

A Conjoint Model of Quantity Discounts

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Quantity discount pricing is a common practice used by business-to-business and business-to-consumer companies. A key characteristic of quantity discount pricing is that the marginal price declines with higher purchase quantities. In this paper, we propose a choice-based conjoint model for estimating consumer-level willingness to pay (WTP) for varying quantities of a product and for designing optimal quantity discount pricing schemes. Our model can handle large quantity values and produces WTP estimates that are positive and increasing in quantity at a diminishing rate. In particular, we propose a tractable WTP function that depends on both product attributes and product quantity and that captures diminishing marginal WTP. We show how such a function embeds standard WTP functions in the quantity discount literature as special cases. We also demonstrate how to use the model to estimate the consumer value potential, which is the product of the premium a consumer is willing to pay and her volume potential. Finally, we propose a parsimonious experimental design approach for implementation.

We illustrate the model using data from a conjoint study of online movie rental services. The empirical results show that the proposed model has good fit and predictive validity. In addition, we find that marginal WTP in this category decays rapidly with quantity. We also find that the standard choice-based conjoint model results in anomalous WTP distributions with negative WTP values and nondiminishing marginal willingness-to-pay curves. Finally, we identify four segments of consumers that differ in terms of magnitude of WTP and volume potential, and we derive optimal quantity discount schemes for a monopolist and a new entrant in a competitive market.

Key words: quantity discounts; willingness to pay; choice models; mixed logit; conjoint analysis *History*: Received: July 20, 2010; accepted: December 2, 2011; Wayne DeSarbo served as the guest editor-in-chief, then Preyas Desai served as the editor-in-chief, and Pradeep Chintagunta served as associate editor for this article.

1. Introduction

Quantity discounts represent a popular pricing practice used by business-to-business and businessto-consumer companies. For example, Blockbuster charges \$8.99, \$13.99, and \$16.99 for one, two, and three DVDs out-at-a-time plans, respectively. Disney charges admission rates for Disney World that depend on the number of days. For a 1-day admission, Disney charges adults \$79, and for 10 consecutive days, it charges \$243. Similarly, consumer goods companies often charge lower per-unit prices for large packages of products, such as detergents, beers, and paper towels (Allenby et al. 2004). Based on a sample of 472 brands, Gerstner and Hess (1987) find that a large majority (91.5%) were sold at quantity discounts in a supermarket in North Carolina. Other examples include print advertising rates that vary with respect to the number of ads placed per year and express mail service rates that depend on shipment volume. One key aspect of quantity discount pricing is that the per-unit or marginal price declines with higher purchase quantity.¹

From a demand perspective,² the rationale for quantity discounts is that often consumers' marginal willingness to pay (WTP) decreases with increasing quantity. A pricing scheme that mirrors consumers' WTP patterns is more profitable to the firm than a mere uniform price that charges the same price regardless of the number of purchased units (Dolan and Simon 1997). A second rationale for quantity discounts is consumers' heterogeneity in WTP: heavy users have higher marginal WTP for large quantities

¹ There are several forms of nonlinear pricing such as multipart tariff, multiblock tariff, and price points (see Dolan and Simon 1997, p. 164). In this paper, we focus on the price-points form of quantity discounts.

² There is also a supply-side rationale for quantity discounts that stems from the supplier's cost savings (e.g., reduced production, inventory, and transportation costs) when selling larger quantities (Dolan and Simon 1997).

than light users (Dolan and Simon 1997, p. 174; Wilson 1993). Thus knowledge of *consumer-level* WTP for successive units of a product or service is critical for designing optimal quantity discount schemes.

Conjoint analysis (Green and Srinivasan 1990) has been gainfully utilized to assess the impact of price on demand and estimate consumer WTP for products and services. Kohli and Mahajan (1991) introduce an approach for measuring reservation price, which corresponds to the price that equates the utility of a new product to that of a status quo product. Jedidi and Zhang (2002) further develop this method to allow for the effect of new product introduction on category-level demand. Chung and Rao (2003) and Jedidi et al. (2003) describe methods for estimating consumer WTP for product bundles. More recently, Ding et al. (2005) and Park et al. (2008) propose incentive-compatible conjoint procedures for eliciting consumer WTP for product attributes. Miller et al. (2011) compare the performance of four commonly used approaches to measure consumers' WTP to real purchase data. They find that conjoint analysis does well in inferring the true demand curve and determining the right pricing decision.

Most pricing applications of conjoint analysis do not include quantity as an attribute in the design. They implicitly assume that a consumer buys one unit of a product at a single price and that consumer purchase rates do not depend on price (Iyengar et al. 2008, Kim et al. 2004). Although it may seem trivial to add quantity as a factor, there are several design and analysis issues that traditional conjoint analysis may encounter when estimating WTP for successive units of a product.

First, the traditional conjoint design requires as many price factors as quantity levels (i.e., a price factor for the first unit and a discount factor for each of the subsequent quantity levels). For example, for a product with six quantity levels, one needs to create six corresponding price factors. If each price/discount factor has three levels, then the full factorial is 6×3^6 . Thus a traditional conjoint design may work in situations where the range of quantity offered is limited, but it may not be efficient when the range is large, making the respondent task tedious. Second, the conjoint part-worth function, although flexible, may not result in WTP measures that are positive, monotonically increasing in quantity, and characterized by diminishing return. These properties are required for a proper WTP estimation (see Haab and McConnell 1998). Failure to enforce these constraints can lead to nonsensical measures of WTP and erroneous demand curves. For example, in a conjoint study on midsize sedans, Sonnier et al. (2007) obtain negative WTP estimates for between 13% and 23% of the participants. In our study, the standard choice-based conjoint model resulted in only two respondents (out of 250) with WTP estimates that satisfy the constraints of diminishing marginal WTP and positivity.

Recently, a few models have been proposed to account for volume in conjoint analysis. Kim et al. (2004) introduce a volumetric conjoint model in which product attributes are related to satiation parameters. Iyengar et al. (2008) propose a choice-based conjoint model that infers consumer usage levels as functions of the product features and the price components of a three-part tariff. Schlereth et al. (2010) use a WTP function approach to derive optimal two-part tariffs. However, none of these models is built to directly handle quantity discounts in conjoint analysis.

In this paper, we build on this emerging literature and propose a choice-based conjoint model for estimating consumer-level WTP values for varying quantities of a product and for designing optimal quantity discount pricing schemes. Our model can handle large quantity values and produces WTP estimates that are positive and increasing in quantity at a diminishing rate. We show how the proposed WTP function embeds two standard functions used in the quantity discount literature as special cases. In particular, we show how to use the WTP function to estimate the consumer value potential, which we decompose in terms of WTP for the first unit (which captures price premium) and a WTP multiple (which captures volume potential).

We also propose a parsimonious experimental design approach for implementation that can handle a large number of quantity and price levels. Two critical features of the design are needed for determining the WTP values for different quantities of a product: (i) the experiment must include purchase quantity of the product as an attribute, and (ii) all choice sets in the conjoint experiment must include the no-purchase option. This latter feature is critical for obtaining unambiguous dollar-metric estimates of WTP (Haaijer et al. 2000).

We test our proposed model using data from a conjoint experiment involving consumer choice of online movie rental plans and compare our WTP distributions to those obtained from a standard choice-based conjoint (CBC) model. We find that the marginal WTP in this category decays rapidly with quantity. We also find that the standard CBC model results in anomalous WTP distributions with negative WTP values and nondiminishing marginal WTP estimates. For example, 16% of the respondents would not purchase a one-DVD plan from Netflix when offered for free, even though 63% of these respondents are current Netflix subscribers. We identify four segments of consumers that differ in terms of their WTP premium and purchase volume potential. An online movie rental company could use such information to target its customers based on their value potential to the firm.

Finally, we use the parameter estimates to characterize consumer demand for online movie rental services and to design optimal quantity discount schemes that maximize gross contribution.

The rest of this paper is organized as follows. In §2, we describe the proposed model. In §3, we report an application of the model to the pricing of online movie rental services. Section 4 discusses the empirical results. In §5, we use the estimation results to characterize consumer demand for online movie rental services, and in §6, we use them to derive optimal quantity discount schedules. Section 7 concludes the paper.

2. The Conjoint Model

In this section, we first model consumer surplus as a difference between consumer willingness to pay and price. Next, we propose a WTP function for measuring the maximum amount that a consumer is willing to pay for a given quantity of a product (Wilson 1993). Finally, we present the Bayesian multilevel procedure we use for model estimation.

2.1. The Surplus Model

Consider a choice set consisting of *J* alternatives. Each choice alternative j (j = 1, ..., J) represents a product or a service that is described in terms of attribute levels, product size or quantity of service offered, and price (e.g., a movie rental plan for two DVDs out at a time from Blockbuster for \$13.99 a month). Thus, in contrast to standard conjoint analysis, consumer choice is based on both product attributes and quantity offered. Embedding such a quantity component in the conjoint design is a critical part of our measurement of consumers' WTP for successive units.

Let q_j be the quantity offered for product alternative *j*. We assume that consumer *i* (*i* = 1, ..., *I*) cannot choose more than one alternative. Let $p(q_j)$ be the price associated with q_j units of product *j*. The price schedule $p(q_j)$ represents a quantity discount scheme whereby the marginal price successively decreases with quantity. For example, Netflix charges a monthly fee of p(1) =\$9.99 for a one-movie-at-a-time plan and p(2) =\$14.99 for a two-movie-at-a-time plan. We specify the following surplus equation for consumer *i* and product *j*:

$$S_{ij}(q_j, p(q_j)) = \text{WTP}_{ij}(q_j) - p(q_j) + \varepsilon_{ij}, \quad (1)$$

where $\text{WTP}_{ij}(q_j)$ is the willingness to pay that consumer *i* associates with q_j units of product *j*, and ε_{ij} is an error term that is observable to consumers but unobservable to the researcher.

Let j = 0 denote the no-choice option. We set the willingness to pay for zero quantity to 0 (i.e., WTP_{*ii*}(0) = 0). Then using Equation (1), the surplus

corresponding to the no-choice option for consumer *i* is $S_{i0}(0, 0) = \varepsilon_{i0}$.

The surplus model specified in Equation (1) is derived from a quasi-linear utility function, which is free of wealth effects. (The derivation is straightforward and can be obtained from the authors upon request.) Thus, in terms of utility, the WTP or reservation price in Equation (1) refers to the price that equates the utility of q units of product j to the utility of the no-choice option (see Jedidi and Zhang 2002). The assumption of quasi-linearity of the utility function is reasonable for products and services whose prices are relatively small compared with the total budget but may not be adequate for products whose demand depends on income such as cars (Nevo 2000, p. 518). We check for the robustness of such an assumption in our empirical application.

2.1.1. The WTP Function. We assume that the willingness to pay that consumer *i* associates with q_i units of product *j*, WTP_{ii}(q_i), increases with quantity but at a decreasing rate. Such an assumption stems from consumer diminishing marginal utility or satiation and has been a cornerstone of utility theory (e.g., Baucells and Sarin 2007). Specifically, we propose a WTP function where the marginal WTP from the *q*th unit (q > 1) of product *j* is a fraction of the marginal WTP from the (q - 1)th unit. Let $w_{ij1} > 0$ be consumer *i*'s WTP for the first unit of product *j*, and let λ_{iq}^{j} be her decay parameter for product j and quantity q. Suppose for now that this decay parameter is invariant across products; i.e., $\lambda_{iq}^{\prime} = \lambda_{iq} \ (0 < \lambda_{iq} \le 1) \ \forall j = 1, \dots, J$. (Later, we discuss the more general case where the decay parameter can vary as a function of product features.) Then the marginal WTP for the second unit is $\lambda_{i2}w_{ii1}$, and the marginal WTPs for the third, fourth, ..., qth units are $\lambda_{i2}\lambda_{i3}w_{ij1}, \lambda_{i2}\lambda_{i3}\lambda_{i4}w_{ij1}, \ldots, \lambda_{i2}\lambda_{i3}, \ldots, \lambda_{iq}w_{ij1}$ respectively. Summing the marginal WTPs up to the *q*th unit, we obtain the following WTP function:

WTP_{ij}(q_j) =
$$w_{ij1} \sum_{k=1}^{q_j} \prod_{m=1}^k \lambda_{im}, \quad q_j \ge 1,$$
 (2)

where $0 < \lambda_{im} \le 1$ is the WTP decay parameter for the *m*th unit (*m* > 1) of product *j*. By definition, $\lambda_{i1} = 1$.

As a special case, suppose the decay parameter $\lambda_{im} = \lambda_i$ (for all m > 1) does not vary with quantity. Then the marginal WTPs for the first, second, ..., *q*th units are w_{ij1} , $\lambda_i w_{ij1}$, $\lambda_i^2 w_{ij1}$, ..., $\lambda_i^{q-1} w_{ij1}$, respectively. Note that as $0 < \lambda_i \leq 1$, the marginal WTPs are positive and decreasing. In addition, the closer λ_i is to 1 (0), the smaller (larger) is the diminishing of the marginal WTP. Note that the WTP function in Equation (2) reduces to WTP_{ij}(q_j) = w_{ij1} when consumers buy only one unit of a product.

The WTP for q units in Equation (2) is the discounted sum of marginal WTPs for each unit of product *j* and is additively separable across quantities. Such a WTP specification is suitable for product categories characterized by diminishing marginal utility of consumption. This assumption holds for most products and is commonly made in the economics literature (e.g., Gerstner and Hess 1987, Wilson 1993). The WTP specification, however, may not be suitable for addictive product categories (Gordon and Sun 2010) or products that command quantity premia. Furthermore, some product categories require a minimum number of units of the product for the purchase to have meaningful value to consumers. For example, shoes and earrings have no value unless bought in pairs. In these situations, one needs to redefine quantity q_i in terms of minimum purchase sizes (e.g., pairs).

Substituting Equation (2) for $\text{WTP}_{ij}(q_j)$ in Equation (1), we obtain the following, fully specified, surplus function for consumer *i* and product *j*:

$$S_{ij}(q_j, p(q_j)) = w_{ij1} \sum_{k=1}^{q_j} \prod_{m=1}^k \delta_{im} - p(q_j) + \varepsilon_{ij}$$
$$= S_{ij}(q_j, p(q_j)) + \varepsilon_{ij}, \qquad (3)$$

where $s_{ij}(q_j, p(q_j))$ is the systematic component of surplus.

2.1.2. Special Cases. Our proposed WTP function is general and subsumes standard WTP functions commonly used in the quantity discount literature as special cases. Two such functions are the power WTP function (e.g., Shugan 1985) and the quadratic WTP function (e.g., Lambrecht et al. 2007).

The power WTP function is defined as

$$WTP_{ij}(q_j) = w_{ij1}q_j^{\beta_i}, \quad q_j \ge 1,$$
(4)

where $w_{ij1} > 0$ is the WTP for the first unit and $0 < \beta_i \le 1$ is a parameter that enforces the diminishing marginal willingness to pay. Translated in terms of Equation (2), the power function implies the following pattern of decay parameters:

$$\lambda_{iq} = \frac{(q)^{\beta_i} - (q-1)^{\beta_i}}{(q-1)^{\beta_i} - (q-2)^{\beta_i}}, \quad q > 1.$$
(5)

Note that $\lambda_{iq} \ge 0$ increases with *q*. For example, if $\beta_i = 0.6$, then $\lambda_{i2} = 0.52$, $\lambda_{i3} = 0.87$, $\lambda_{i4} = 0.90$, This means that the marginal WTP decays at a slower rate with increasing quantity.

The quadratic WTP function is defined as

$$\text{WTP}_{ij}(q_j) = \begin{cases} \beta_{i0} + \beta_{i1}q_j - 0.5\beta_{i2}q_j^2, & \text{if } q_j \le \frac{\beta_{i1}}{\beta_{i2}}, \\ \beta_{i0} + \frac{(\beta_{i1})^2}{2\beta_{i2}} & \text{if } q_j > \frac{\beta_{i1}}{\beta_{i2}}, \end{cases}$$
(6)

where β_{i0} is an intercept term and $\beta_{i1} \ge 0$ and $\beta_{i2} \ge 0$ are parameters whose ratio represents a threshold beyond which marginal WTP is 0. The quadratic WTP function implies the following pattern of decay parameters:

$$\lambda_{iq} = \begin{cases} \frac{\beta_{i1} - 0.5\beta_{i2}((q)^2 - (q-1)^2)}{\beta_{i0}I_{q=2} + \beta_{i1} - 0.5\beta_{i2}((q-1)^2 - (q-2)^2)} \\ \text{if } 2 \le q \le \frac{\beta_{i1}}{\beta_{i2}}, \\ 0 \quad \text{if } q > \frac{\beta_{i1}}{\beta_{i2}}, \end{cases}$$
(7)

where $I_{q=2}$ is an indicator variable that takes a value of 1 if q = 2 and 0 otherwise. Note that when $\beta_{i0} = 0$, the decay parameter λ_{iq} is a decreasing function of q. For example, if $\beta_{i0} = 0$, $\beta_{i1} = 3.0$, and $\beta_{i2} = 0.5$, then $\lambda_{i2} = 0.82$, $\lambda_{i3} = 0.77$, $\lambda_{i4} = 0.71$, This means that the marginal WTP decays at a faster rate with increasing quantity. When $\beta_{i0} > 0$, the decay parameter λ_{iq} has a lower value at q = 2 but then decreases afterward.

A priori, we do not know whether the decay rate is constant, declining, or increasing over successive quantities. Past research has indicated that the rate at which consumers satiate differs across products, people, and contexts (e.g., Redden 2008). For instance, research on eating behavior has found that people satiate at a lower rate with food when they have wine or beer as aperitifs compared with when they have water or fruit juice (Westerterp-Plantenga and Verwegen 1999). Thus the quadratic WTP function may be a better model for consumers in the water or fruit juice condition, whereas the power WTP function may be a better model for those in the wine or beer condition. Restricting the analysis to either the power WTP function or the quadratic WTP function may therefore result in an erroneous conclusion about consumer willingness to pay.³ The WTP function in Equation (2) is sufficiently flexible to capture any pattern of diminishing marginal willingness to pay.

To summarize, the power WTP function implies an increasing λ pattern over successive quantities, whereas the quadratic function implies a decreasing pattern. Therefore using either of these functions implicitly imposes a certain pattern of decay on the data. Thus, our WTP specification can be useful in empirical applications where the pattern of decay is unknown a priori and/or where the researcher is interested in testing certain hypotheses about decay pattern.

³ Simulation results show that the quadratic WTP model does poorly in fitting data generated from a power WTP model, and vice versa.

2.1.3. Properties of the WTP Function. The proposed WTP function in Equation (2) has desirable properties. It admits nonnegative values and is an increasing function of quantity but at a decreasing rate (Wilson 1993). In addition, the WTP for *q* units is a multiple of the WTP for the first unit, w_{ii1} . For example, in the special case $\lambda_{im} = \lambda_i$ for all m > 1, the WTP for *q* units is given by $w_{ii1} \times (1 + \lambda_i + \lambda_i^2 + \dots + \lambda_i^{q-1})$. One benefit of this property is that one can compute a WTP multiple for an infinite quantity *q*. For the case $\lambda_{im} = \lambda_i$, this multiple is $\lambda_i/(1-\lambda_i) = (1+\lambda_i+\lambda_i^2+$ $\lambda_i^3 + \cdots$). To illustrate, for a consumer with $\lambda_i = 0.5$, this WTP multiple is equal to 2. That is, this consumer is willing to pay a maximum of twice his or her WTP for the first unit for an offer with an infinite number of units. Thus one could segment consumers based on their WTP for the first unit, w_{ii1} , and their WTP multiple. The first component captures price premium, and the second captures volume potential. In addition, one could score consumers based on their value potential, which is the product of the WTP for the first unit and the WTP multiple.

2.2. The Impact of Product Attributes on WTP

We now discuss how the product attributes (e.g., brand name, product features) impact consumers' willingness to pay for the first unit and the decay parameters.

To capture the impact of product attributes on the WTP of the first unit, we reparametrize w_{ij1} as follows:

$$w_{ij1} = \exp\left(\sum_{l=1}^{L} \alpha_{il} x_{jl}\right)$$
 for $i = 1, ..., I$,
 $j = 1, ..., J$, and $l = 1, ..., L$, (8)

where x_{jl} is the value of product *j* on attribute *l*, and α_{il} measures the impact of x_{jl} on w_{ij1} (part-worth). The use of the exponential function ensures the positivity of WTP for the first unit.

In Equation (2), we specify different decay parameters for different quantities. This nonparametric specification works well for products sold in small quantities but is infeasible for products sold in large quantities. Moreover, we assume a common decay parameter for all product variants (j = 1, ..., J). Although this assumption may be acceptable for some products (e.g., online DVD plans), it may not be reasonable for others where the decay rate can differ across product variants (e.g., light versus dark beers). To accommodate these issues and ensure that the decay parameters fall in the (0, 1] interval, we reparametrize them as a logistic function of both quantity and product attributes. That is,

$$\lambda_{iq}^{j} = 1 / \left(1 + \exp\left(\theta_{i0} + \theta_{i1}q + \theta_{i2}q^{2} + \sum_{l=1}^{L} \gamma_{il}x_{il}\right) \right)$$

for all $i, q > 1$, (9)

where θ_{i0} is an intercept; and θ_{i1} , θ_{i2} , and γ_{il} (l = $1, \ldots, L$) capture the impact of quantity and product features and variants, respectively, on the alternativespecific decay parameter. The specification allows different product variants (e.g., brands) to have different decay parameters. In addition, it allows for different decay patterns of WTP over quantities. For example, if both θ_{i1} and θ_{i2} are 0, then the decay rate is constant across quantities (i.e., $\lambda_i^j = 1/(1 + 1)$ $\exp(\theta_{i0} + \sum_{l=1}^{L} \gamma_{il} x_{il}))$. However, if θ_{i1} is positive (negative) and θ_{i2} is 0, then the decay coefficient becomes smaller (larger) with increasing quantity (i.e., λ_{iq}^{\prime} = $1/(1 + \exp(\theta_{i0} + \theta_{i1}q + \sum_{l=1}^{L} \gamma_{il}x_{jl})))$. Such a specification captures the quadratic (power) WTP model. In the empirical application, we test for these different nested versions and a more general nonparametric specification as in Equation (2).

2.3. Model Estimation

Consider a sample of *I* consumers, each choosing at most one product alternative from a set of *J* alternatives. Let *t* indicate a choice task. If consumer *i* contributes T_i such observations, then the total number of observations in the data is given by $T = \sum_{i=1}^{I}$. Let $z_{ijt} = 1$ if the choice of alternative *j* is recorded for choice task *t*; otherwise, $z_{ijt} = 0$. Let j = 0 denote the index for the no-choice alternative. Thus, $z_{i0t} = 1$ if the consumer chooses none of the alternatives.

We assume that consumers are surplus maximizers.⁴ On choice task *t*, let $S_{ijt} = S_{ijt}(q_j, p(q_j)) = s_{ijt}(q_j, p(q_j)) + \varepsilon_{ijt}$ and $S_{i0t} = \varepsilon_{i0t}$ denote the surplus from alternative *j* and the no-choice option, respectively. Thus, a consumer would choose alternative *j* in choice task *t* if it has the maximum surplus $\{S_{ijt} > S_{ikt}, k = 0, ..., J, k \neq j\}$ and would choose none of the alternatives if the no-choice option (j = 0) has the maximum surplus $\{S_{i0t} > S_{ijt}, j = 1, ..., J\}$.

We assume that ε_{ijt} follows an independent and identically distributed extreme value distribution with scale parameter $\mu_i > 0$ (see Ben-Akiva and Lerman 1985, pp. 104–105).⁵ The scale parameter μ_i is necessary because the price coefficient is normalized to 1 in the surplus Equation (3). Therefore, consumer *i*'s choice probability for product *j* on choice

⁴ Our consumer surplus maximization problem can lead to an interior solution. That is, it is optimal for a consumer to spend a fraction of her budget on purchasing q units of product j (inside good) and the remaining budget on other goods (outside good). This is because the surplus function in Equation (3) is nonlinear in quantity, and the budget set implied by the quantity discount p(q) is convex. The proof is available from the authors upon request.

⁵ We assume that the errors are independent because of the cyclical design approach that we use for constructing the choice sets (see §3.1). This is consistent with past work in choice-based conjoint (e.g., Iyengar et al. 2008).

occasion *t*, Pr_{ijt} , and no-choice probability, Pr_{i0t} , are given by

$$\Pr_{ijt} = \frac{\exp(\mu_i s_{ijt}(q_j, p(q_j)))}{1 + \sum_{k=1}^{J} \exp(\mu_i s_{ikt}(q_k, p(q_k)))} \quad \text{and} \\ \Pr_{i0t} = \frac{1}{1 + \sum_{k=1}^{J} \exp(\mu_i s_{ikt}(q_k, p(q_k)))}.$$
(10)

As we model consumer surplus, the parameter estimates directly provide the *indifference* reservation price that makes a consumer indifferent between buying and not buying a certain quantity (i.e., a 50% chance of buying). We can also use the parameter estimates to calculate reservation prices that correspond to other levels of probability of purchase. For instance, we can compute a *floor* reservation price at or below which a consumer would buy q units of product jwith almost certainty (e.g., a 95% chance of buying). We compute this price by setting the no-choice probability (Pr_{i0t}) to 5% and solving for p(q). Similarly, we can compute a *ceiling* reservation price that would make a consumer almost certainly not buy the product (e.g., a 5% chance of buying). Thus, we can compute a WTP range for each consumer and quantity level.6

For an individual *i*, let $\alpha_i = (\alpha_{i1}, \ldots, \alpha_{iL})'$, $\gamma_i = (\gamma_{i1}, \ldots, \gamma_{iL})'$, $\theta_i = (\theta_{i0}, \theta_{i1}, \theta_{i2})'$, and $\psi_i = (\alpha_i, \gamma_i, \theta_i, \mu_i)$ be the joint vector of parameters. We use the choice data to estimate the vector of parameters, ψ_i , for each individual. Because it is not possible to obtain sufficient choice data to estimate separate models for each individual, we use a Bayesian multilevel structure (Gelman and Hill 2007) that specifies how the individual-level parameters vary in the population and thereby statistically pool information across individuals. We assume that

$$\psi_i \sim \mathcal{N}(\bar{\psi}, \Sigma), \tag{11}$$

where ψ and Σ are population-level parameters to be estimated.

The model parameters are estimated using a standard Bayesian estimation procedure using Markov chain Monte Carlo (MCMC) methods (see Online Appendix A at http://mktsci.journal.informs.org/). This mixed logit procedure allows one to compute WTP measures as part of the MCMC iteration process and provides confidence intervals for WTP for different quantities and at different levels of aggregation. Hence managers can use such information to design optimal quantity discount schemes or customized pricing strategies for each consumer or consumer segment.

3. An Empirical Application

We illustrate the model using data from a choicebased conjoint experiment on DVD movie rentals by mail. Subscribers to this service rent movies online, receive them in DVD format by mail, and return them by mail free of charge after watching. The sample consists of 250 consumers. The online DVD rental category was chosen for several reasons. DVD rental is a product category that most consumers are familiar with. In addition, consumers are familiar with the various DVD rental plans offered by the two major competitors (Netflix and Blockbuster).

3.1. Design of Conjoint Experiment

We used four attributes to create online movie rental plans (conjoint profiles): (1) service provider, (2) number of movies out at a time offered under the plan, (3) monthly price of the plan, and (4) Blu-ray movie availability (yes or no). These are the same attributes that online movie rental companies (e.g., Netflix) use to describe their plans at the time of the study.

The service provider attribute has three levels: a hypothetical new service with the generic name MovieMail and the two leading brand names in the category (Netflix and Blockbuster Online). These two leading brands jointly accounted for 77.42% market share of the online DVD rental market in 2008.⁷ We included a hypothetical new service to examine the impact of brand name on the WTP curve. This new service was described to respondents as follows:

MovieMail.com is a new online movie rental service about to enter the market. Like Netflix and Blockbuster Online, MovieMail operates by mail and promises to have the same movie selection, search capabilities, and mail delivery time.

Note that the attribute-level details of MovieMail (e.g., price) were not included in the description; however, they were included as treatment variables in the conjoint experiment.

The number of movies out at a time, q, has three levels: low (one or two DVDs out at a time), medium (three or four DVDs out at a time), and high (five or six DVDs out at a time). Note that though each level has two values, respondents will see only one of these values in a particular DVD plan. For example, if the number of movies at a time is "low" in a

⁶ Note that our model-based WTP range is distinct from the ICERANGE proposed by Wang et al. (2007), as the latter arises from consumer-level uncertainty in WTP.

⁷ The U.S. DVD and video sales and rental market was valued at \$7.6 billion in 2008; brick-and-mortar stores claimed 69% of the revenue share. Mail-order companies such as Blockbuster and Net-flix together commanded 24% of the market, whereas kiosks had a mere 6% share and online streaming or download options an even smaller 1% (see Mitra 2009). Thus Netflix and Blockbuster Online commanded a 77.42% (= 24/31) share of the online DVD rental market.

particular conjoint profile, then we assign the respondent a value of either one or two DVDs out at a time randomly.

The monthly price attribute p(q) has two components: the base price level of the plan and the depth of quantity discount. The base price for a plan *p* is based on the price of the one-DVD-out-ata-time plan and has three levels: low (\$5.99 or \$6.99), medium (\$7.99 or \$8.99), and high (\$9.99 or \$10.99). The monthly price for a plan with no quantity discount (i.e., uniform pricing) is p(q) = p * q. One way to capture quantity discounts is through $p(q) = p * q^{b}$, where b < 1 (decreasing block) measures the percent increase in total monthly price when quantity increases by 1%. We specify three levels for the depth of quantity discount: low (b = 0.88 or 0.84), medium (b = 0.80 or 0.76), and high (b = 0.71 or 0.67). Similar to the attribute number of movies out at a time, only one base price value and one quantity discount rate appear in a particular conjoint profile. When q = 6, the *b* values correspond to the following quantity discount rates: low (20% or 25% discount), medium (30% or 35%), and high (40% or 45%). Suppose that q = 3, p =\$9.99, and b = 0.88 for a particular conjoint profile. Then the monthly rate for such a plan is $p(q) = \$9.99 * 3^{0.88} = \$26.14.$

Our experimental design has two novelties. First, because price and quantity can take large sets of values, we adopt a randomized-block-design-type approach where we initially establish low, medium, and high intervals (the blocks) for each of the factors and then randomly assign specific values from the intervals for each respondent. This approach ensures that each quantity, price, and discount value is tested in the experiment. Second, unlike traditional conjoint, we do not specify different prices for different quantities. Instead, we decompose the price variable into two components: the price of the first unit and the depth of discount. This will result in a more parsimonious experimental design. In the context of our study where we have three brand levels, two levels for Blu-ray availability, and six quantity levels, a full-blown traditional conjoint design would necessitate six price/discount factors (one for each quantity level). Assuming three levels for each price factor, this results in a $3 \times 2 \times 6 \times 3^6$ design (26,244 profiles in the full factorial), whereas our design is only $3 \times 2 \times$ $3 \times 3 \times 3$ (135 possible profiles). Even if one reduces the quantity levels to three, the traditional design still results in $486 = 3 \times 2 \times 3 \times 3^3$ full factorial profiles. However, this design does not allow the testing of every quantity and price value.

We used a cyclic design approach for constructing choice sets (see Huber and Zwerina 1996). We first generated six orthogonal designs of 18 profiles each from the $3^4 \times 2$ full factorial using Proc Optex in SAS.

For each orthogonal plan, we then used the cyclic design procedure to generate 18 choice sets with three online movie rental plans each.

In the literature, the D_0 -error is the most widely used measure of efficiency of a conjoint design (e.g., Huber and Zwerina 1996). The lower the D_0 error, the higher the efficiency of the design and therefore the greater the asymptotic efficiency of the parameter estimates. For our conjoint design, the D_0 error is 0.09, which indicates a high level of efficiency.

Each participant in the study was randomly assigned to one of the six choice designs. After the conjoint task was explained, each participant was presented a sequence of 18 choice sets of movie rental plans in show-card format. The participant's task was to choose at most one of the three alternatives (i.e., nochoice is possible) from each choice set shown. See Figure 1 for an example of a choice set that we used in the conjoint experiment. We controlled for order effects by randomizing the order of profiles across subjects. We randomly selected 15 of the 18 choice sets for model estimation and the remaining three for holdout prediction.

3.2. Descriptive Results

As part of the conjoint survey, we also collected information about respondents' demographics (e.g., income), their current movie rental provider, the type of plan they subscribe to, and how many DVDs they actually receive by mail in a month. Of the 250 respondents, 73.2% (25.2%) have Netflix (Blockbuster) as their current provider. The remaining 1.6% subscribe to other online movie rental companies. Overall, 29% of these respondents have a plan for one DVD out at a time, 25% have a plan for two DVDs out at a time, 38% have a plan for three DVDs out at a time, 4% have a plan for four DVDs out at a time, and the remaining 4% have plans for five or more DVDs out at a time. This percentage breakdown compares very well with that reported in FeedFliks⁸ and suggests that our sample is representative of online DVD rental users. Finally, we find that respondents on a plan with one DVD out at a time receive an average of 4.9 DVDs per month from their service provider. Those with plans for two, three, and four DVDs out at a time receive an average of 7.7, 10.4, and 13.8 DVDs per month, respectively. Thus consumers with plans for fewer DVDs are costlier to serve (per DVD) than those with plans for more DVDs.

3.3. Model Specifications

We used the data from the conjoint experiment to estimate four nested models. The models were selected

⁸ FeedFliks.com collects self-stated information from their registered users on various plan features such as number of DVDs out at a time, the average rental period, and typical queue sizes.

Figure 1 An Example of a Choice Set

If you were shopping for an online movie rental service today and these three plans were your only options, which would you choose? *Choose by clicking one of the buttons below.*

Attribute	Plan 1	Plan 2	Plan 3
Service Provider	MovieMail	Netflix	Blockbuster
Movies At-a-			
Time	2	3	6
Blu-Ray Movies	Yes	No	Yes
Monthly Plan			
Cost	\$15.99	\$17.99	\$33.99
NONE: I wouldn't			
choose any of these	Plan 1	Plan 2	Plan 3
0	0	0	0

to investigate various patterns in the decay of the marginal WTP for successive quantities. In all models, we initially specify decay parameters that vary over quantities but not product features (e.g., brand). That is, $\lambda_{iq}^j = \lambda_{iq}$ ($0 < \lambda_{iq} \le 1$) $\forall j = 1, ..., J$. Later, we generalize the models to allow the decay parameters to vary by product features as well. Let MM_j , NF_j , and BB_j be 0/1 dummy variables indicating whether MovieMail, Netflix, or Blockbuster, respectively, is the service provider of plan *j*. Let BR_j indicate whether Blu-ray movies are offered in plan *j*. Then the general model is specified as

$$s_{ij}(q_j, p(q_j)) = w_{ij1} \sum_{k=1}^{q_j} \prod_{m=1}^k \lambda_{im} - p(q_j), \qquad (12)$$

$$w_{ij1} = \exp(\alpha_{i0} + \alpha_{i1}NF_j + \alpha_{i2}BB_j + \alpha_{i3}BR_j, \qquad (13)$$

where q_j is the number of DVDs out at a time offered under plan j and $p(q_j)$ is the monthly price for the plan. Note that MovieMail is used as the base service provider in Equation (13). Thus the brand coefficients should be interpreted relative to MovieMail.

The models vary in terms of how we specify the decay parameters. The most general model is nonparametric, with decay parameters represented by five separate coefficients. That is,

$$\lambda_{im} = 1/(1 + \exp(\gamma_{im}))$$
 for $m = 2, ..., 6$, (14)

where γ_{im} (m = 2, ..., 6) are individual-specific parameters. Note that the logistic function ensures that the decay coefficients fall in the (0, 1] interval. We refer to this model, defined by Equations (12)– (14), as the *nonparametric decay* model. Because of its nonparametric form, this decay function is flexible. One drawback, however, is that the specification is not parsimonious, especially in cases where the quantity variable takes a large set of values. In such cases, it is difficult to estimate a model with a decay coefficient for each quantity unit. The next model is a nested parametric form where the decay coefficients are reparametrized as a quadratic function of quantity. That is,

$$\lambda_{im} = 1/(1 + \exp(\theta_{i0} + \theta_{i1}m + \theta_{i2}m^2)),$$

$$m = 2, \dots, 6, \qquad (15)$$

where the parameters θ_{i0} , θ_{i1} , and θ_{i2} capture how the decay coefficients vary with quantity. For comparison purposes, we refer to the model in Equations (12), (13), and (15) as the *quadratic decay* model. A comparison of fit of this model relative to the nonparametric model provides evidence for the suitability of the parametric form of the decay function.

To test other patterns in the decay of the marginal WTP of successive quantities, we estimate two other nested versions. The first model sets both θ_{i1} and θ_{i2} to 0. In this model, for a consumer *i*, the decay coefficient is constant across quantities; i.e., $\lambda_{im} =$ $\lambda_i = 1/(1 + \exp(\theta_{i0}))$ for m > 1. Thus, the marginal WTP for the first, second, third, ..., *q*th unit is, respectively, w_{ij1} , $\lambda_i w_{ij1}$, $\lambda_i^2 w_{ij1}$, ..., $\lambda_i^{q-1} w_{ij1}$. We call this model the constant decay model. The second model sets θ_{i2} to 0. In this model, the decay coefficient varies with increasing quantity. If θ_{i1} is positive (negative), the decay coefficient becomes smaller (larger) with increasing quantity, and hence the marginal WTP decays at a faster (slower) rate. We refer to this specification as the linear decay model. This model represents a quadratic (power) WTP function if θ_{i1} is positive (negative). Note that the words "linear" and "quadratic" refer to the linear and quadratic terms, respectively, in the exponential function in Equation (15). They do not connote that marginal WTP decays linearly or quadratically.

4. Results

We used MCMC methods for estimating the models (see Online Appendix A). For each model, we ran sampling chains for 50,000 iterations. Convergence was assessed by monitoring the time series of the draws and by assessing the Gelman-Rubin statistics (Gelman and Rubin 1992). In all cases, the Gelman-Rubin statistic was less than 1.1, suggesting that convergence was satisfactory.⁹ We report the results based on 30,000 draws retained after discarding the initial 20,000 draws as burn-in iterations.

4.1. Model Comparisons

We use log Bayes factor (log BF) to compare the models. This measure accounts for model fit and automatically penalizes model complexity (Kass and Raftery 1995). In our context, log BF is the difference between the log-marginal likelihood of the nonparametric model (LML_{M1}) and that of a nested model (LML_{M2}). We use the MCMC draws to obtain an estimate of the log-marginal likelihood for each of the models. Table 1 reports the results for all four models.

Kass and Raftery (1995) suggest that a value of log $BF = LML_{M2} - LML_{M1}$ greater than 5 provides strong evidence for the superiority of a model. Hence the LML results in Table 1 provide strong evidence for the superiority of the linear decay model relative to all other models.

The constant decay model, where $\lambda_{im} = \lambda_i$ for all *i* and *m*, performed relatively poorly (lowest LML). This result suggests that the rate of decay in marginal WTP, λ_i , is not constant over successive quantities. The superiority of the quadratic decay model over the nonparametric decay model suggests that there is no need to estimate a decay coefficient for each quantity unit. Similarly, the superiority of the linear decay model over the quadratic decay model suggests that a linear specification is sufficient for capturing the pattern of decay in marginal WTP. Thus this parametric specification is not only parsimonious but does very well in capturing the shape of the WTP function.

4.2. Predictive Validity

To assess predictive validity, we calculate the holdout hit rate and validation log-likelihood (VLL) for each model. This latter statistic has been used in the Bayesian literature for assessing predictive validity (e.g., Montoya et al. 2010). The estimated parameters for each model were used to test that model's predictive validity for holdout samples. Recall that the calibration data for each respondent included 15 choice sets, and the holdout sample included three choice sets. The results in Table 1 indicate that the selected

Table 1 Model Performance Comparison

Model specification	LML	Log BF	Holdout hit rate	Holdout LL	Actual plan hit rate
Nonparametric decay	-2,144.45	—	70.2	-514.14	56.1
Quadratic decay	-2,127.95	16.50	71.2	-510.30	57.4
Linear decay ^a	- 2 ,099.33	45.12	72.8	-485.58	60.1
Constant decay	-2,210.92	-66.47	69.7	-529.46	48.3

Notes. LML denotes log-marginal likelihood. Holdout LL denotes holdout loglikelihood.

^aSelected model. The LML is highlighted in boldface.

linear decay model has the highest holdout hit rate and VLL. The smaller differences in predictive validity among the linear, quadratic, and nonparametric decay models are expected because the first model is a special case of the latter two models. However, the improvement in predictive validity for the linear decay model over the constant decay model is more noticeable when measured by VLL versus the holdout hit rate. This is expected because VLL is a more sensitive measure, which explains its use in practice.

As a further validation, we use the individual-level parameters and market prices for the available online DVD plans to predict the respondents' actual plans. The two major players in the market are Blockbuster and Netflix. In our sample, both companies account for 98.4% of the market. Blockbuster offers three plans (one to three DVDs out at a time), whereas Netflix offers four (one to four DVDs out at a time). Thus there are seven choices available to consumers. Using the MCMC draws, we predicted the choice probability of each subject for each of these plans given the monthly fee and brand name. Consistent with the holdout task, we find that the linear decay model predicts real behavior well: a 60% hit rate compared to a 14% chance criterion and 35% maximum chance criterion. This performance fares well with the more general models and is superior to the constant decay model, which results in a 48% hit rate. See Table 1.¹⁰

4.3. Robustness Checks

We conducted two robustness checks. The first checks for the robustness of the quasi-linear utility assumption underlying our model. The second tests whether the decay parameters vary over product variants.

4.3.1. Robustness of the Quasi-Linear Utility Assumption. We checked for the robustness of the quasi-linearity assumption by comparing our empirical results to those obtained using WTP functions that

⁹ The Gelman-Rubin statistic relies on running multiple MCMC chains to test whether they all converge to the same posterior distribution. It takes a value of 1 when the within-chain variance of the parameters is equal to the between-chain variance. Large (much greater than 1) Gelman-Rubin statistics indicate that the between-chain variance is substantially greater than the within-chain variance so that a longer simulation is needed.

¹⁰ This validation exercise is a test of consistency rather than a test of predictive validity. A more stringent test entails a delayed holdout task involving real behavior and using a data collection format that is different from the one used in the calibration task. We thank an anonymous reviewer for raising these points.

are derived from non-quasi-linear utility functions. Specifically, we use two non-quasi-linear utility specifications. The first specifies the utility from the outside good in a logarithmic form (Sudhir 2001), whereas the second uses a power function. The estimation results show no incremental gain from relaxing the quasilinearity assumption. In fact, both models have worse log-marginal likelihood and validation log-likelihood values than our proposed quasi-linear model, which indicates overparametrization. Thus, the results of both analyses suggest that our assumption of quasilinearity is robust. The details of these analyses can be obtained from the authors upon request.

4.3.2. Assessing the Impact of Plan Features on the Decay Parameters. To test whether the decay parameters vary across product variants, we reestimated the models with decay parameters varying in terms of *both* quantity and plan features (i.e., MovieMail, Netflix, Blockbuster, and Blu-ray). See Equation (9). In all four models, we find that none of the plan features significantly impacts the decay parameters. In addition, all the LMLs are worse than the corresponding values reported in Table 1. For instance, the linear decay model with plan features in the decay coefficients has LML and VLL equal to -2,109.3 and -493.87, respectively. Both quantities are significantly worse than those of the linear decay model (see Table 1). Thus, in this application, the decay coefficients do not appear to vary across brands or to be affected by whether the plan has Blu-ray availability or not.

4.4. Parameter Values

We now discuss the parameter estimates from the models. Table 2 summarizes the posterior distributions of the parameters by reporting their posterior means and 95% posterior intervals. The middle panels report the estimates for the three parametric decay models, and the rightmost panel reports those for the nonparametric decay model.

4.4.1. Scale Parameter. All models result in scale parameter estimates that are statistically indistinguishable (i.e., their 95% posterior intervals overlap).

4.4.2. WTP for First Unit. All the models produce parameter estimates that are similar in magnitude. Netflix (the market leader) has the highest mean part-worth value. The mean part-worth value for Blockbuster is not significantly different from that of the unbranded online movie rental service MovieMail, which we use as the base brand. The mean part-worth value for Blu-ray is positive and significant (zero value is outside the 95% posterior interval). Translated in WTP values, for the selected linear decay model, consumers are willing to pay an average of \$12.47, \$11.35, and \$11.37 (\$11.78, \$10.73,

and \$10.74) for a one-DVD-out-at-a-time plan with (without) Blu-ray that is offered, respectively, by Netflix, Blockbuster, and MovieMail.¹¹ Thus, on average, consumers are willing to pay an additional \$1.10 for Netflix compared with MovieMail or Blockbuster and about \$0.65 to have movies in Blu-ray format. Currently, Netflix charges an additional \$2 for the Bluray option in its one-DVD-out-at-a-time plan, whereas Blockbuster charges no additional fees. The free Bluray option may indicate a strategic move by Blockbuster to compensate for its weaker brand equity. See Reisinger (2009).

4.4.3. Decay Parameters. In the constant decay model, the decay parameter is not significantly different from zero. As $\lambda_{im} = \lambda_i = 1/(1 + \exp(\theta_{i0}))$ for m > 1, this means that the average decay rate in the sample is about $\lambda = 0.5$. Thus the marginal WTP for the second unit is half of the WTP of the first unit, the marginal WTP for the third unit is one-fourth of the first unit, and so on. One could use this decay pattern to estimate a WTP multiple by calculating the sum of the geometric series $1 + \lambda + \lambda^2 + \lambda^3 + \cdots = 1/(1 - \lambda)$. Thus the constant decay model implies a WTP multiple of 2 (= 1/(1 - 0.5)). That is, on average, consumers are willing to pay a maximum of twice their WTP for the first unit for a plan that offers an "infinite" number of movies out at a time.

The linear decay model shows that the decay rate increases rapidly with larger quantity. On average, we find that the marginal WTP for the second unit is 79% of the WTP of the first unit (i.e., the average of $\lambda_{i2} = 1/(1 + \exp(\theta_{i0} + 2\theta_{i1}))$ in the sample), and the marginal WTP for the third unit is $\lambda_2\lambda_3 = 38\%$ of the WTP of the first unit. For the fourth, fifth, and sixth units, the marginal WTP is, respectively, $\lambda_2\lambda_3\lambda_4 = 7\%$, $\lambda_2\lambda_3\lambda_4\lambda_5 = 1\%$, and $\lambda_2\lambda_3\lambda_4\lambda_5\lambda_6 = 0.1\%$ of the WTP of the first unit. Summed over an infinite quantity, the linear decay model results in a WTP multiple of 2.25 (= $1 + \lambda_2 + \lambda_2\lambda_3 + \lambda_2\lambda_3\lambda_4 + \cdots$). That is, on average, consumers are willing to pay a maximum of 2.25 times the WTP for the first unit for a plan offering an "infinite" number of DVDs out at a time.

The quadratic (nonparametric) decay model resulted in decay rates similar to those from the linear decay model. For the quadratic (nonparametric) decay model, the marginal WTP of the second, third, fourth, fifth, and sixth units is, respectively, 0.75, 0.37, 0.12, 0.04, and 0.01 (0.76, 0.31, 0.15, 0.10, and 0.07) of the WTP of the first unit. For the quadratic (nonparametric) model, the average WTP multiple is estimated to be 2.30 (2.40). Figure 2 depicts the

¹¹ The corresponding 95% posterior intervals for Netflix, Blockbuster, and MovieMail with Blu-ray are, respectively, (12.44, 12.49), (11.32, 11.37), and (11.34, 11.39). Without Blu-ray, they are, respectively, (11.75, 11.81), (10.69, 10.75), and (10.71, 10.76).

Parameter	Parameter label	Constant decay	Linear decay	Quadratic decay	Parameter label	Nonparametric decay
WTP first-unit parameters						
Intercept	α ₀	2.51 (2.39, 2.62)	2.36 (2.27, 2.45)	2.41 (2.31, 2.52)	α ₀	2.37 (2.28, 2.43)
Netflix	α ₁	0.08 (0.05, 0.12)	0.09 (0.06, 0.13)	0.09 (0.06, 0.12)	α ₁	0.09 (0.06, 0.12)
Blockbuster	α2	-0.02 (-0.06, 0.02)	-0.01 (-0.04, 0.03)	0.00 (-0.03, 0.03)	α2	0.00 (-0.04, 0.03)
Blu-ray	α_3	0.05 (0.02, 0.08)	0.06 (0.03, 0.09)	0.06 (0.03, 0.09)	α_3	0.06 (0.03, 0.09)
Decay parameters						
Intercept	θ_0	-0.01 (-0.24, 0.25)	- 4.24 (-5.68, -2.61)	- 4.99 (-8.31, -1.68)	γ_2	- 1.19 (-1.59, -0.73)
q	θ_1		1.68 (1.33, 1.90)	2.45 (0.19, 4.78)	γ_3	0.37 (-0.19, 0.92)
q^2	θ_2			-0.26 (-0.65, 0.11)	γ_4	-0.25 (-2.19, 0.66)
				. ,	γ_5	-0.79 (-4.59, 0.84)
					γ_6	-4.16 (-4.59, -1.07)
Scale parameter	μ	0.42 (0.33, 0.51)	0.48 (0.39, 0.58)	0.47 (0.39, 0.57)	μ	0.45 (0.39, 0.52)

Table 2 Parameter Estimates: Posterior Means and 95% Posterior Intervals

Notes. Coefficients for which 0 lies outside the 95% interval are highlighted in boldface. The 95% posterior confidence intervals for parameters are shown in parentheses.

decay rates for all four estimated models. As the figure shows, all the models except the constant decay model have decay functions that are similar. The constant decay model appears to understate the decay rate for the first few units and overstate it for the larger units.

4.4.4. WTP Range. Recall that we can use the parameter estimates to calculate the floor (ceiling) reservation price below (above) which a consumer would almost certainly buy (not buy) a plan with



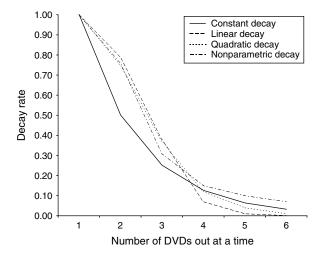


Table 3	Floor, Indifference, and Ceiling Reservation Prices for
	Netflix Plans Without Blu-ray

		-	
Number of DVDs out at a time	Floor price (\$)	Indifference price (\$)	Ceiling price (\$)
1	5.59	11.78	17.96
2	14.94	21.12	27.31
3	19.51	25.69	31.88
4	20.41	26.59	32.77
5	20.48	26.67	32.84
6	20.51	26.73	32.86

q DVDs out at a time. To illustrate, Table 3 reports the average floor and ceiling reservation prices for Net-flix plans without Blu-ray that we obtained using the selected linear decay model parameter estimates. For completeness, the table also reports the average indifference reservation prices, or WTP.

In summary, the empirical results show that the linear decay model has the best statistical fit and predictive validity. These results suggest that the marginal WTP in the online movie rental service category decays rapidly with quantity.¹²

¹² To test whether there are any systematic differences between Netflix and Blockbuster customers, we estimated the models only on respondents who are current Netflix subscribers. The estimation results show that the parameters of the full sample and those of the Netflix sample are statistically indistinguishable.

5. Demand Analysis

We now use the individual-level parameter estimates to examine the extent of consumer heterogeneity in WTP and characterize consumer demand for online movie rental services.

5.1. Consumer Heterogeneity

To explore the extent of heterogeneity in the WTP for the first unit and the WTP multiple in the sample, we used the MCMC draws of the selected linear decay model parameters to compute the posterior mean values of these statistics for each consumer in the sample. To illustrate, Figure 3 depicts the consumer-level estimates for Netflix. Across consumers, the average WTP for the first DVD without Blu-ray is \$11.78, and the 95% heterogeneity interval is (\$3.72, \$23.67); the WTP multiple has a posterior mean of 2.25 and 95% heterogeneity interval of (1.0, 5.77).

We used *K*-means clustering to segment consumers in our sample based on their mean WTP for first unit and WTP multiple.¹³ We identified four segments of consumers shown in Figure 3 based on a scree plot of the percentage of variance explained by the clusters. To profile these segments, we use self-stated behavioral data that we collected in our survey. Table 4 reports the descriptive statistics of the four segments.

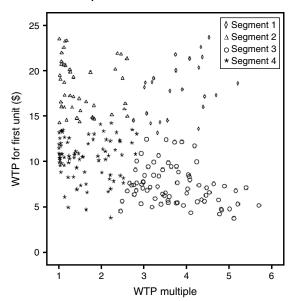
Segment 1 consists of 10.7% of the consumers in the sample who have a high WTP for the first unit (mean = \$17.93) and a high WTP multiple (mean = 3.81). Multiplying each consumer's WTP multiple by his or her WTP for the first unit is a measure of the value potential of the consumer. Thus segment 1 consumers are the most attractive with an average value potential of \$68.99 per consumer. We call this segment the "high value" segment.

Segment 2 consists of light users who have a high WTP for the first unit. The mean WTP for the first unit in this segment is \$18.22, and the mean WTP multiple is 1.54. Of the total number of consumers in the sample, 19.1% belong to this segment. The average value potential per customer in this segment is \$27.81. We label this segment the "high premium" segment.

Segment 3, which represents 33.3% of the consumers, consists of heavy users with a low willingness to pay. The mean WTP multiple for this segment is 3.72, and the mean WTP for the first unit is \$7.31. Thus the average value potential per customer in this segment is similar to Segment 2 and is equal to \$26.71. We name this segment the "high volume" segment.

Segment 4, the least attractive segment, embodies 36.9% of the consumers. The mean WTP for the first

Figure 3 WTP for the First DVD from Netflix Without Blu-ray and WTP Multiple



unit is \$10.40, and the mean WTP multiple is 1.63. Hence the mean value potential for this segment is \$16.93. We call this segment the "low value" segment.

The segmentation results appear to be concordant with respondents' self-stated behavior. Respondents in the high volume segments (1 and 3) currently subscribe to plans with a higher number of DVDs out at a time and appear to watch more movies than respondents in the low volume segments (2 and 4). These results give some face validity for the proposed segmentation scheme.

5.2. Willingness-to-Pay Distribution for Successive Units

To further explore heterogeneity, Figure 4(a) displays the cumulative WTP distribution for each successive DVD of an online movie rental plan without Blu-ray offered by Netflix (i.e., the percentage of consumers whose WTP for the *q*th DVD is greater than a given price) that we obtained from the selected linear decay

Table 4 S	Segments'	Description
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Segment	1 (High value)	2 (High premium)	3 (High volume)	4 (Low value)
Segmentation bases				
WTP of first unit (\$)	17.93 ^a	18.22	7.31	10.40
WTP multiple	3.81	1.54	3.72	1.63
Behavioral descriptors				
Number of DVDs in plan	3.50	2.09	2.94	1.76
Number of movies per week	3.83	2.80	3.65	2.61
Segment value potential (\$)	68.99	27.81	26.71	16.93
Segment size (%)	10.70	19.10	33.30	36.90

^aAverage across consumers in a segment.

¹³ Equivalently, we could segment consumers based on their WTP for plans for 1, 2, ..., 6 DVDs out at a time. We could also segment them based on their WTP range for successive units.

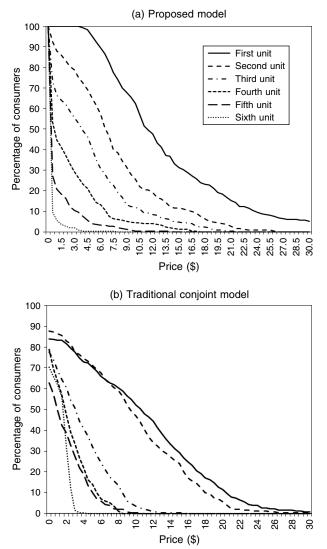


Figure 4 Cumulative WTP Distributions for Successive Units

model. From the figure, we can determine that 68% of consumers have a WTP greater than or equal to \$9.00 for the first DVD from Netflix. Similarly, 65.6% of consumers have a WTP greater than or equal to \$5.00 for the second DVD. Note that, for the first DVD, one could determine the potential demand at any given price. The demand for the second DVD, however, depends jointly on the prices of the first and second DVDs. Similarly, the demand for the *q*th DVD depends on the prices of $1, \ldots, q$ DVDs. Consequently, the information Figure 4(a) should not be construed as demand curves for successive units.

Figure 4(b) shows the corresponding WTP distributions that we obtained using the standard choice-based conjoint model.¹⁴ The figure illustrates

the limitations of using traditional conjoint analysis for measuring WTP over successive units that we discussed in §1.

First, at zero price, not all consumers purchase the online DVD service from Netflix. Thus there are 16% of consumers who would not purchase (i.e., have a negative WTP for) a one-DVD plan when it is offered for free even though 63% of these respondents are current Netflix subscribers. Similarly, 12.4%, 22.4%, 21.2%, 37.2%, and 29.6% would not purchase a second, third, ..., sixth DVD, respectively, if offered for free. This anomalous result occurs because traditional conjoint analysis does not constrain WTP to be positive. This finding is consistent with past research (e.g., Sonnier et al. 2007). In contrast, our proposed model ensures that WTP is always positive, as illustrated in Figure 4(a).

Second, note that the WTP curves in Figure 4(b) intersect each other. For instance, at a price of \$2.50, 79% of the consumers would purchase the first unit and 82% would purchase the second unit. This is anomalous because one would expect the demand for the second unit to be lower than the demand for the first unit. This happens because traditional conjoint analysis does not impose diminishing marginal WTP. Researchers in economics (e.g., Baucells and Sarin 2007, Wilson 1993) have emphasized the need to impose such a restriction. Without it, it is possible to find situations such as the one described above, where consumers may be willing to pay a higher amount for a successive unit than for a previous one. In contrast, our proposed model explicitly accounts for such a restriction.

5.3. Demand Profile

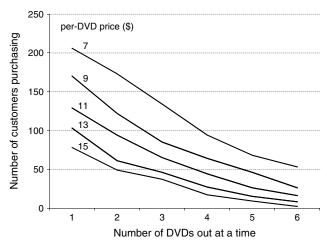
One approach to depict the demand curve for successive units is to use Wilson's (1993, p. 50) demand profile method, which specifies, for each (per-unit) price *p*, the number of consumers purchasing at least *q* units. Figure 5 presents the demand profile for Netflix online movie rental plans without Blu-ray for p = \$7, \$9, \$11, \$13, and \\$15.

For each price *p*, the demand profile represents the distribution of purchase sizes *q* at that price. For example, when the per-unit price is \$9, 122 out of 250 (or 48%) consumers would be willing to buy Netflix plans for two or more DVDs out at a time. Similarly, for each unit *q*, the demand profile reveals the distribution of marginal WTP for that unit. For example, 65 out of 250 (or 26%) consumers have marginal WTP greater than or equal to \$11 for the third (*q* = 3) unit.

Following Wilson (1993, p. 50), we used the demand profile information to compute the price elasticity for each successive unit of demand. For Netflix, an average price elasticity of demand for the first DVD is -1.15. That is, if Netflix increases its price

¹⁴ In this analysis, we use five dummies to indicate the six quantity levels, two dummies for brand name, one dummy for Blu-ray, and price is treated as a continuous variable.





by 1%, its demand for one DVD out at a time would decrease by 1.15%. For the second and third DVDs, we find an average elasticity of -1.42. For the fourth, fifth, and sixth DVDs, the price elasticities are -1.96, -2.25, and -3.24, respectively. For comparison, we also computed the average price elasticities for Blockbuster. For their plans, these elasticities are -1.17 (for one DVD at a time), -1.65 (for two DVDs), -1.79 (for three DVDs), -2.29 (for four DVDs), -2.57 (for five DVDs), and -3.57 (for six DVDs). As expected, consumers have a higher price sensitivity for Blockbuster than they do for Netflix.

In summary, the demand analysis results illustrate the kind of managerial insights that can be derived from our proposed model. We now discuss how to use the estimation results to design optimal quantity discount schemes.

6. Quantity Discount Schedule Design

In this section, we use the demand profile method (Wilson 1993) to design a quantity discount schedule for a monopolist. Online Appendix B discusses the design of a discount schedule for a new entrant in a competitive setting. In both analyses, we assume that the DVD rental service is available to all consumers in the market and enjoys full awareness immediately after launch.

Suppose that MovieMail is a monopolist and is considering offering four online movie rental plans. What quantity discount schedule should it offer? To examine this question, we need to estimate the variable cost that MovieMail would incur while serving customers in different plans. Currently, Netflix incurs a marginal cost of \$1.22 per rented DVD. This cost includes mailing costs, packaging costs, and royalty fees.¹⁵ Recall that our survey results indicate that consumers rent, on average, 4.9, 7.7, 10.4, and 13.8 DVDs per month under plans for 1, 2, 3, and 4 DVDs out at a time, respectively (see §3.2). Thus, assuming that MovieMail has a cost structure similar to Netflix, the plan-specific marginal costs would be $c_1 = 5.98 (= \$1.22 * 4.9), $c_2 = 9.39 , $c_3 = 12.69 , and $c_4 = 16.84 for plans for one, two, three, and four DVDs out at a time, respectively.

MovieMail will choose a price discount scheme that maximizes its gross contribution. To solve this problem, we use the price-point method suggested by Wilson (1993). Table 5 reports the demand profile for MovieMail for q = 1 to 4 DVDs and unit prices varying from \$6 to \$14 per DVD. At unit price p =\$6, for example, 215 consumers would subscribe to a plan for one DVD out at a time or more from MovieMail, and 193 of these consumers would subscribe to at least a two-DVD-out-at-a-time plan. Thus the number who would subscribe to exactly one DVD out-at-a-time plan is 22 (= 215 - 193). Table 5 also reports the marginal cost of each plan.

We use the demand profile and marginal cost information for each unit to determine the optimal price for each successive DVD that maximizes gross contribution. For the first DVD, the profit maximizing price is \$13 with a gross contribution of \$638.82 (= (13 - 5.98) * 91). Thus, under a monopolist scenario, MovieMail would achieve a market penetration of 36.4% (= 91/250). That is, 36.4% of the consumers would subscribe to at least a one-DVD plan if it is priced at \$13. Similarly, for the second DVD, the profit-maximizing price is \$10 with a gross contribution of 672.18 (= (10 - 3.41) * 102). Thus, for a two-DVD plan, the optimal price is \$23 (= \$13 + \$10). Using the information from Table 5, we can estimate how many (among 91) consumers will subscribe to at least the two-DVD plan. The table indicates that there are 77 consumers willing to pay \$22 (= 2×11) and 63 consumers willing to pay $24 = 2 \times 12$ for a two-DVD plan. Interpolating between these two demand predictions, we note that there are about 70 (among 91) consumers willing to pay at least \$23 for a two-DVD plan. Similarly, the optimal price for a three-DVD plan is determined to be \$31. This price appeals to about 58 (among 70) consumers. Finally, the optimal price for a four-DVD plan is \$39 and would attract 40 (among the 58) consumers. Under this discount scheme, 21 consumers (or 8.4%) would subscribe to the one-DVD plan, 12 consumers (or 5%) would subscribe to a two-DVD plan, 18 consumers (or 7%) to a three-DVD plan, and 40 consumers (or 16%)

¹⁵ Netflix reports that it mails about two million DVDs per day (Netflix 2008). There are 313 mailing days (i.e., excluding

Sundays) per year. Therefore Netflix ships a total of 626 million DVDs per year. The total subscription cost, which includes mailing, packaging, and royalty fees, is reported to be \$761,133,000. Therefore the cost per DVD is \$1.22.

Demand for Price per DVD 1st DVD 2nd DVD 3rd DVD 4th DVD \$6 215 193 145 106 \$7 190^a 145 116 82 \$8 173 127 93 63 \$9 155 115 76 47 102 40 \$10 139 58 51 30 \$11 120 77 \$12 106 63 42 26 \$13 91 51 37 17 \$14 78 42 27 12 Marginal unit cost (\$) 5.98 3.41 3.30 4.15^b Optimal gross 638.82 672.18 437.10 242.68 contribution (\$) 10 8 Optimal marginal price (\$) 13 8 Optimal plan price (\$) 13 23 31 39

Table 5 Demand Profile of MovieMail and Optimal Discount Schedule for a Monopolist

Note. Entries in boldface have maximum gross contribution and correspond to optimal choices for the prices.

^aReads as follows: 190 consumers would subscribe to a plan of one DVD or more if the per-DVD price is \$7.

^bThis is the difference between the cost of a four DVDs out-at-a-time plan and three DVDs out-at-a-time plan.

to a four-DVD plan. We obtain a similar discount schedule when we use finer price intervals with \$0.50 increments.

For comparison, suppose MovieMail is entering a market where Netflix and Blockbuster are incumbents. Presently, Netflix (Blockbuster) offers four (three) online movie rental plans. Both firms charge \$8.99, \$13.99, and \$16.99 for the one-, two-, and three-DVD plans, respectively. For the four-DVD plan, Netflix charges \$23.99. Suppose MovieMail decided to offer four online movie rental plans. Then the optimal prices under this competitive scenario are \$8.22, \$12.69, \$16.40, and \$21.82 for the one- to four-DVD plans, respectively (see Online Appendix B). As expected, competitive prices are much lower than the ones under a monopolist setting. For example, for the two-DVD plan, a monopolist charges about \$11.50 per DVD (i.e., \$23 for the plan), whereas the new entrant charges about \$6 per DVD (i.e., \$12 for the plan). Thus, compared with the monopolist case, consumers receive a price reduction of about \$5 per DVD because of competition.

7. Conclusions

Quantity discount pricing is commonly used by firms. This pricing scheme charges consumers a per-unit price that declines with purchase quantity. The critical information for designing such a quantity discount scheme is knowledge of consumers' WTP for successive units of a product.

In this paper, we propose a choice-based conjoint model for estimating consumer-level WTP values for

varying quantities of a product. We use a novel WTP function that embeds standard WTP functions in the literature as special cases. The derived WTP function can handle large quantity values and allows WTP to be positive and to increase with quantity at a decreasing rate. A key benefit of this formulation is that it enables the segmentation of consumers in terms of WTP potential for the first unit (which measures price premium) and WTP multiple (a measure of volume potential) and the scoring of consumers based on their value potential to the firm. We also propose a parsimonious experimental design approach for implementation that does not entail as many price/discount and quantity factors as required by a standard conjoint design.

We estimate four variants of the proposed model using data we collected in a conjoint experiment involving consumer choice of online movie rental plans. We find that the linear decay model has the best fit and predictive validity. Although parsimonious, the constant decay model has poorer performance, which suggests that WTP decays rapidly with larger quantity.

We use the parameter estimates to quantify the distribution of WTP and WTP multiple in the sample. For Netflix, the average WTP for a one-DVD plan is about \$11 and the average WTP multiple is 2.25, which suggests an average of about \$300 value potential per customer per year. Additionally, we find that consumers are willing to pay an extra \$1.10 for a one-DVD plan from Netflix relative to MovieMail (a fictitious brand name) and an extra \$0.65 for the Bluray option. Blockbuster, however, has no differential brand value relative to MovieMail.

We compare the WTP distributions from our model with those from a standard choice-based conjoint model. The latter model gives anomalous WTP distributions with negative WTP values and nondiminishing marginal WTP values. These results illustrate the need to enforce the positivity and the diminishing marginal WTP in a conjoint model when estimating WTP.

We identify four consumer segments that vary in terms of WTP, WTP multiple, and value potential. An online movie rental company could use such information to target its customers based on their value potential to the firm. We also illustrate how to build a demand profile (Wilson 1993) using the consumer-level WTP estimates. Finally, we use the parameter estimates to design an optimal quantity discount scheme that maximizes gross contribution. We consider two scenarios: a monopolist and a new entrant in a competitive market. As expected, the optimal monopolist schedule resulted in higher per-DVD prices than the competitive schedule.

There are several avenues for future research. From a measurement perspective, our choice task did not provide any incentive for respondents to be truthful. Although CBC is found to do well in inferring WTP, future applications of the method should consider incentive alignment when collecting calibration and holdout choice data. In this paper, we assume a quasi-linear utility function. Whereas this assumption is standard in the WTP literature and is reasonable in the context of our application, it may not be reasonable for products whose demand is affected by income. Future research should generalize our model by fitting non-quasi-linear utility functions. Our proposed conjoint model assumes only an allocative effect of price. Other research has found evidence for both informative and allocative effects of price (e.g., Rao 1984). Future research should generalize the model to capture both effects (e.g., by conducting two separate conjoint studies as in Rao and Sattler 2003). Finally, an area of managerial interest would be to apply the model in other product categories and especially in business-to-business settings, where the quantity variable takes large values.

Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mktsci.journal .informs.org/.

Acknowledgments

The authors thank Rajan Sambandam and Christi Clark from TRC Market Research for their support in data collection. The authors also thank Andre Bonfrer, Eric Bradlow, Stephanie Finnel, Arun Gopalakrishnan, Rajeev Kohli, Abba Krieger, Leonard Lodish, Eric Schwartz, and Venkatesh Shankar for their insightful comments.

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