence for a causal relationship between the reference duration and change-point.

The parameter estimates of the first latent-change-point distribution are $Beta(25.1, 119.2)$ and of the second latent-change-point distribution are $Beta(6.53, 3.83)$, respectively. The average mean squared error of the model with two latent change-points and hyperbolic discounting is 0.436 and the error variance is estimated to be 1.869. As before in Study 1 and Study 2 - "No Priming" Condition, comparing the DIC of models without change-point, we find that exponential discounting is superior to hyperbolic discounting. Please see Table 9 for estimates of our proposed model with exponential discounting.

Table 8: Experiment 2b (Experiment with 12-Month Priming) -
Parameter Estimates and Model Fit (considering Hyperbolic Discounting)

Model without latent change-point			Model with 1 latent change-point			Model with 2 latent change-points		
Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*	Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*	Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*
α	0.04 *(0.03, 0.05)*	0.01 *(0.00, 0.02)*	α	0.04 *(0.02, 0.09)*	0.27 *(0.00, 0.61)*	α	0.31 *(0.14, 0.71)*	0.23 *(0.05, 0.82)*
β	0.05 *(0.05, 0.06)*	0.00 *(0.00, 0.00)*	β	0.03 *(0.03, 0.03)*	0.00 *(0.00, 0.00)*	β	0.22 *(0.16, 0.67)*	0.01 *(0.00, 0.02)*
ν	29.5 *(28.98, 30.04)*	8.70 *(6.46, 11.60)*	$\nu_{(1)}$	27.28 *(26.77, 27.81)*	8.96 *(6.68, 11.81)*	$\nu_{(1)}$	31.63 *(30.12, 33.81)*	7.76 *(4.78, 11.25)*
			$\nu_{(2)}$	18.94 *(17.82, 20.14)*	40.27 *(28.67, 55.94)*	$\nu_{(2)}$	18.21 *(17.08, 19.53)*	25.14 *(17.85, 34.95)*
			τ_1	11.11 *(10.87, 11.36)*	0.52 *(0.00, 0.02)*	$\nu_{(3)}$	6.92 *(5.50, 8.45)*	29.02 *(9.04, 61.65)*
						$\tau_{(1)}$	11.36 *(11.12, 11.51)*	0.43 *(0.16, 1.15)*
						$\tau_{(2)}$	22.06 *(20.23, 23.91)*	5.07 *(3.19, 6.76)*
DIC	5466.04		DIC	3779.16		DIC	3331.05	

Table 9: Experiment 2b (Experiment with 12-Month Priming) -
Parameter Estimates and Model Fit (considering Exponential Discounting)

Model without latent change-point			Model with 1 latent change-point			Model with 2 latent change-points		
Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*	Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*	Parameter	Posterior mean *(Posterior 95% interval)*	Posterior variance *(Posterior 95% interval)*
β	0.04 *(0.04, 0.05)*	0.01 *(0.01, 0.01)*	β	0.04 *(0.02, 0.05)*	0.01 *(0.01, 0.01)*	β	0.03 *(0.03, 0.03)*	0.01 *(0.01, 0.01)*
ν	28.46 *(27.99, 28.95)*	8.82 *(6.69, 11.58)*	$\nu_{(1)}$	36.58 *(17.29, 49.24)*	12.54 *(7.87, 27.94)*	$\nu_{(1)}$	27.66 *(27.22, 28.11)*	7.67 *(5.83, 10.18)*
			$\nu_{(2)}$	28.42 *(26.34, 29.70)*	8.19 *(6.02, 11.09)*	$\nu_{(2)}$	25.5 *(24.38, 26.69)*	17.33 *(9.55, 28.09)*
			τ_1	21.91 *(10.68, 28.81)*	11.73 *(0.29, 18.16))*	$\nu_{(3)}$	10.01 *(6.75, 13.31)*	42.01 *(7.95, 148.11)*
						$\tau_{(1)}$	13.46 *(13.01, 13.92)*	1.54 *(0.94, 2.38)*
						$\tau_{(2)}$	17.88 *(17.09, 18.8)*	4.70 *(2.84, 7.93)*
DIC	5352.7		DIC	4240.15		DIC	3545.64	

6 Discussion

Firms typically offer pricing contracts as a means of accessing a product or service. Subscriptions (or flat rate plans) with long contractual durations are ideal as customer lock-in can be achieved. For consumers, the primary incentive for choosing a subscription with a long contractual duration comes from the associated price discounts. Beyond that the consumers' choice of a specific plan involves an intertemporal decision, as they have to discount expected future utility from the service. While evaluating the overall benefit of a service, it is likely that the consumers' instantaneous utility is time dependent, especially with long contracts, as they may expect changes in technology or in their personal life (e.g., moving due to changing jobs), which can render a service less attractive. Incorporating such changes in instantaneous utility will be important when determining how consumers discount future benefits.

In this paper we show that individuals' discounting of future benefits from a service is influenced by latent changes in their expected instantaneous utility and that capturing these changes leads to a substantial impact on the nature of the identified discount function. For doing so, we gather data from studies in which participants had to answer a set of matching tasks, i.e. to state a price that would make them indifferent between a short baseline contract of an Internet access service (e.g., one-month) and one with longer duration (e.g., a six-month contract). To identify discounting patterns from the matching tasks, we propose a model of individuals' intertemporal discounting that accounts for changes in their expected instantaneous utility from consuming the service via latent change-points. Our model identifies individuals' discounting pattern due to the interplay of contract length, and any changes in their expected instantaneous utility for the service.

We have several important results. We find that models with latent change-points are empirically superior to models without latent change-points. There is conceptual superiority of including latent change-points as these are highly correlated with typical reference durations that consumers may consider in their decisions (e.g., time to graduation). Interestingly, while a consumers' discounting pattern is consistent with exponential (or constant) discounting in the absence of change-points, an inclusion of change-points allows us to identify that individuals discount future benefits with discounts rates following a hyperbolic pattern. Finally, we find that our proposed model that accommodates change-points in the expected instantaneous utility from a service fits the data significantly better than a model that allows for change-points in the discount function.

Our work will be of interest to researchers who study anomalies in consumers' intertemporal decisions. For instance, in the classic work of Thaler (1981), subjects showed present-biased preferences i.e., hyperbolic discounting, a pattern that has been found in many subsequent studies (Loewenstein and Prelec 1992; Laibson 1997; Ariely and Zauberman 2000; Ariely and Loewenstein 2000; Frederick et al. 2002). Collectively, these studies show that people do not discount in a rational manner by employing a constant discount rate independent of time as assumed in the discounted utility model proposed by Samuelson (1937). We add to this stream of work by showing that allowing for changes in the expected instantaneous utility can be critical for uncovering the hyperbolic discounting pattern.

For marketing practice, our results provide new ways in which customers could be segmented and consequently which type of contracts should be targeted towards what type of customers. While information regarding customers' planning horizon is typically not available, their psychographics and demographics can provide reasonable proxies. For instance, students will have much shorter planning horizons as they may be graduating and relocating. Similarly, single professionals, e.g., consultants, are more likely to relocate frequently. Such customers will value contracts with shorter lengths and firms can optimally price these contracts to make them more appealing. On the other hand, our research can also help firms in selling longer contract periods (e.g. longer subscription or flat-rate plans) to customers. Past work suggests that determining the correct discount rate is important for pricing (Yao et al. 2012; Dubé et al. 2014). Our results suggest that an even larger price discount per time unit should be offered for subscriptions with contract periods beyond the reference duration that consumers consider.

There are several avenues for future research. From a measurement perspective, we used matching tasks which is a common procedure for collecting data to study individuals' intertemporal discounting. Further research could show the impact of reference durations when discount rates are estimated using other data collection methods such as choice tasks or field experiments.

We analyzed consumers' intertemporal decisions using subscription plans for Internet service. Corroboration of these novel findings by subsequent research in other product and service categories, and possibly with subjects of different demographics, would be quite useful. Finally, we have considered services in which the payment is upfront. There are contexts in which consumers do pay a monthly fee even while subscribing for a yearly contract. For instance, many gym memberships involve monthly payments even if one is on an annual contract. For such cases, our model could be extended to incorporate the discounting of monthly fees as well. We hope this paper encourages work in these and related directions.

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Technical Appendix

As shown in the main text of the paper, the estimated model without latent change-points is defined by

$$\frac{p_i^o(T_j)}{T_j} = \frac{\nu_{i1}}{T_j\, k_i\, (\alpha_i - \beta_i)} \left[(1+\alpha_i T_j)^{1-\frac{\beta_i}{\alpha_i}} - 1 \right] + \frac{p_i^o(1)}{T_j} - \frac{\nu_{i1} \left[(1+\alpha_i)^{1-\frac{\beta_i}{\alpha_i}} - 1 \right]}{T_j\, k_i\, (\alpha_i - \beta_i)} + \varepsilon_{iT_j} \tag{A.1}$$

The estimated model with one latent change-point is defined by

$$\frac{p_i^o(T_j)}{T_j} = \begin{cases} \frac{\nu_{i1}}{T_j\, k_i\, (\alpha_i - \beta_i)} \left[\lambda(T_j) - 1\right] + \eta_i + \varepsilon_{iT_j}, & \text{if } T_j < \tau_i \\ \\ \frac{\nu_{i1}}{T_j\, k_i\, (\alpha_i - \beta_i)} \left[\lambda(\tau_i) - 1\right] + \frac{\nu_{i2}}{T_j\, k_i\, (\alpha_i - \beta_i)} \left[\lambda(T_j) - \lambda(\tau_i)\right] + \eta_i + \varepsilon_{iT_j}, & \text{if } T_j \geq \tau_i \end{cases} \tag{A.2}$$

$$\text{with} \qquad \lambda(t) = (1+\alpha_i t)^{1-\frac{\beta_i}{\alpha_i}} \qquad \text{and} \qquad \eta_i = \frac{p_i^o(1)}{T_j} - \frac{\nu_{i1}\, [\lambda(1) - 1)]}{T_j\, k_i\, (\alpha_i - \beta_i)}$$

The estimated model with two latent change-points is defined by

$$\frac{p_i^o(T_j)}{T_j} = \begin{cases} \frac{\nu_{i1}}{T_j\ k_i\ (\alpha_i-\beta_i)}\left[\lambda(T_j)-1\right]+\eta_i\ +\ \varepsilon_{iT_j}, & \text{if } T_j < \tau_{1i} \\ \frac{\nu_{i1}\ [\lambda(\tau_{1i})-1]}{T_j\ k_i\ (\alpha_i-\beta_i)} + \frac{\nu_{i2}\ \left[\lambda(T_j)-\lambda(\tau_{1i})\right]}{T_j\ k_i\ (\alpha_i-\beta_i)} + \eta_i\ +\ \varepsilon_{iT_j}, & \text{if } \tau_{1i} \le T_j < \tau_{2i} \\ \frac{\nu_{i1}\ [\lambda(\tau_{1i})-1]}{T_j\ k_i\ (\alpha_i-\beta_i)} + \frac{\nu_{i2}\ [\lambda(\tau_{2i})-\lambda(\tau_{1i})]}{T_j\ k_i\ (\alpha_i-\beta_i)} + \frac{\nu_{i3}\ \left[\lambda(T_j)-\lambda(\tau_{2i})\right]}{T_j\ k_i\ (\alpha_i-\beta_i)} + \eta_i + \varepsilon_{iT_j}, & \text{if } \tau_{2i} \le T_j \end{cases} \tag{A.3}$$

$$\text{with} \qquad \lambda\left(t\right) = \left(1+\alpha_i t\right)^{1-\frac{\beta_i}{\alpha_i}} \qquad \text{and} \qquad \eta_i = \frac{p_i^o(1)}{T_j} - \frac{\nu_{i1}\ [\lambda(1)-1)]}{T_j\ k_i\ (\alpha_i-\beta_i)}.$$

Below we describe the details for estimating the model with one latent change-point. The estimation of a model without latent change-points or with two latent change-points is similar. The model was coded in WinBUGS 1.4 (Bayesian inference Using Gibbs Sampling; Spiegelhalter et al. 2003). All codes are available from the authors upon request.

Likelihood Let consumer i provide J observations. We assume that ε_{iT_j}, which captures measurement error from the survey is i.i.d. across plans and respondents. Let ε_{iT_j} be normally distributed with mean 0 and variance σ_ϵ. For an observation j, let $g(\frac{p_i^o(T_j)}{T_j}|\nu_{i1}, \nu_{i2}, \alpha_i, \beta_i, \sigma_\epsilon)$ denote the probability density function for a normal distribution evaluated with mean defined by equation A.2 and variance σ_ϵ. Let L_i denote the likelihood for data from consumer i. Then,

$$L_i = \prod_{j=1}^{J} g(\frac{p_i^o(T_j)}{T_j}|\nu_{i1}, \nu_{i2}, \alpha_i, \beta_i, \sigma_\epsilon) \tag{A.4}$$

Let there be N consumers. As observations across consumers are assumed to be independent, the overall likelihood of the data is:

$$L = \prod_{i=1}^{N} L_i \tag{A.5}$$

The model contains several individual-level coefficients. We specify heterogeneity across consumers by assuming a distributional specification. For model estimation, we reparameterize α_i as exp(ψ_i^α) and β_i as exp(ψ_i^β) to ensure that they are positive. We assume that ψ_i^α (ψ_i^β) is normally distributed with mean $\mu_\alpha(\mu_\beta)$ and variance σ_α (σ_β). Similarly, we reparameterize the parameter ν_{i1} as exp(ζ_{i1}) and ν_{i2} as exp(ζ_{i2}) to ensure they are positive. We assume that ζ_{i1} (ζ_{i2}) is normally distributed with mean $\mu_{\nu 1}$ ($\mu_{\nu 2}$) and variance $\sigma_{\nu 1}$ ($\sigma_{\nu 2}$).

For the Bayesian estimation, we use the following set of priors for the population level parameters. For both parameters of the discount function (α, β) related to the mean of heterogeneity distribution, namely, {μ_α, μ_β} we set univariate normal prior with mean 0 and variance 1 and for both utility parameters, namely, $\mu_{\nu 1}$, $\mu_{\nu 2}$ we set univariate normal prior with mean 3 and variance 1. These prior values allow for a reasonable range for the log-normal distribution. For all parameters related to the variance of the heterogeneity distributions, namely, {σ_α, σ_β, $\sigma_{\nu 1}$, $\sigma_{\nu 2}$}, we set independent inverse Gamma prior IG (0.01, 0.01). Finally, we also assume that σ_ϵ has an inverse Gamma prior IG (0.01, 0.01).

Posterior Distributions

Individual-level parameters: We carried out the estimation by sequentially generating draws of ($\zeta_{i1}, i = 1, 2, ...n$), ($\zeta_{i2}, i = 1, 2, ...n$), ($\psi_i^\alpha, i = 1, 2, ...n$), ($\psi_i^\beta, i = 1, 2, ...n$), ($\tau_i, i = 1, 2, ...n$) from their posterior distributions conditional on other parameters. For each individual-level parameter, we used the Metropolis-Hastings algorithm with a random walk chain (Chib and Greenberg 1995, p330).

We used the step function in WinBUGS to implement the piecewise defined model in the WinBUGS framework and to estimate the posterior probability distribution of the latent change-point τ_i on the interval $(1, t)$. The change-point parameter τ_i has a continuous beta distribution as forming the prior density Beta(a, b), where a and b have a log-normal prior with values (2,1). We multiplied the beta prior with T (in our application with 60, please see below the numerical simulation study for a sensitivity analysis of the used values). The step() function can be used in WinBUGS as an indicator (Boolean) variable. This function returns 1 if its argument is greater than zero, and 0 otherwise (see Spiegelhalter et al. (2003b) for more details about the step() function).

Population-level parameters: We generated $(\mu_\alpha, \sigma_\alpha, \mu_\beta, \sigma_\beta, \mu_{\nu 1}, \sigma_{\nu 1}, \mu_{\nu 2}, \sigma_{\nu 2})$ and σ_ϵ given the draws of individual-level parameters using the standard Gibbs Sampler (Gelman, Carlin, Stern, and Rubin 1995).

For each model, we ran sampling chains for 400,000 iterations and assessed convergence by monitoring every 10th value of the time series of the draws. We report the results based on 30,000 draws retained after discarding the initial 10,000 draws as burn-in iterations.

We measured model fit with the Deviance Information Criterion (DIC) (Ando 2007; Spiegelhalter et al. 2003a), the log marginal density calculated using Newton and Raftery (1994, p. 21) importance sampling method and the Bayesian Information Criterion (BIC) (Schwarz 1978).

Simulation Study

The objectives of our simulation study are threefold. First, we assess how well our proposed model is able to identify latent change-points in consumers' intertemporal preferences. Second, we demonstrate how well our proposed model is able to recover the true parameters of the simulation. Third, we assess the sensitivity of our Bayesian estimation approach to the assumed priors.

For generating the observed monthly price indifference data for contracts of different lengths ($\frac{p_i^o(T_j)}{T_j}$), we specified the utility change-point model of intertemporal preferences as used in the empirical application. The simulated parameter values of α, β and ν are drawn from log-normal distributions. See Table A1 for the specific distribution values used to simulate the parameters of the utility change-point models of intertemporal preferences. We chose these values as they are similar to the estimated values in the empirical applications.

To demonstrate that our estimation approach can identify the correct number of change-points in consumers' flow utility, we simulated observed indifference prices for 200 individuals for ten different contract periods (3, 6, 9. 12, 18, 24, 30, 36, 48, 60 months) where the underlying flow utility ν has either zero, one or two latent change-points. In other words, we generated observed data for individuals assuming that the true model for all individuals has either zero, one or two latent change-points in their flow utility. For each true model of consumers' intertemporal preferences, we performed the data generation as follows. First, we simulated the individual-specific parameters for each individual using α, β, $\nu_{(1)}$, $\nu_{(2)}$ and $\nu_{(3)}$ (if applicable depending on the true model) as well as τ_1 and τ_2 (if applicable depending upon the true model). Next, based on these parameters, we computed individual-specific latent monthly indifference prices ($\frac{p_i^*(T_j)}{T_j}$) for each of the ten contract periods. Finally, we added a stochastic error term to each observation to create the observed data $\frac{p_i^o(T_j)}{T_j}$ on indifference prices. For each data generating model (containing zero, one or two change-points in the flow utility), we used ten Monte Carlo replications, which differ only in the stochastic error term i.e., the individual-specific parameters were held fixed across the ten datasets. These ten Monte Carlo replications help us to study the variations of parameter estimates across the generated samples. Thus, there are three data generating models for consumers' intertemporal preferences and ten datasets for each model resulting in a total of 30 datasets.

We estimated three models of consumers' intertemporal preferences (with zero, one or two latent change-points) on each of the 30 simulated datasets. For example, we estimated a model with zero, one or two latent change-points when the true data generation mechanism had zero change-points. Thus, we estimated 90 models on the 30 simulated datasets. Table A2 provides an overview of the log-marginal likelihoods (DICs) of the 90 models estimated on the 30 datasets. The results provide evidence that our estimation approach can identify the correct number of change-points in consumers' flow utility. Specifically, when the true data generating mechanism has zero change-points, the DIC of the models with zero change-points is better than models that impose one or two change-points. Similarly, when the true data generating mechanism has one (two) change-point(s), the DIC of the models with one (two) change-points is better than other model specifications.

To demonstrate how well our proposed model is able to recover the true parameters in the simulated data, we analyzed the estimates based on the correct model specification i.e., if the true data generating mechanism has one change-point, we assessed the parameters after estimating a one change-point model. We used the same priors that we presented in the "Model "Estimation" subsection of our experiments. Table A3 shows the true value and the respective average estimates across the ten Monte Carlo replications. The table also includes the variance of all parameter estimates across these ten datasets. A comparison of the true (simulated) values and the average posterior mean for all parameters shows that our proposed model recovers the parameters quite well.

Furthermore, to assess our model's ability to recover the true parameters with different priors of the change-point distribution, we simulated data of the one change-point model specification with different values for a and b of $Beta(a, b)$ distribution. Table A4 shows the true value and the respective estimated posterior mean and posterior variances of models with different $Beta(a, b)$ distributions such as $Beta(2, 2)$, $Beta(3, 2)$ and $Beta(.5, .5)$. For each case, we multiplied the $Beta(a, b)$ distribution by $T = 60$. A comparison of the true (simulated) values shows that our proposed model recovers the parameters of the change-point distribution quite well.

Finally, to assess how sensitive our estimation is to other assumed priors, we re-estimated a one change-point model specification with different prior specifications. Thus, in the results presented below, the true data generating mechanism has a single change-point and we varied the priors in the Bayesian estimation of a model that also imposes a single change-point. Table A5 shows the true parameter values of the simulation together with the estimated parameter values of the baseline model (see results in Table A3 for model with one latent change-point). We estimated six models with different prior specifications, namely (1.) a model with a more diffused prior for the variance of the stochastic error term (i.e. IG (0.1, 0.1)), (2.) a model with a more diffused prior for the population-level parameters α, β, ν_1 and ν_2 (IG (0.1, 0.1)), (3.) a model with a large prior mean for α and β (Prior mean of α and β is 1), (4.) a model with a large prior mean for ν_1 and ν_2 (Prior mean of ν_1 and ν_2 is doubled to 6), (5.) a model with a diffuse prior specification for the change-point in that it has mass points beyond the longest contract duration (i.e., the $Beta(a, b)$ distribution of the latent change-point is multiplied by 120 instead of 60). Table A5 shows the robustness of our estimation to different prior specifications and clearly indicates that the model is able to recover the true parameters.

Table A1: Parameter Distributions in Numerical Simulation

Model without latent change-point		Model with 1 latent change-point		Model with 2 latent change-points	
α	log-normal distribution meanlog = log(0.75) sdlog = 0.3	α	log-normal distribution meanlog = log(0.75) sdlog = 0.3	α	log-normal distribution meanlog = log(0.75) sdlog = 0.3
β	log-normal distribution meanlog = log(0.15) sdlog = 0.2	β	log-normal distribution meanlog = log(0.15) sdlog = 0.2	β	log-normal distribution meanlog = log(0.15) sdlog = 0.2
ν	log-normal distribution meanlog = log(60) sdlog = 0.1	$\nu_{(1)}$	log-normal distribution meanlog = log(60) sdlog = 0.1	$\nu_{(1)}$	log-normal distribution meanlog = log(60) sdlog = 0.1
		$\nu_{(2)}$	log-normal distribution meanlog = log(40) sdlog = 0.1	$\nu_{(2)}$	log-normal distribution meanlog = log(40) sdlog = 0.1
		τ_1	normal distribution mean = 20 sd = 5 $\tau_1 > 1$	$\nu_{(3)}$	log-normal distribution meanlog = log(10) sdlog = 0.1
				τ_1	normal distribution mean = 20 sd = 5 $\tau_1 > 1$
				τ_2	normal distribution mean = 40 sd = 5 $\tau_1 + 3 < \tau_2 < 60$

Table A2: Numerical Simulation Study - DIC Model Comparision

Simulation		Model 0CP			Model 1CP			Model 2CP	
Estimation	Model 0CP	Model 1CP	Model 2CP	Model 0CP	Model 1CP	Model 2CP	Model 0CP	Model 1CP	Model 2CP
1. Estimation	3361.15	3390.70	3405.90	5764.27	3628.98	3643.51	6882.64	5019.33	3682.15
2. Estimation	3383.73	3394.10	3403.75	5666.57	3607.57	3651.87	6794.68	4836.89	3374.12
3. Estimation	3318.74	3349.08	3351.78	5646.54	3444.10	3468.30	6905.01	4849.21	3510.06
4. Estimation	3376.82	3391.17	3398.36	5714.54	3605.85	3629.11	6864.94	4975.09	3611.60
5. Estimation	3340.63	3373.69	3380.33	5698.11	3565.57	3577.44	6846.52	4845.23	3544.55
6. Estimation	3278.07	3291.33	3298.65	5663.04	3556.38	3589.39	6849.52	4677.82	3345.57
7. Estimation	3165.22	3192.83	3199.40	5617.89	3529.20	3560.29	6851.04	5117.38	3615.91
8. Estimation	3277.98	3307.43	3325.76	5737.38	3645.11	3683.12	6825.53	4638.41	3601.34
9. Estimation	3350.57	3369.04	3382.18	5678.77	3419.69	3534.56	7012.93	4670.71	3530.24
10. Estimation	3219.67	3237.32	3262.86	5683.66	3689.44	3702.62	6823.47	4937.11	3562.86
Average	3307.26	3329.67	3340.90	5687.08	3569.19	3604.02	6865.63	4856.72	3537.84

Table A3: Numerical Simulation Study - Parameter Recovery For Simulated Data Using The Proposed Model

Model without latent change-point			Model with 1 latent change-point			Model with 2 latent change-points		
Parameter	True Value	Average Posterior Mean *Average Posterior Variance*	Parameter	True Value	Average Posterior Mean *Average Posterior Variance*	Parameter	True Value	Average Posterior Mean *Average Posterior Variance*
α	0.788	0.786 *(0.154)*	α	0.788	0.785 *(0.206)*	α	0.788	0.787 *(0.351)*
β	0.154	0.155 *(0.025)*	β	0.154	0.153 *(0.030)*	β	0.154	0.152 0.047
ν	60.09	60.15 *(1.37)*	$\nu_{(1)}$	60.09	60.13 *1.65*	$\nu_{(1)}$	60.09	60.38 *(2.03)*
			$\nu_{(2)}$	40.21	40.28 *(2.416)*	$\nu_{(2)}$	40.21	40.24 *(3.256)*
			τ_1	21.14	21.14 *(1.660)*	$\nu_{(3)}$	10.098	10.037 *(1.263)*
						τ_1	19.98	19.636 *(1.988)*
						τ_2	39.51	39.636 *(1.972)*

Table A4: Numerical Simulation Study - Parameter Recovery For Simulated Data with Different Change-Point Distributions (with 1 latent change-point)

	Simulation of Change-Point $\tau \sim Beta(2,2)$		*Simulation of Change-Point* $\tau \sim Beta(3,2)$		*Simulation of Change-Point* $\tau \sim Beta(.5,.5)$	
Parameter	True Value	Posterior Mean *Posterior Variance*	True Value	Posterior Mean *Posterior Variance*	True Value	Posterior Mean *Posterior Variance*
α	.788	.785 *.04*	.788	.786 *.05*	.788	.788 *.03*
β	.154	.154 *.00*	.154	.155 *.00*	.154	.153 *.00*
$\nu_{(1)}$	60.093	59.922 *4.11*	60.093	59.998 *4.25*	60.093	60.072 *4.30*
$\nu_{(2)}$	40.205	40.292 *3.08*	40.205	40.189 *2.42*	40.205	42.203 *8.02*
τ_1	33.174	33.101 *17.01*	34.018	33.993 *13.91*	27.136	27.306 *3.38*
$a.beta$	2	2.006 *0.26*	3	2.997 *.33*	.5	.504 *.07*
$b.beta$	2	2.012 *.25*	2	2.039 *.32*	.5	.505 *.19*

Table A5: Numerical Simulation Study - Sensitivity of Parameter Recovery From Prior Values in Model Estimation (with 1 latent change-point)

Parameter	True Value	*Baseline Model* Posterior Mean *Posterior Variance*	*Model w/ large error term* Posterior Mean *Posterior Variance*	*Model w/ large precision* Posterior Mean *Posterior Variance*	*Model w/ large prior α & β* Posterior Mean *Posterior Variance*	*Model w/ large prior ν_1 & ν_2* Posterior Mean *Posterior Variance*	*Model w/ large prior change-point* Posterior Mean *Posterior Variance*
α	.788	.783	.778	.793	.793	.784	.782
		.205	*.186*	*.197*	*.184*	*.198*	*.195*
β	.154	.152	.152	.158	.156	.158	.157
		.029	*.027*	*.026*	*.025*	*.027*	*.027*
$\nu_{(1)}$	60.093	60.171	60.151	60.414	60.336	60.362	60.343
		1.641	*1.572*	*1.611*	*1.510*	*1.559*	*1.552*
$\nu_{(2)}$	40.205	40.524	40.516	40.603	40.595	40.617	40.580
		2.438	*2.430*	*2.446*	*2.427*	*2.434*	*2.433*
τ_1	21.143	21.140	21.144	21.130	21.139	21.140	21.145
		1.732	*1.733*	*1.723*	*1.730*	*1.734*	*1.752*